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APPLICATION OF CUBIC SPLINE NUMERICAL MODELING ON DISPLACEMENT MECHANISM

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Abstract

Estimation of incremental oil recovered in a successful enhanced oil recovery (EOR) project has always been done over the years using already established formulae, undermining substantially some inherent challenges. To address some of these posed shortcomings, however, necessitates the need to model the EOR curve.

This research paper presents the formulation and application of a highly sophisticated numerical model to model the incremental oil recovery curve in order to obtain improved values of the incremental oil recovered . Rate-time curves from laboratory data for surfactant and polymer flooding were used. The methodology used was cubic spline numerical modeling. "OUR" algorithm was used as the solution method to tridiagonal system of equations formed. Different and continuous equations were derived for each interval between successive data points (knots) and then joined together piecewise to form the composite equation to represent the EOR process. The incremental oil recovered was then obtained by applying the cubic spline to quadrature (numerical integration).

The results showed that the incremental oil obtained by the cubic spline model was 2.7% and 5.6% more than that obtained by the trapezoidal rule in the surfactant and polymer flooding respectively. The trapezoidal rule would always give less amount of the incremental oil because the exactitude of its results is dependent on the linearity of the function being approximated. This suggests that the cubic spline model gives better results.

Key Words: *Cubic interpolant formulation; Cubic spline; Equation of curvature; First derivative; Incremental oil recovered; Linear or Natural Spline Boundary Condition; piecewise curve; Second derivative and Tridiagonal system of equations.*

1. Introduction

A universal technical measure of the success of an EOR project is the amount of incremental oil recovered [1-2]. Over the years, ways by which this is inferred ranged from graphical procedures to highly sophisticated numerical models. One of the former, the decline curves method and one of the later, often the trapezoidal rule. Both pose challenges of inaccuracy in shifting [3-4] and approximation of the integral under a curve with that under a straight line segment [5] respectively. These substantially undermine the exactitude of the amount of the incremental oil obtained by both methods, and to which less attention has been paid over the years . However, the application of cubic spline numerical modeling concept significantly addresses some of these posed issues.

Spline concept by different authors of several literatures were studied. None actually worked on the application of the concept on recovery mechanism but some did on other engineering disciplines. However, a brief review of the concept is encapsulated below.

A spline is simply a curve. In mathematics, it's a special function defined piecewise by polynomials [6-9]. In computer science, the term spline refers to a piecewise polynomial curve [6].

The solution was to place metal weights (called knots) at the control points, and bend a tin metal or wooden beam (a spline) trough the weights. A piecewise polynomial function

$f(x)$ is obtained by dividing the independent variable x into contiguous intervals and representing the function in each interval by a separate or different and continuous polynomial. These polynomials are then joined together at the interval endpoints (knots) in such a way that a certain degree of smoothness of the resulting or composite function is guaranteed [6].

Cubic spline is a spline constructed of piecewise third-order polynomials which pass through a set of n control data points (x_1, y_1), (x_2, y_2),....... (x_n, y_n)[6]. That is, the cubic spline function approximations are composite approximations. In the mathematical approach to the determination of the spline function approximation through the n data points. x_i are the nodes of the approximations and the corresponding points y_i , where the contiguous curves meet are called the knots of the approximation [7]. Its application to quadrature involves the use of integral function approximations [12].

In the application of cubic spline numerical modelling on recovery mechanism, our objective is to get better results of the incremental oil recovered in the EOR project by modelling the curve of the incremental oil recovered. However, the details of the procedure for obtaining the experimental data as well as the plotted graph used were not included in the research. Additionally, although there are several solution methods to tridiagonal system of equations but "OUR" algorithm was used in the modelling.

2. Models Formulation

To apply cubic spline numerical modelling on recovery mechanism procedurally entails derivations based on first-order langrage interpolating polynomials, calculus of finite differences, use of solution method to tridiagonal system of equations and application to quadrature [9-10].

Mathematical requirements to be satisfied by a cubic spline function [7]:

1. Each curve through the contiguous points is a cubic.
2. The composite curve over the entire interval x_1 and x_n must interpolate the data by passing through each knot.
3. The curve itself and the first and second derivatives of the composite curve must be continuous at the nodes x_i.
4. Conditions must be prescribed at the end points x_1 and x_n of the interval, depending on whether the data points indicate that beyond these points the extrapolation curve is required to approach a straight line or a parabola, or exhibit some other behavior such as periodicity over the interval $x_1 \le x \le x_n$.

2.1 Cubic interpolant formulation

A cubic spline is one that spans n knots. Denoting $f_{i,i+1}(x)$ as the cubic polynomial that spans the interval between knots i and $i+1$, we note that the spline is a piecewise cubic curve, assembled together to form the $n-1$ cubics $f_{1,2}(x)$, $f_{2,3}(x), \ldots\ldots, f_{n-1,n}(x)$, all of which have different coefficients.

Using Lagrange's two-point interpolation, the second derivative which is a linear function can be expressed as:

$$f''(x) = \left(\frac{x_{i+1}-x}{x_{i+1}-x_i}\right) f''(x_i) + \left(\frac{x-x_i}{x_{i+1}-x_i}\right) f''(x_{i+1}) \qquad for\ x_i \le x \le x_{i+1} \qquad (2.1)$$

And denoting the second derivative of the knot at i by k_i and with the condition of its continuity, it's required that:

$$f''_{i-1,1}(x_i) = f''_{i+1,1}(x_i) = k_i \qquad (2.2)$$

Integrating twince with respect to x and with the condition that $f(x)$ is required to pass through (x_i, y_i) and (x_{i+1}, y_{i+1}) gives:

$$f_{i,i+1}(x) = \frac{k_i}{6}\left[\frac{(x-x_{i+1})^3}{x_i-x_{i+1}} - (x-x_{i+1})(x_i-x_{i+i})\right] - \frac{k_{i+1}}{6}\left[\frac{(x-x_i)^3}{x_i-x_{i+1}} - (x-x_i)(x_i-x_{i+i})\right] + \frac{y_i(x-x_{i+1}) - y_{i+1}(x-x_i)}{x_i-x_{i+1}} \qquad (2.3)$$

Consequet upon this, there must be some formulations to determine the second derivatives (k_i) at the interior knots. This leads to the yet unused condition of continuity of the first derivative with which the equations of curvatures are formulated.

2.2 Equations of curvatures

Here, we note that the second derivatives (k_i) of the spline at the interior knots are found from the slop continuity conditions:

$$f'_{i,i-1}(x_i) = f'_{i,i+1}(x_i) \qquad i = 1, 2, 3, \dots\dots, n-1 \tag{2.4}$$

Applying the conditions given by Eq.(2.4) in Eq.(2.3) gives the following simultaneous equations:

$$k_{i-1}(x_{i-1} - x_i) + 2k_i(x_{i-1} - x_{i+1}) + k_{i+1}(x_i - x_{i+1}) = 6\left[\frac{y_{i-1} - y_i}{x_{i-1} - x_i} - \frac{y_i - y_{i+1}}{x_i - x_{i+1}}\right] \quad i = 2, 3, \dots n-1 \tag{2.5}$$

But if the data are evenly spaced at interval (h), then we can write:

$$h = x_{i-1} - x_i = x_i - x_{i+1} \tag{2.6}$$

Then, Eq. (2.5) becomes:

$$k_{i-1} + 4k_i + k_{i+1} = \frac{6}{h^2}[y_{i-1} - 2y_i + y_{i+1}] \qquad i = 2, 3, \dots\dots, n-1 \tag{2.7}$$

Eq. (2.7) is a set of $n-2$ linear simultaneous equations for the n derivatives (k_i), and when these are known, the spline function approximation formed by the set of functions in Eq.(2.5) defined over the consecutive intervals $x_i \le x \le x_{i+1}$ can be constructed. It is crucial to the practical use of splines that this system of equations being nonsigunar,and that an extremely efficient algorithm be available to solve it.

As the of values of k_1 and k_n cannot be found from the condition that $f'(x)$ is continuous across the nodes x_1 and x_n, these values must be specified as additional conditions.

The choice of values for k_1 and k_n prescribed as end conditions must be made intuitively, based on the way the data points indicate the interpulated curve is most likely to behave (be extrapolated) beyond the end points of the interval $x_1 \le x \le x_n$. Three typical choices are the *natral or linear spline* end condition, the *parabolic spline* end condition and *periodic spline* end conditions [6]. We used linear boundary condition in this research.

2.3 End or Boundary conditions

1. *Natural or linear spline end conditions .*This choice of end condition involves setting.

$$k_1 = k_n = 0. \tag{2.8.}$$

2. *Parabolic Spline end conditions.* This choice of end condition involves setting.

$$k_1 = k_2 \text{ and } k_{n-1} = k_n. \tag{2.9}$$

3. *Periodic Spline end conditions*. This choice of end condition involves setting

$$f(x_1) = f(x_{n-1}) \text{ and } f'(x_n) = f'(x_2). \tag{2.10}$$

2.4 "OUR" Algorithm

"OUR" Algorithm is an extremely efficient solution method to tridiagonal system of equations whose formulations are as follows:

$$-A_i K_{i-1} + B_i K_i - C_i K_{i+1} = D_i \qquad i = 2, 3, \dots., n-1 \tag{2.11}$$

Eq. (2.11) is the general form of the set of linear simultaneous equations formed by Eq (2.7), where A_i, B_i and C_i are the coefficients of the second derivatives at the knots and D_i equals the right hand side of the equation.

Setting $\propto(2) = B(2)$ and $S(2) = D(2)$ (2.12)

From $I = 3 \ to\ M$, Where M is the last interior position vector. We can write:

$$\propto(I) = B(I) - \frac{A(I)}{\propto(I-1)} \times C(I-1) \tag{2.13}$$

and

$$S(I) = D(I) + \frac{A(I)}{\propto(I-1)} \times S(I-1) \quad (2.14)$$

For the second derivatives k_i, at the last interior knot, we have:

$$K(M) = \frac{S(M)}{\propto(M)} \quad (2.15)$$

But for $J = M - 1 \ to \ 1$, we have:

$$K(J) = \frac{S(J) + C(J) \times K(J+1)}{\propto(J)} \quad (2.16)$$

where $A(I)$, $B(I)$, $C(J)$, $D(I)$ and $S(I)$ are "OUR" algorithm parameters, I and J are position vectors, M is the last position vector and K is the second derivative.

3. Applications

To apply the formulated models, there must be recovery profile either from laboratory or field data. And these are given in Table 3.1 and 3.2.

Table 3.1 Surfactant flooding data

Time *Sec x 60*	Oil recovered *$m^3 x\ 1.10^{-6}$*	Water/surfactant collected, *$m^3 x\ 1.10^{-6}$*	Total *$m^3 x\ 1.10^{-6}$*	PV, Surfactant injected	Cumulative oil recovered % OOOP
4:00	0.7	7.9	8.6	1.690955	63.46667
8:00	1	8.1	9.1	1.919598	66.13333
12:00	0.6	8.2	8.8	2.140704	67.73333
16:00	0.4	8.2	8.6	2.356784	68.8
TOTAL	2.7				

Table 3.2 Polymer flooding data

Time *Sec x 60*	Oil recovered *$m^3 x\ 1.10^{-6}$*	Water/polymer collected, *$m^3 x\ 1.10^{-6}$*	Total *$m^3 x\ 1.10^{-6}$*	PV, polymer injected	Cumulative oil recovered % OOOP
4:00	0.2	8.4	8.6	2.572864	69.33333
8:00	0.3	8.7	9.0	2.798995	70.13333
12:00	0.3	8.7	9.0	3.025126	70.93333
16:00	2.8	6.0	8.8	3.246231	78.4
20:00	1.8	6.5	8.3	3.454774	83.2
24:00	0.4	8.6	9.0	3.680905	84.26667
28:00	0.2	8.4	8.6	3.896985	84.8
TOTAL			6.0		

The tabulated data in Table 3.1 and 3.2 were obtained from laboratory experiments for surfactant and polymer flooding. The graphical depiction of the recovery profile on which our model was applied is given in Fiq 3.1.

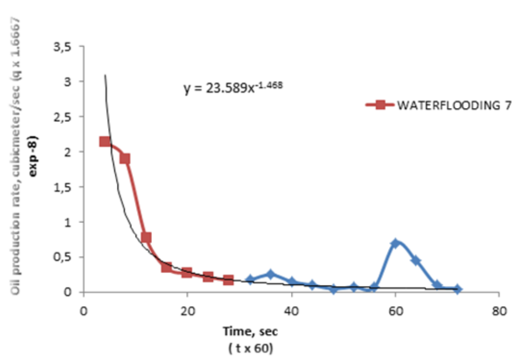

Fig 3.1 Incremental oil recovery curve from laboratory data.

Appliying the cubic spline numerical model, we firstly identified the EOR curve, the incremental oil recovery curve ,the boundries, chosed the boundary conditions, determined the second derivatives at the interior knots and modeled the curve. With the models, we determined the cumulative oil recovered per interval and joined them piecewise to obtain the cumulative oil recovered during the process by applying them to quadrature (numerical integration). That is, Each cubic is integrated per its interval of validity to obtain the amount of cumulative oil recovered per interval. And upon this, that of the composite function was obtained. We then obtained the incremental oil recovered by subtracting the integral under the curve of the projected decline from the integral under the corresponding incremental oil recovery curve. A brief analytical approach of the procedure is shown below.
The models are:

$$f_{i,i+1}(t) = \frac{k_i}{6}\left[\frac{(t-t_{i+1})^3}{t_i - t_{i+1}} - (t-t_{i+1})(t_i - t_{i+1})\right] - \frac{k_{i+1}}{6}\left[\frac{(t-t_i)^3}{t_i - t_{i+1}} - (t-t_i)(t_i - t_{i+1})\right] + \frac{q_i(t-t_{i+1}) - q_{i+1}(t-t_i)}{t_i - t_{i+1}} \qquad (3.1)$$

Eq. (3.1) $(Interpolant)$ is the cubic that spans each interval between two knots. Where q and t are recovery rate and time repectively.

It should be noted that the values of k_i are determined using "OUR" algorithm. However, any other solution method to tridiagonal system of equations is applicable.

The cumulative oil recovered per interval is given by:

$$\int_{t_i}^{t_{i+1}} f_{i,i+1}(t)\,dt = \int_{t_i}^{t_{i+1}} \left(\frac{k_i}{6}\left[\frac{(t-t_{i+1})^3}{t_i - t_{i+1}} - (t-t_{i+1})(t_i - t_{i+1})\right] - \frac{k_{i+1}}{6}\left[\frac{(t-t_i)^3}{t_i - t_{i+1}} - (t-t_i)(t_i - t_{i+1})\right] + \frac{q_i(t-t_{i+1}) - q_{i+1}(t-t_i)}{t_i - t_{i+1}}\right) dt \qquad (3.2)$$

$$The\ incremental\ oil = N_p(a) - N_p(b) - \int_a^b f\,(x)dx \qquad (3.3)$$

where $N_p(a)$ is the cumulative oil produced $(in\ m^3)$ at the lower limit, $N_p(b)$ is the cumulative oil produced $(in\ m^3)$ at the upper limit, a is the lower limit $(in\ secs)$, b is the upper limit $(in\ secs)$ and $f(x)$ is the equation of the projected decline.

3.1 Surfactant flooding process

For the surfactant flooding process as shown in Fig 3.1, we have.

Setting $k_1 = k_4 = 0$ for *linear or natural boundary contion*, the cubic spline or composite function is given by:

$$S(t) = -\frac{k_2}{6}\left[\frac{(t-t_1)^3}{t_1 - t_2} - (t-t_1)(t_1 - t_2)\right] + \frac{q_1(t-t_2) - q_2(t-t_1)}{t_1 - t_2} + \frac{k_2}{6}\left[\frac{(t-t_3)^3}{t_1 - t_3} - (t-t_3)(t_2 - t_3)\right] - \frac{k_3}{6}\left[\frac{(t-t_2)^3}{t_2 - t_3} - (t-t_2)(t_2 - t_3)\right] + \frac{q_2(t-t_3) - q_3(t-t_2)}{t_2 - t_3} + \frac{k_3}{6}\left[\frac{(t-t_4)^3}{t_3 - t_4} - (t-t_4)(t_3 - t_4)\right] + \frac{q_3(t-t_4) - q_4(t-t_3)}{t_3 - t_4} \qquad (3.4)$$

The cumulative oil recoverd during the process is given by:

$$N_p(2640) - N_p(1920) = \int_{1920}^{2160}\left(-\frac{k_2}{6}\left[\frac{(t-t_1)^3}{t_1-t_2} - (t-t_1)(t_1-t_2)\right] + \frac{q_1(t-t_2)-q_2(t-t_1)}{t_1-t_2}\right)dt + \int_{2160}^{2400}\left(\frac{k_2}{6}\left[\frac{(t-t_3)^3}{t_1-t_3} - (t-t_3)(t_2-t_3)\right] - \frac{k_3}{6}\left[\frac{(t-t_2)^3}{t_2-t_3} - (t-t_2)(t_2-t_3)\right] + \frac{q_2(t-t_3)-q_3(t-t_2)}{t_2-t_3}\right)dt + \int_{2400}^{2640}\left(\frac{k_3}{6}\left[\frac{(t-t_4)^3}{t_3-t_4} - (t-t_4)(t_3-t_4)\right] + \frac{q_3(t-t_4)-q_4(t-t_3)}{t_3-t_4}\right)dt \quad (3.5)$$

Finally,

$$The\ incremental\ oil = N_p(2640) - N_p(1920) - \int_{1920}^{2640} 23.589x^{-1.468}\,dx = 7.4\times10^{-6}m^3 \quad (3.6)$$

3.2 Polymer flooding process

For the polymer flooding process as shown in Fig 3.1, we have:

Setting $k_1 = k_5 = 0$ for *linear or natural boundary contion*, the cubic spline or composite function is given by:

$$S(t) = -\frac{k_2}{6}\left[\frac{(t-t_1)^3}{t_1-t_2} - (t-t_1)(t_1-t_2)\right] + \frac{q_1(t-t_2)-q_2(t-t_1)}{t_1-t_2} + \frac{k_2}{6}\left[\frac{(t-t_3)^3}{t_1-t_3} - (t-t_3)(t_2-t_3)\right] - \frac{k_3}{6}\left[\frac{(t-t_2)^3}{t_2-t_3} - (t-t_2)(t_2-t_3)\right] + \frac{q_2(t-t_3)-q_3(t-t_2)}{t_2-t_3} + \frac{k_3}{6}\left[\frac{(t-t_4)^3}{t_3-t_4} - (t-t_4)(t_3-t_4)\right] - \frac{k_4}{6}\left[\frac{(t-t_3)^3}{t_3-t_4} - (t-t_3)(t_3-t_4)\right] + \frac{q_3(t-t_4)-q_4(t-t_3)}{t_3-t_4} + + \frac{k_4}{6}\left[\frac{(t-t_5)^3}{t_4-t_5} - (t-t_5)(t_4-t_5)\right] + \frac{q_4(t-t_5)-q_5(t-t_4)}{t_4-t_5} \quad (3.7)$$

The cumulative oil recoverd during the process is given by:

$$N_p(4320) - N_p(3360) = \int_{3360}^{3600}\left(-\frac{k_2}{6}\left[\frac{(t-t_1)^3}{t_1-t_2} - (t-t_1)(t_1-t_2)\right] + \frac{q_1(t-t_2)-q_2(t-t_1)}{t_1-t_2}\right)dt + \int_{3600}^{3840}\left(\frac{k_2}{6}\left[\frac{(t-t_3)^3}{t_2-t_3} - (t-t_3)(t_2-t_3)\right] - \frac{k_3}{6}\left[\frac{(t-t_2)^3}{t_2-t_3} - (t-t_2)(t_2-t_3)\right] + \frac{q_2(t-t_3)-q_3(t-t_2)}{t_2-t_3}\right)dt + \int_{3840}^{4080}\left(\frac{k_3}{6}\left[\frac{(t-t_4)^3}{t_3-t_4} - (t-t_4)(t_3-t_4)\right] - \frac{k_4}{6}\left[\frac{(t-t_3)^3}{t_3-t_4} - (t-t_3)(t_3-t_4)\right] + \frac{q_2(t-t_3)-q_3(t-t_2)}{t_3-t_4}\right)dt \int_{4080}^{4320}\left(\frac{k_4}{6}\left[\frac{(t-t_5)^3}{t_4-t_5} - (t-t_5)(t_4-t_5)\right] + \frac{q_4(t-t_5)-q_5(t-t_4)}{t_4-t_5}\right)dt \quad (3.8)$$

Finally,

$$The\ incremental\ oil = N_p(4320) - N_p(3360) - \int_{3360}^{4320} 23.589x^{-1.468}\,dx = 21.3439\times10^{-6}m^3 \quad (3.9)$$

4. Results and discussion

Figure 4.1 shows that the incremental oil obtained by the cubic spline model is 2.7% and 5.6% more than that obtained by the trapezoidal rule in the surfactant and polymer flooding respectively. This suggests that the incremental oil recovered by the trapezoidal rule is always less than the actual amount because the curve is approximated with straight line segments and therefore does not reflect the actual success of the the EOR project. Since this is a bit misleading, the use of cubic spline numerical model is highly encouraged.

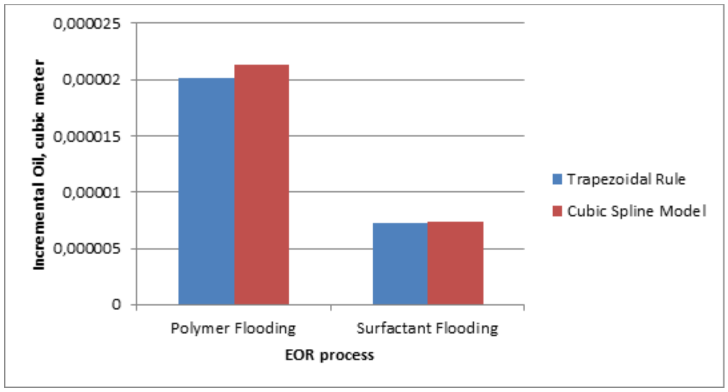

Fig 4.1 Comparison between the results of the cubic spline model and those of the trapezoidal rule.

Based on observations, it's noted that the EOR process defines a function whose equation is unknown, but only understood behaviourally from the plotted graph. Since the cubic spline model represents this function equation wise and the area under rate-time plot is the amount, The model gives better result of the incremental oil if well applied.

Figure 1.1 clearly defines incremental oil. Imagine a field, reservoir or well whose oil rate is declining from A to B. At B, an EOR project is initiated and if successful, the rate should show a deviation from the projected decline at some point after B. Incremental oil is the difference between what was actually recovered, B to D, and what would have been recovered had the proccess not been initiated, B to C [1].

As simple as the concept of cubic spline is, EOR is difficult to determine in practice. There are several reasons for this[1].

1. Combined (comingled) production from EOR and non EOR well. Such production makes it difficult to allocate the EOR-produced oil to the EOR project. Comingling occurs when, as is usually the case, the EOR is phased into a field undergoing other types of recovery.
2. Oil from other sourses. Usually the EOR project has experienced substatial well cleanup or other improvements before startup. The oil produced as a result of such treatment is not easily differentiated from the EOR oil. The hypothetically projected curve must be accurately estimated. But since it did not occur, there is no way of accessing this accuracy.
3. Inaccurate estimate of hypothetical declines

Figure 3.1 shows the behaviour of the data. Where Eq (4.1) defines the curve of the projected decline.

$$y = 23.589x^{-1.468} \quad (4.1)$$

This shows that after 1320 seconds there was the need to initiate the EOR proccess. However, the effect began at about 1920 seconds and ended at 2640 seconds, therefore, the boundaries were taken as such. And natural end conditions were assumed. That is the second derivatives at the end knots are zero. This represents the surfactant flooding proccess. The second is polymer flooding, it occures between 3480 and 4320 seconds, and the same spline end conditions were applied. The choice of values for k_1 and k_n prescribed as end conditions must be made intuitively, based on the way the data points indicate the interpulated curve is most likely to behave (be extrapolated) beyond the end points of the interval $x_1 \leq x \leq x_n$.

Three typical choices are the *natral or linear spline* end condition, the *parabolic spline* end condition and *periodic spline* end conditions [6].

Eq (3.4) is the interpolant that spans the surfactant flooding proccess and it spans between four knots $(data\ points)$, two interior and two end knots $(k_1 = k_4 = 0)$, and it gives a total inrecmental oil of $7.4 \times 10^{-6} m^3$. While in the polymer flooding procces, Eq (3.7) is the interpolant, and it spans five knots $(data\ points)$, three interior and two end knots $(k_1 = k_5 = 0)$. And it gives a total inrecmental oil of $21.3439 \times 10^{-6} m^3$.

Eq (2.7) shows that It is crucial to the practical use of splines that this system of equations being nonsigunar,and that an extremely efficient algorithm be available to solve it. When applied, It reveals that the value of the second derivative at each interior knot depends on the curvature of the function.

5. Conclusion

The main goal of this study is to devise a means by which better results of the incremental oil recovered in any successful EOR project are obtainable. Based on the procedure involved and the results obtained, the following conclusions were drawn:

1. Though the model is a highly sophisticated numerical model, however, it gives a more reliable value of the incremental oil.
2. The amount of incremental oil recovered by the cubic spline model is always more than that obtained by the trapezoidal rule.
3. The distinction between the results obtained by the cubic spline model and that of the trapezoidal rule might represent the error incurred in trpezoidal rule if the boundary conditions are properly applied.
4. The difference between the integral under the incremental oil recovery curve and the integral under the corresponding curve of the projected decline represents the incremental oil.
5. The value of the second derivative at each interior knot depends on the curvature of the function.

Symbols used

A_i	$[-]$	*Coefficient of the second derivative*
$A(I)$	$[-]$	*"OUR" algorithm parameter*
B_i	$[-]$	*Coefficient of the second derivative*
$B(I)$	$[-]$	*"OUR" algorithm parameter*
C_i	$[-]$	*Coefficient of the second derivative*
$C(I)$	$[-]$	*"OUR" algorithm parameter*
D_i	$[-]$	*Coefficient of the second derivative*
$D(I)$	$[-]$	*"OUR" algorithm parameter*
EOR	$[-]$	*Enhanced oil recovery*
$f_{i,i+1}(x)$ and $f_{i,i-1}(x)$	$[-]$	*Interpolant or cubic*
$f'_{i,i+1}(x)$	$[-]$	*First derivative*
$f''_{i,i+1}(x)$	$[-]$	*Second derivative*
h	$[M]$	*Height*
i , I and J	$[-]$	*Position vector*
IOR	$[M^3]$	*Incremental oil recovered*
K or k_i	$[-]$	*Second derivative*
k_n	$[-]$	*Second derivative at the last or end knot*
k_1	$[-]$	*Second derivative at the first or end knot*
M	$[-]$	*Position vector*
N_P	$[M^3]$	*Cumulative oil produced*
q	$\left[M^3/_{SEC}\right]$	*Recovery rate*
$S(I)$ and $S(J)$	$[-]$	*"OUR" algorithm parametrs*
$S(t)$	$[-]$	*Spline function*
t	$[SEC]$	*Time*
x	$[-]$	*Independent variable*
x_i	$[-]$	*Node*

x_1	$[-]$	*end or first node*
x_n	$[-]$	*end or last node*
y	$[-]$	*dependent variable*
y_i	$[-]$	*knot*
y_1	$[-]$	*end or first knot*
y_n	$[-]$	*end or last knot*
$\propto (I)$	$[-]$	*"OUR" algorithm parameter*

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References

[1] Russel J. Defining Enhanced Oil Recovery. 2010.
[2] Don WG, Paul WG. Enhanced Oil Recovery. Texas: Richardson; 1998.
[3] John MC. Oil property Evaluation. Englewood Cliffs: Prentice -Hall Inc; 1960.
[4] Thusen HG. Engineering Economy. Englewood Cliffs: Prentice -Hall Inc; 1950.
[5] Steven C, Chapra, Raymond PC. Numerical Methods For Engineers. 6th ed. Tufts University and University of Michigan; 2010.
[6] Jerome F, Trevor H, Robert T. The Element of Statistical Learning. Basis Expansion and Regularization 2001; 115-164.
[7] Alan Jeffrey. Advanced Engineering Mathematics. University of Newcastle: Upon-Tyne; 2002.
[8] Jeffrey RC. Introduction to Numerical Methods. The Hong Kong University of Science and Technology; 2012.
[9] Richard W, Daniels. An Introduction to Numerical Methods and Optimization technique. North-Holland. New York: Oxford; 1978.
[10] Autar Kaw, Micheal Kateltas. Spline Method of Interpolation. General Engineering; 2009.
[11] Dukkipati RV. Numerical Methods. FairField University; 2010.
[12] Rathod HT, Shrivalli HY, Nagaraja KV, Kesavulu Nardu. In. Journal of Math. Analysis. Vol 4 2010; 28: 1387-1415.