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AN IMPROVED VERSION OF CALCULATING THE PRESSURE TRAVERSE IN MULTIPHASE FLOW VERTICAL PIPE

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ABSTRACT

The quandary of predicting pressure losses in multiphase flow systems is not recent to the industry and it still does not have a complete solution that can cater for all types of flow conditions. This has brought about many particular solutions for limited flow conditions. This is as a result of the intricacy of multiphase flow and also the difficulty in analyzing even flow conditions that are limited. The new model estimated the Pressure losses and compressibility factor at each point of conduit as the fluid flows through the pipe rather than assumed average constant pressure and compressibility factor. The new method proposed was used to obtain the pressure traverse in three (3) different flow conditions and the results obtained are more accurate as compared with those obtained using the method proposed by other investigators: Poettmann and Carpenter; Fancher and Brown; Hagedorn and Brown.

Keywords: pressure losses; multiphase flow; flow pattern; density; viscosity.

1. Introduction

The petroleum and chemical industries are interested in accurately calculating the pressure losses that occur for multiphase flow in tubing and pipe lines. The importance of accurate calculations of pressure losses in pipes takes root due to the fact that practically all oil well production design involves multiphase flow. Studies on multiphase flow in vertical pipe have sought to develop a technique with which the pressure drop can be calculated. Pressure losses in flow of gas and liquid phase (two-phase) are quite different from those encountered in dry gas phase (single-phase) alone .This effect has been widely studied [5,7,8,9,10]. Accurate predictions of pressure losses in pipes insure good well design. As regards flow of reservoir fluids, an important physical property to be monitored closely is the **"*pressure*".** The accurate forecast of pressure at any location in a flow string is very critical; not only in optimizing production, but also to facilitate apt designs of flow strings and artificial lift installations. Finding a single correlation that can accurately predict pressure losses for multiphase flow for different cases and tubing sizes has not been possible. The reason is because describing the different relationships between liquids and gases is not easy. The different physical properties of the fluids such as viscosity, density, and interfacial tension change as a function of pressure and temperature. Besides, liquids and gases normally present different flow patterns when they flow together in a pipe. In some cases, gases move at velocities much higher than the liquid and as a result, the density of the multiphase mixture increases and gives rise to disparities in the density calculated from the gas-liquid ratio (GLR) of the produced stream. In other scenarios, the velocity of the liquid phase varies along the pipe wall over a short distance and results in inconsistent friction loss. Also, the liquid may be completely entrained in the gas bringing about a negligible effect on the frictional loss on the walls of the pipe.

One dominant factor affecting multiphase flow systems is the thermodynamic behavior of the flowing hydrocarbon mixture. This is chiefly because the magnitude of the physical properties of both the gas and liquid phases are dictated by pressure and temperature. Severe investigators such as Poettmann and Carpenter [8], and Tek [10], Orkiszewski [7] and Ros [9] have developed model on pressure drop or pressure gradient along the tubing, which might only be approximate solutions. They may not accurately provide information about pressure conditions at the bottom

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of the well due to the fluid column consisting of two or more fluid phase. Their models treated the liquid and gas as a homogenous single-phase flow without accounting for dissolved gas in oil. A method was presented that incorporated the pressure changes at different length of pipe, the effect of solution gas in the liquid phases and produced gas specific gravity. This paper presents a model for predicting the bottom hole flowing pressure in multiphase system, where oil/gas or oil/water/gas are flowing together. The model is a modification of Hagedorn and Brown method. The modification predicts the bottom hole flowing pressure in multiphase system as function of operational and fluid/pipe parameters. It devised a method of predicting the pressure at each point of conduit and treat compressibility factor as a function of pressure rather than a constant value.

2 Mathematical model

2.1 Properties of fluid mixtures

2.1.1 Density of a multiphase mixture

The average density of the flowing multiphase mixture $\overline{\rho_m}$ is given by:

$$\overline{\rho_m} = \overline{\rho_L}\, H_L + \overline{\rho_G}\,(1 - H_L) \tag{1}$$

The average density of the liquid (oil and water) is expressed as:

$$\overline{\rho_L} = \left[\frac{\gamma_O(62.4) + 0.0136 R_S \gamma_g}{B_o}\right]\left(\frac{1}{1+WOR}\right) + \left[\gamma_w(62.4)\left(\frac{WOR}{1+WOR}\right)\right] \tag{2}$$

The average density of the gas is expressed as:

$$\overline{\rho_G} = 2.703\gamma_g \left(\frac{\overline{P}}{\overline{\overline{TZ}}}\right) \tag{3}$$

Putting (2) and (3) into (1) above Thus, the density of the mixture is:

$$\overline{\rho_m} = \left\{\left[\frac{\gamma_O(62.4) + 0.0136 R_S \gamma_g}{B_o}\right]\left(\frac{1}{1+WOR}\right) + \left[\gamma_w(62.4)\left(\frac{WOR}{1+WOR}\right)\right]\right\} H_L + \left[2.703\gamma_g \left(\frac{\overline{P}}{\overline{\overline{TZ}}}\right)(1 - H_L)\right] \tag{4}$$

2.1.2 Velocity of a multiphase mixture

Recall, the velocity of the mixture is given by:

$$u_m = u_{SL} + u_{SG} \tag{5}$$

The velocity of the superficial liquid u_{SL} is given by:

$$u_{SL} = 0.000065 \frac{q_L}{A_p}\left\{ B_o\left(\frac{1}{1+WOR}\right) + B_w\left(\frac{WOR}{1+WOR}\right)\right\} \tag{6}$$

The velocity of the superficial gas , u_{SG} is given by:

$$u_{SG} = 0.000000327 \,\frac{q_L\left[GLR - R_S\left(\frac{1}{1+WOR}\right)\right]}{A_p}\left(\frac{\overline{T\,Z}}{\overline{P}}\right) \tag{7}$$

Substituting eqn (6) and (7) into (5) above

$$u_m = 0.000065 \frac{q_L}{A_p}\left\{ B_o\left(\frac{1}{1+WOR}\right) + B_w\left(\frac{WOR}{1+WOR}\right)\right\} + 0.000000327 \,\frac{q_L\left[GLR - R_S\left(\frac{1}{1+WOR}\right)\right]}{A_p}\left(\frac{\overline{T\,Z}}{\overline{P}}\right) \tag{8}$$

The basic flow equation in symbolic differential form based on one-pound mass of flowing fluid is given by:

$$144\,\frac{g_c}{g} V_m\, dp + dh + \frac{u_m du_m}{g} + dW_f + dW_e = 0 \tag{9}$$

Assuming no work is done by the flowing fluid, equation (9) becomes:

$$144\,\frac{g_c}{g} V_m\, dp + dh + \frac{u_m du_m}{g} + dW_f = 0 \tag{10}$$

Defining the frictional loss according to Darcy – Weisbach's equation

$$dW_f = \frac{f dh \overline{u_m^2}}{2dg} \tag{11}$$

Substituting (11) into (10) above, we obtain

$$144\ \frac{g_c}{g} V_m\, dp + \ dh + \frac{u_m du_m}{g} + \frac{f dh \overline{u_m^2}}{2dg} = 0 \qquad (12)$$

At this point, recall that the flow is multiphase (simultaneous flow of liquid, gas and solid). $144\ \frac{g_c}{g} V_m\, dp + \ dh + \frac{u_m du_m}{g} + \frac{f dh \overline{u_m^2}}{2dg} = 0$ (13)

Integrate equation (13) above and collect the like terms, equation (13) becomes:

$$144\ \frac{g_c}{g} \int_{P_1}^{P_2} V_m dp + \frac{u_{m2}^2 - u_{m1}^2}{2g} + (h_2 - h_1)\left[1 + \frac{f\overline{u_m^2}}{2gd}\right] = 0 \qquad (14)$$

Assuming the volume of the mixture remains constant between downstream pressure P_1 and upstream pressure P_2 at an average value $\overline{V_m}$

The integral $\int_{P_1}^{P_2} V_m dp = \overline{V_m}\,(P_2 - P_1)$ (15)

Put equation (15) into equation (14) above

$$144\ \frac{g_c}{g} \overline{V_m}\,(P_2 - P_1) + \frac{u_{m2}^2 - u_{m1}^2}{2g} + (h_2 - h_1)\left[1 + \frac{f\overline{u_m^2}}{2gd}\right] = 0 \qquad (16)$$

Solving equation (16) for $(h_2 - h_1)$

$$(h_2 - h_1) = \frac{144\,\frac{g_c}{g}\overline{V_m}\,(P_1 - P_2) - \frac{(u_{m2}^2 - u_{m1}^2)}{2g}}{\left[1 + \frac{f\overline{u_m^2}}{2gd}\right]} \qquad (17)$$

Also, recall for one pound mass of flowing fluid,

$$\overline{V_m} = \frac{1}{\rho_m} \qquad (18)$$

Substituting equation (18) into (17) above. Equation (13) can be re-written as:

$$\Delta h = \frac{144\,\frac{g_c}{g\overline{\rho_m}}\,\Delta p - \Delta(\frac{\overline{u_m^2}}{2g})}{\left[1 + \frac{f\overline{u_m^2}}{2gd}\right]} \qquad (19)$$

If g is assumed to be numerically equal to g_c and the conversion units commonly used in the field is made: Therefore,

$$\Delta h = \frac{144\Delta P - \overline{\rho_m}\,\Delta(\frac{\overline{u_m^2}}{2g_c})}{\overline{\rho_m} + \frac{f w^2}{2.9652 \times 10^{11} d^5 \overline{\rho_m}}} \qquad (20)$$

Thus,

$$\Delta h = \Delta h_1 + \Delta h_2 + \ \dots\dots\dots + \Delta h_n \qquad (21)$$

Hagedorn and Brown calculated average pressure as:

$$\overline{P} = \frac{P_w + P_b}{2} \qquad (22)$$

The proposed method involves discretizing the pressure points along the length of the conduit; obtain the average pressure between each step change in pressure and subsequently the average compressibility factor at these points.

$$\overline{P} = \frac{P_i + P_{i+1}}{2} \qquad (23)$$

The assumptions made in this model include:

1. Flow of fluid occurs at steady state conditions.
2. Temperature of the fluid remains constant at some average value.
3. Frictional factor is constant along the length of the conduit.
4. Solution gas is assumed constant throughout the pipe length.

3. Analysis of results

This study demonstrates a new approach to estimate pressure traverses in multiphase flow vertical pipes and tubing. This approach is proficient in providing satisfactory results once points along the conduit are discretized and the compressibility factor is not treated as a constant but rather as a pressure dependent variable.

Figure 1 depicts the average pressure against the compressibility factor. Hagedorn and Brown [5] used average pressure as well as constant compressibility factor over the entire pipe length to calculate pressure traverse. The new model is an improved version of Hagedorn and Brown in that compressibility factor is treated as pressure dependent variable not as a constant. The new model also shows the variation of compressibility factor with varying pressure.

Figure 2 and 3 depict the deviation of the pressure traverse obtained using the proposed model and that obtained using other models; Hagedorn and Brown [5], Poettmann and Carpenter [8] and Fancher and Brown[4]. The distance between 500psig and 1000psig for 2in tubing and a flowing stream with GLR of 500 Scf/Stk bbl was found to be 1783ft by the proposed model. This compares to 1870ft obtained by Hagedorn and Brown method, 2425ft obtained by Poettmann and Carpenter Method and 2350ft obtained by the Fancher and Brown method. For a 1.25in tubing with the same flowing conditions as the stream above, the distance was found to be 1122ft as compared with 1250ft obtained by the Hagedorn and Brown method, 1650ft obtained by the Poettmann and Carpenter method and 1465ft obtained by Fancher and Brown method. These show that Hagedorn and Brown, Poettmann and Carpenter and Fancher and Brown overpredict the pressure traverse in the pipe as a result of average constant in their models.

Figure 4 establishes the digression of the pressure traverse from that obtained by the Hagedorn and Brown method for a flowing stream of oil and gas only with GOR of 1000Scf/Stbo. The distance as a result of a 500psig and 1000psig pressure points, flow rate of 600 Stbo/Day with a GOR of 1000Scf/Stk bbl is 3034ft. The distance obtained for this flow condition using the Hagedorn and Brown method is 3579ft.

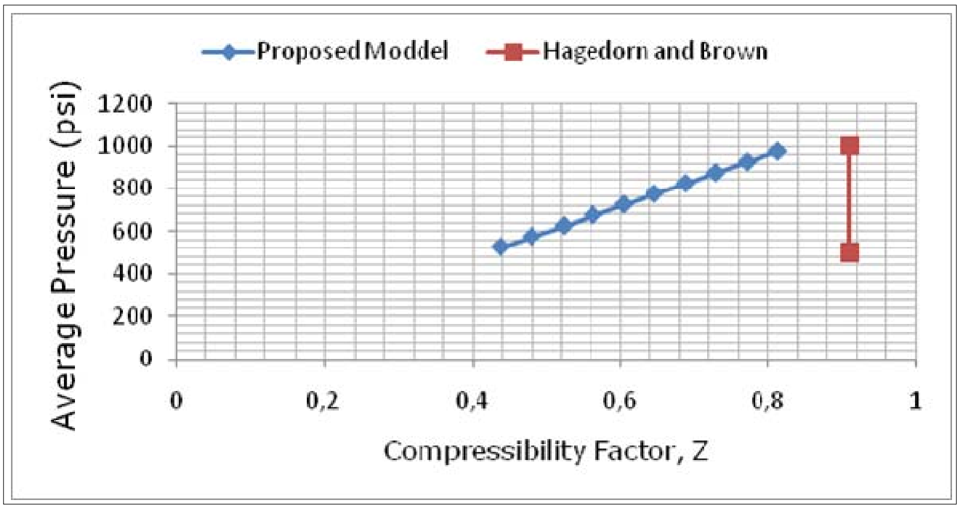

Figure 1 Average pressure against compressibility factor

Case 1

d=1.995 in.; P_1 = 500psig; P_2 = 1000psig; T_1 = 120°F; T_2 = 150°F; γ_g = 0.65; γ_w = 1.07; ρ_o=22°API; q_o=400stk bpd; q_w=600stk bpd; μ_g =0.018cp; σ_o =30dynes/cm; σ_w=70dynes/cm;GLR=500Scf/Stkbb

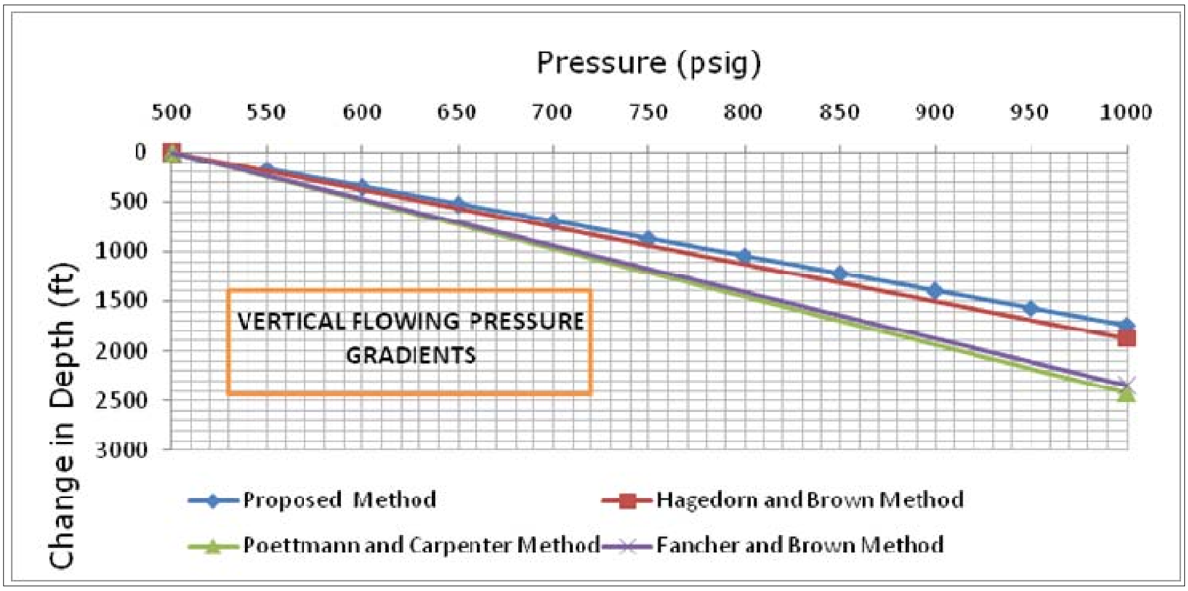

Figure 2 Pressure Traverse between 500 and 1000psig for a 2inch tubing

Case 2

d=1.25 in.; P_1 = 500psig; P_2 = 1000psig; T_1 = 120°F; T_2 = 150°F; γ_g = 0.65; γ_w = 1.07; ρ_o = 22°API; q_o = 400stk bpd; q_w = 600stk bpd; μ_g = 0.018cp; σ_o = 30dynes/cm; σ_w = 70dynes/cm; GLR = 500 Scf/Stk bbl.

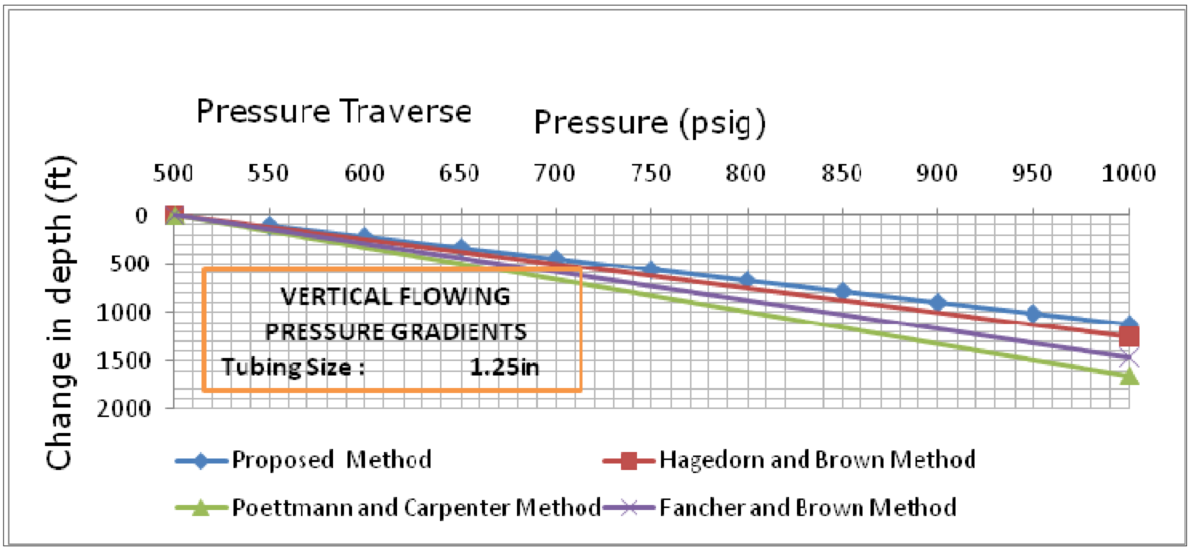

Figure 3 Pressure Traverse between 500 and 1000 psig for a 1.25inch tubing

Case 3

d=1.995 in.; P_1 = 500psig; P_2 = 1000psig; T_1 = 120°F; T_2 = 180°F; γ_g = 0.65; γ_w = 1.07; ρ_o = 42°API; q_o = 600stk bpd; q_w = 0stk bpd; μ_g = 0.02cp; σ_o = 30dynes/cm; σ_w = 70dynes/cm; GLR = 1000 Scf/Stk bbl.

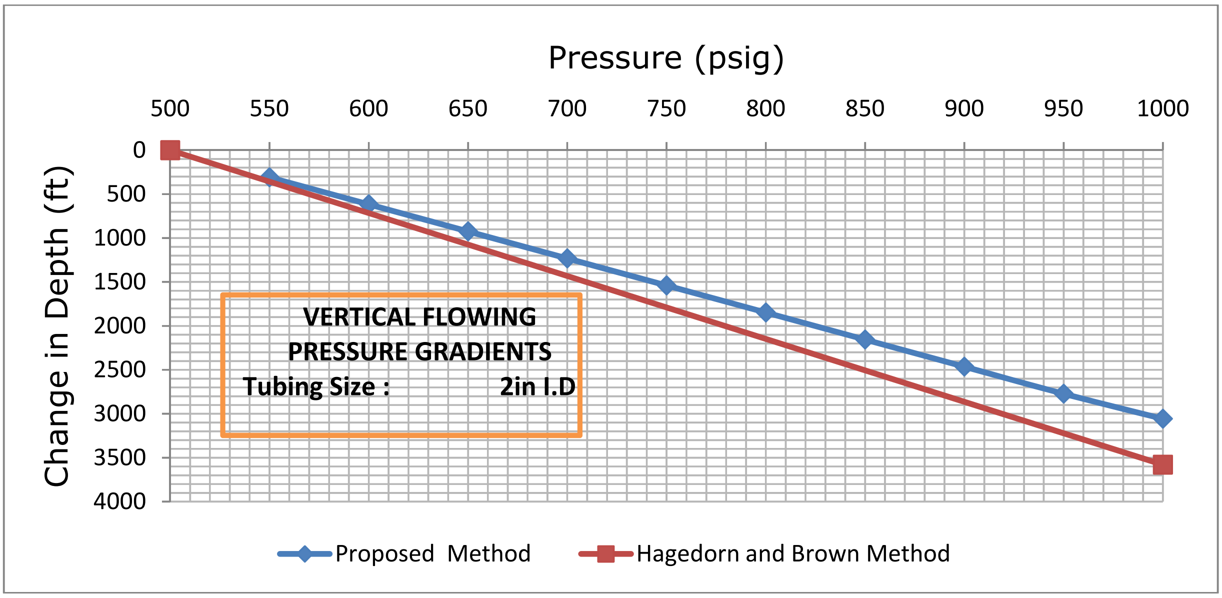

Figure 4 Pressure Traverse between 500 and 1000 psig for a 2inch tubing with GLR 1000 Scf/Stb

4. CONCLUSION

The following conclusions were drawn from the results of this study:

1. The method proposed helped to reduce the errors normally encountered in making approximations across the whole length of the conduit. Also, the incorporation of the varying the compressibility factor of the gas along with the discretized points using the Hall and Yarborough method aided in the reduction of errors in the results obtained to the barest minimum. This is because the values of the compressibility factor obtained using the above mentioned method is very close to that obtained from experiments.
2. The analysis showed that the pressure and compressibility factor vary at every point of pipe.

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NOMENCLATURE

Symbols	Description	Unit
A_p	Cross-Sectional Area of pipe	ft^2
Bo	Oil Formulation Volume Factor	rb/STB
d	Pipe Diameter	ft
g	Acceleration Due to Gravity	ft/sec^2
g_c	Conversion Constant (32.17)	Ib_m ft/Ib_f
H_L	Liquid Hold-up	
P	Pressure	psi
$\overline{P}$	Average Pressure	psi
q_L	Liquid Flow rate	Stk bbl
R_s	Solution Gas Ratio	scf/bbl
$\overline{T}$	Average Temperature	^oR
u	Velocity	ft/sec
u_m	Velocity of Mixture	ft/sec
$\overline{u_m}$	Average Velocity of the Mixture	ft/sec
u_{SG}	Superficial Gas Velocity	ft/sec
$\overline{u_{SG}}$	Average Superficial Gas Velocity	ft/sec
u_{SL}	Superficial Liquid Velocity	ft/sec

$\overline{u_{SL}}$	Average Superficial Liquid Velocity	ft/sec
V	Volume of Fluid	ft3
$\overline{V_m}$	Average Volume of Mixture	ft3
w	Mass Flow rate of Mixture	Ibm/stk bbl
W_e	Work done by the Fluid Mixture	
W_f	Frictional Work	
$\overline{Z}$	Average Compressibility factor	
P_w	Wellhead pressure	psig
P_b	Bottom hole pressure	psig

Abbreviations

GLR	Gas-Liquid Ratio
WOR	Water-Oil Ratio

Subscripts

g	Gas
L	Liquid
m	Mixture
o	Oil
w	Water

Greek Letters

$\overline{\rho_g}$	Average Density of Gas Phase	Ib/cu.ft
$\overline{\rho_L}$	Average Density of the Liquid Phase	Ib/cu.ft
$\overline{\rho_m}$	Average Density of the Mixture	Ib/cu.ft
γ_g	Gas specific gravity	
γ_o	Oil Specific gravity	
γ_w	Water Specific gravity	

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APPENDIX A

Derivation of equation (20)

Velocity of a multiphase mixture

Recall, the velocity of the mixture is given by:

$$u_m = u_{SL} + u_{SG} \quad \text{(A.1)}$$

The velocity of the superficial liquid, u_{SL} is given by:

$$u_{SL} = \frac{5.615 q_L}{86400 A_p}\left\{ B_o\left(\frac{1}{1+WOR}\right) + B_w\left(\frac{WOR}{1+WOR}\right)\right\} \quad \text{(A.2)}$$

Simplifying,

$$u_{SL} = 0.000065\frac{q_L}{A_p}\left\{ B_o\left(\frac{1}{1+WOR}\right) + B_w\left(\frac{WOR}{1+WOR}\right)\right\} \quad \text{(A.3)}$$

The velocity of the superficial gas u_{SG} is given by:

$$u_{SG} = \frac{q_L\left[GLR - R_s\left(\frac{1}{1+WOR}\right)\right]}{86400 A_p}\left(\frac{14.7}{\bar{P}}\right)\left(\frac{\bar{T}}{520}\right)\left(\frac{\bar{Z}}{1}\right) \quad \text{(A.4)}$$

Simplifying,

$$u_{SG} = 0.000000327\,\frac{q_L\left[GLR - R_s\left(\frac{1}{1+WOR}\right)\right]}{A_p}\left(\frac{\overline{T\,Z}}{\bar{P}}\right) \quad \text{(A.5)}$$

The average velocity $\overline{u_m}$ is as follows:

$$\overline{u_m} = \overline{u_{SL}} + \overline{u_{SG}} \quad \text{(A.6)}$$

$$W = m q_L \quad \text{(A.7)}$$

$$m = 350\gamma_{O}\left(\frac{1}{1+WOR}\right) + 350\gamma_{W}\left(\frac{WOR}{1+WOR}\right) + 0.0764(GLR)(\gamma_G) \quad \text{(A.8)}$$

$$\sigma_L = \sigma_o\left(\frac{1}{1+WOR}\right) + \sigma_W\left(\frac{WOR}{1+WOR}\right) \quad \text{(A.9)}$$

$$\mu_L = \mu_O\left(\frac{1}{1+WOR}\right) + \mu_W\left(\frac{WOR}{1+WOR}\right) \quad \text{(A.10)}$$

Substituting equations (4) and (8) into (20)

$$\Delta h = \frac{144\Delta P - \left(\left\{\left[\frac{\gamma_O(62.4) + 0.0136 R_S \gamma_g}{B_O}\right]\left(\frac{1}{1+WOR}\right) + \left[\gamma_W(62.4)\left(\frac{WOR}{1+WOR}\right)\right]\right\}H_L + \left[2.703\gamma_g\left(\frac{\bar{P}}{\overline{\overline{TZ}}}\right)(1-H_L)\right]\right)\Delta\left(\frac{\left(\frac{0.000065\frac{q_L}{A_p}\left\{B_O\left(\frac{1}{1+WOR}\right) + B_W\left(\frac{WOR}{1+WOR}\right)\right\} + 0.000000327\frac{q_L\left[GLR - R_S\left(\frac{1}{1+WOR}\right)\right]}{A_p}}{\left(\frac{\overline{T\,Z}}{\bar{P}}\right)}\right)^2}{2g_c}\right)}{\left\{\left[\frac{\gamma_O(62.4) + 0.0136 R_S \gamma_g}{B_O}\right]\left(\frac{1}{1+WOR}\right) + \left[\gamma_W(62.4)\left(\frac{WOR}{1+WOR}\right)\right]\right\}H_L + \left[2.703\gamma_g\left(\frac{\bar{P}}{\overline{\overline{TZ}}}\right)(1-H_L)\right] + \frac{fw^2}{2.9652\times 10^{11} d^5\left(\left\{\left[\frac{\gamma_O(62.4) + 0.0136 R_S\gamma_g}{B_O}\right]\left(\frac{1}{1+WOR}\right) + \left[\gamma_W(62.4)\left(\frac{WOR}{1+WOR}\right)\right]\right\}H_L + \left[2.703\gamma_g\left(\frac{\bar{P}}{\overline{\overline{TZ}}}\right)(1-H_L)\right]\right)}} \quad \text{(A.11)}$$

$$\Delta h = \frac{144\Delta P - \left(\left\{\left[\frac{A_5 + A_6}{B_O}\right](A_2) + A_4 A_3\right\}H_L + \left[C_3\left(\frac{\bar{P}}{\overline{\overline{TZ}}}\right)(1-H_L)\right]\right)\Delta\left(\frac{\left(\frac{C_1\{B_O A_2 + B_W A_3\} + C_2\{GLR - C_4\}}{\left(\frac{\overline{T\,Z}}{\bar{P}}\right)}\right)^2}{2g_c}\right)}{\left\{\left[\frac{A_5 + A_6}{B_O}\right](A_2) + [A_4 A_3]\right\}H_L + \left[C_3\left(\frac{\bar{P}}{\overline{\overline{TZ}}}\right)(1-H_L)\right] + \left(\frac{A_1}{\left\{\left[\frac{A_5 + A_6}{B_O}\right](A_2) + A_4 A_3\right\}H_L + \left[C_3\left(\frac{\bar{P}}{\overline{\overline{TZ}}}\right)(1-H_L)\right]}\right)} \quad \text{(A.12)}$$

Let:

$$A_1 = \frac{fw^2}{2.9652\times 10^{11} d^5} \quad \text{(A.13)}$$

$$A_2 = \frac{1}{1+WOR} \quad (A.14)$$

$$A_3 = \frac{WOR}{1+WOR} \quad (A.15)$$

$$A_4 = \gamma_w(62.4) \quad (A.16)$$

$$A_5 = \gamma_o(62.4) \quad (A.17)$$

$$A_6 = R_s\gamma_g(0.0136) \quad (A.18)$$

$$C_1 = \frac{0.000065q_L}{A_p} \quad (A.19)$$

$$C_2 = \frac{0.000000327q_L}{A_p} \quad (A.20)$$

$$C_3 = 2.703\gamma_g \quad (A.21)$$

$$C_4 = R_sA_2 \quad (A.22)$$