Petroleum and Coal

Article Open Access

Application of numerical signal differentiation methods to determine stationarity of a process

A.V. Maystrenko[1], A.A. Svetlakov[1], T.V. Gandzha[1], V.M. Dmitriev[1], Natalia V. Aksenova[2]*

[1] Department of Electronic Automation and Control Systems, Tomsk State University of Control Systems and Radio-electronics, Tomsk, Russia
[2] Institute of Cybernetics, Tomsk Polytechnic University, Tomsk, Russia

Received April 2, 2017; Accepted May 31, 2017

Abstract

Among important tasks requiring their solution when creating an integrated mathematical model of a major oil pipeline is a problem to determine stationary character of different processes in the pipeline. Importance of this task is defined by the fact that both mathematical description and analysis of stationary and non-stationary processes may significantly differ. Mathematical models of stationary processes (modes, objects) are as a rule significantly less complex than the models of nonstationary processes. For example, they may often be described by systems of linear equations which are easy to solve, while for nonstationary processes the researcher has to solve a system of differential equations of different orders, which a much more difficult and laborious task. It is obvious that there is a necessity to not just identify the process as a stationary or nonstationary, but also precisely determine the moment when it starts changing its stationary state to the opposite type. Meeting this requirement is linked to necessity to timely employ mathematical apparatus corresponding to the current process. In this paper, we propose a new method for the determination of stationary processes, based on the application of the algorithm of numerical differentiation of signals (NDS) using a moving quadratic approximation and pseudoinverse matrices. A wide spectrum of applications that need to identify the intervals of stationary regimes of observed and/or controlled processes, as well as a variety of conditions in which the mentioned intervals should be estimated, determine the relevance of improvement of existing and creation of new methods for solving the problem under discussion. The essence of the method lies in the fact that, during the analysis of stationarity of processes, we take into account not only the values of the signal itself, but also the values of its first and second derivatives. This approach opens up the opportunities to determine the boundaries of the regime of controlled process more precisely and to predict the behavior of process in time. We present the formulation of the real-time NDS problem and the description of the proposed algorithm for its solution, as well as the description of some of the results of research and a new method of determining intervals of stationary processes based on the proposed NDS algorithm, and comparison of the proposed method with well-known in mathematical statistics method for determining the stationary processes based on the use of the criterion of inversion. The proposed method allows to calculate precisely the values of derivatives, as well as to determine the modes of stationary processes of real objects. The method has significantly higher noise immunity in comparison with the methods based on the use of classical statistical tests of stationarity. We recommend applying the proposed method for the development of mathematical models of complex dynamic objects operating in real time.

Keywords: words: *approximation; signal differentiation; derivative; stationary process; model, mathematical statistics; inversions test; pseudoinverse matrix*.

1. Introduction

Among important tasks arising in mathematical modeling of different processes and objects there is a problem to determine stationarity of modes and their characteristic parameters (signals) [1-2]. Importance of this task is defined by the fact that both mathematical description and analysis of stationary and non-stationary processes may significantly differ. It is obvious that there is a necessity to not just identify a process as a stationary one, but to precisely determine the moment when it starts changing from its stationary mode to the opposite one. Meeting this requirement is linked to timely employment of mathematical apparatus corresponding to the process being currently studied. A wide variety of methods is known for solution of this task, the most common of them is so-called inversions test [2-4]. However, multitude and variety of applications requiring determination of observed and/or controlled process' stationarity, as well as variety of conditions under which such intervals should be defined raise an issue of creating new methods to solve this task and improving the existing ones.

This study implements one possible solution of the above mentioned problem, namely application of Numeric Differentiation of Signals (NDS) based upon application of moving quadratic approximation to the differentiated signal, the approximation being obtained with pseudo-inverse matrices [5-8]. The essence of the proposed method is that in addition to the values of the signal itself values of its first derivative are used as well to analyze the processes for their stationarity. Such an approach opens various possibilities not only for more precise determination of controllable process boundaries (hereinafter a process boundary means a moment when the process experience a change from stationary mode to non-stationary and vice versa), but allows predicting changes of the process mode in time.

Statement of the real-time (on-line) NDS task in given below, as well as a description of a proposed solution algorithm and some results from studying this new NDS-based method to determine intervals of process' stationarity and comparison with a well-known inversions test method [2-4] commonly employed for the same task .

2. Problem statement and solution of NDS task in real-time

As we know from calculus [9-10], the signal differentiation task $s = s(t)$, where $s(t)$ is a function of time *t*, is for any fixed value of $t = t_0$ to be able to calculate the value of derivative ds/dt of this signal as per equation

$$ds/dt = ds(t_0)/dt \,. \quad (1)$$

At that, it is assumed that the derivative ds/dt complies with the formula

$$ds/dt = \lim_{\Delta t \to 0} \Delta s/\Delta t \,, \quad (2)$$

where Δt and Δs are variables which are called increment of argument *t* and increment of signal *S* and are defined with the equations

a) $\Delta t = t - t_0$ and b) $\Delta s = s(t) - s(t_0)$. (3)

It should be noted that the formula (2) is a number of cases allows for precise calculation of its value. In particular, this possibility is implementable in all the cases where the differentiated signal *S* is analytically defined, that is, with a formula, and is available for all the mathematical operations necessary to compute its derivative ds/dt as per formula (2). Under real conditions when the differentiated signal *S* is defined non-analytically, but by a certain curve or a table with discrete values of t_i and corresponding values $s_i = s(t_i)$ of the signal *S*, analytical computation of the value of derivative ds/dt using the formula (2) is impossible [11-12]. This is due to the fact that in all such cases we have but finite values of increment Δt and Δs, found in the right-hand side of the formula (2), and as a result we do not know the values of the signal $s = s(t)$, corresponding to the values of argument *t*, between its values $t_0 - \Delta t$ and t_0 , thus, we are unable to compute the precise value of the limit to which their ratio $\Delta s/\Delta t$ tends when $\Delta t \to 0$. This fact forces us to develop and use so-called numerical or digital method of differentiation of the signal *S*, which are from the beginning intended for application under conditions when increments Δt and Δs have finite value. At that, as it happens under real conditions, it is assumed that the increment Δt can be changed with the purpose to use the NDS method and concerning concrete conditions where we intend to use it.

This paper uses the NDS method based upon moving approximation of the differentiated signal with quadratic polynomials, which are obtained by constructing pseudoinverse matrices [5]. The essence of the method is as follows:

1) it is assumed that around any particular value *t* the incoming signal *S* of the differentiator can be approximated with reasonable accuracy by algebraic polynomials of the form

$$\hat{s}(t) = at^2 + bt + c, \tag{4}$$

whose coefficients *a, b* and *c* are constant in the vicinity $[t - \Delta t, t + \Delta t]$ of this value *t* and change when *t* goes out of this vicinity;

2) at any fixed moment t_0 there is a set of $m+1$ measured values $s(t_0 - m\Delta t), s(t_0 - (m-1)\Delta t), \ldots, s(t_0)$;

3) search for polynomial coefficients is reduced to solution of a system of conditional linear equations

$$A\vec{x}^T \approx \vec{s}^T, \tag{5}$$

where the ≈ symbol signifies conditional (approximate) equation, and matrix *A* and vectors $\vec{x}^T$ and $\vec{s}^T$ of the system are determined by the following equations:

$$\text{a) } A = \begin{pmatrix} t_0^2 & t_0 & 1 \\ (t_0 - \Delta t)^2 & (t_0 - \Delta t) & 1 \\ \cdots & \cdots & \cdots \\ (t_0 - m\Delta t)^2 & (t_0 - m\Delta t) & 1 \end{pmatrix}; \quad \text{b) } \vec{x}^T = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \text{and} \quad \text{c) } \vec{s}^T = \begin{pmatrix} s(t_0) \\ s(t_0 - \Delta t) \\ \cdots \\ s(t_0 - m\Delta t) \end{pmatrix}. \tag{6}$$

4) solution of the system of conditional equations (5) is reduced to minimizing the Euclidean metric $\rho(s, \hat{s})$, defined by the equation

$$\rho(s, \hat{s}) = \left(\sum (s_i - \hat{s}_i)^2 \right)^{1/2}. \tag{7}$$

Here $s_i = s(t_i)$ are measured values of the signal *S* in moments $t_i = t_0 - i\Delta t$, $i = \overline{0, m}$; $\hat{s}_i = \hat{s}(t_i)$ are values of the approximating polynomial (4), corresponding to the same moments t_i; *m* is a certain limited natural number smaller than *M*, where *M* is the upper boundary of allowable values of *m*, which is selected with respect to technical capabilities of the hardware used to implement differentiation of the signal as well as required speed, error level in signal values.

5) as a solution $\vec{x}^T$ of the system (5) we use its pseudosolution $\vec{x}_+^T$ obtained from the equation

$$\vec{x}_+^T = A^+ \cdot \vec{s}^T, \tag{8}$$

where A^+ is pseudoinverse of the matrix *A* . As it is known [6-7], the metric (7) reaches its minimum value in the case when the pseudosolution $\vec{x}_+^T$ is used as a solution of an inconsistent system (5), which justifies its use here.

6) as (66) implies, components x_{1+}, x_{2+}, x_{3+} of the pseudosolution $\vec{x}_+^T$ are coefficients *a, b* and *c* respectively of the polynomial (4) and thus these coefficients are given their numeric values as per the following equations:

$$\text{a) } a = x_{1+}; \quad \text{b) } b = x_{2+} \quad \text{and} \quad \text{c) } c = x_{3+}; \tag{9}$$

7) computation of the derivative ds/dt is performed as per equation

$$ds/dt = 2at_0 + b, \tag{10}$$

which was obtained by differentiation of the polynomial (4) according to time *t*.

8) repeated differentiation of this polynomial allows obtaining a simple equation of the form

$$d^2s/dt^2 = 2a, \tag{11}$$

that defines the second derivative d^2s/dt^2 of the differentiated signal *S*.

The set forth sequence of operations is a full and clear-cut representation of the proposed method and its implementing NDS algorithm. Finishing our consideration of the method let us mark the following two features illustrating its ample opportunities for practical application.

1. Existence and unicity of the matrix A^+, pseudoinverse to any initial matrix *A,* as well as controlled parameters Δt and *m* found in this method allow choosing their numerical values in each concrete situation in such a way that provides maximum accuracy of differentiated signal approximation and possibility to implement the method with the technical means available.
2. Use of the proposed method provides for easy implementation of the signal *S* differentiation in the so-called "running window" mode, or similarly in the on-line mode. As may be seen from (6)-(11), it is necessary and sufficient to provide the implementation algorithm with an operator that provides an "update" of the matrix *A* and vector $\vec{s}^T$ in the equation (6) as new measured values $s(t_0+\Delta t), s(t_0+2\Delta t),\ldots$ of the signal *S* become available to the differentiator.

3. Synthesis of a method to determine stationarity intervals of studied processes from process values and their derivatives

The set forth NDS method opens ample possibilities to synthesize different methods to determine stationarity of a process using its values and values of its derivatives. In particular, this method may be used to propose a method to solve the problem and represented by the following sequence of operations.

1) let there be a single measurement of the signal *S* in each moment of time *t* and each of them has *m*+1 available measurements of the signal .
2) then, we will hold the following two conditions as true:
 a) *m* is a natural number that we can select in accordance with our requirements;
 б) distance between the moments is *Δt* and we can change it, as well as the number *m*, if necessary.
3) in each moment *t* values of the first and the second derivatives of the measured signal *s=s(t)* are computed as per equations (10)-(11).
4) criteria of process stationarity are formulated with regards to requirements of a concrete problem. Deviation of the signal *s=s(t)*, its first ds/dt and/or second d^2s/dt^2 derivative in neighboring moments may serve as such criteria under the following conditions:

$$\begin{cases} s_{i+1}-s_i > \alpha; \\ ds_{i+1}/dt_{i+1} - ds_i/dt_i > \beta; \quad i=\overline{0,m}, \\ d^2s_{i+1}/dt_{i+1}^2 - d^2s_i/dt_i^2 > \gamma; \end{cases} \tag{12}$$

where *α*, *β* and *γ* are some parameters whose numerical values are selected from conditions of the concrete process stationarity problem.

5) group processing of the signal values and their derivatives is possible to determine stationarity mode; when there are more than one simultaneously used value in a group the processing uses so-called "running window" mode.
6) parameters *α*, *β* and *γ* are selected individually for solution of each problem depending on specific requirements to signal deviation and deviation of its derivatives in consecutive moments of time. Indeed, the requirements may differ significantly, for some applications it may be less than one per cent, while for others it may comprise several or even a few dozens of percent. This task is a topic for further studies and there will be no further details given in this paper.

Below some experimental results are given to illustrate advantages of the method in comparison to a well-known statistical method to determine process' stationarity by inversions test.

4. Some results of research into the new NDS-based method to determine stationarity

Use of non-parametric criteria requiring no prior information about type of the process (signal) has high practical importance. Among such non-parametric criteria are runs test and inversions test [2-4]. The latter is more powerful in detection of monotonous trends in observational data. The inversions test may be directly applied to test the stationarity hypothesis. That is why the inversions test was selected as a comparison method for the proposed method.

The number of measured signal values allowing for determination of stationary modes for the inversions test is bounded from below by the number 10. It is due to the fact that all the known distribution tables start from this value. The proposed NDS method for stationarity detection does not have this limitation and allows for solution of the problem for three or more measurements of the signal. At that, the more signal values are processed simultaneously, the higher is the accuracy of derivative calculations and the more accurate is the process stationarity identification. However, it should be noted, that, first, computational complexity increases with increased number of measurements processed [5, 13-15]; second, for fixed value of time increment the possibilities to add measurements are limited; third, with increased number of measurements the boundary (moment) of the mode change becomes blurry.

For the experiment, the number of simultaneously processed signal values was selected in a range from 10 to 80 with the step of 10 and the test was performed in a sliding window mode. The signals were different in their appearance, however the figures show only one type of signal where the measurement number m was equal to 10 and 80.

A different method of signal processing was used for the inversions test as well, where a signal was processed in groups of 10-80 measurements. At that, a signal mode was analyzed at the group boundary where the process was either found to be in the same mode as previously (whether it was stationary or nonstationary) or switched to the opposite mode. The results obtained in this way were somewhat worse than those given here. It was caused by the fact that every time when it is necessary to process he next group of signal measurements the group has to be received, while no signal processing is performed during the accumulation phase; thus a part of useful information is lost. However, it is necessary to mention those results because that is the way the classical inversions test is implemented.

Below some numerical simulation results are given and discussed; the aim of such experiments was to confirm performance capability and advantages of the proposed NDS-based method for solution of the process stationarity determination problem. All the experiments were conducted in *Matlab* package. At that, real signals obtained from pressure sensors of the major pipeline pump unit (driven from a high-power electric motor) were used as differentiated signals.

Let us consider a number of graphed dependencies given in Fig.1-5 and illustrating advantages of the proposed method in comparison to the inversions test method.

The results of experimental studies given in Fig. 1 and 3 are functions of the signal and its derivative on time and are denoted as s and ds, respectively [16]. Figures 2, 4 and 5 show stationary parts of the signal obtained during the same moments with the proposed criterion and the inversions test method, they are given in intervals form 1 to 1.5 and from 0 to 0.5 on Y axis and denoted as $Minv$ and $Mnew$ respectively. At that, stationary mode is the topmost level (1.5 and 0.5 respectively). In the legend, the m designates number of measurements analyzed.

A parameter β (the second line of formula (12)) in the figures' titles sets the maximum value of current derivative deviation from its previous moment thus allowing for independent selection of boundaries where the signal is held as a stationary. For each concrete process, this parameter may be set individually in accordance with the requirements and limitations of the process. During the experiments, this interval was selected randomly without considerations for any factors with the aim to demonstrate capabilities of the method.

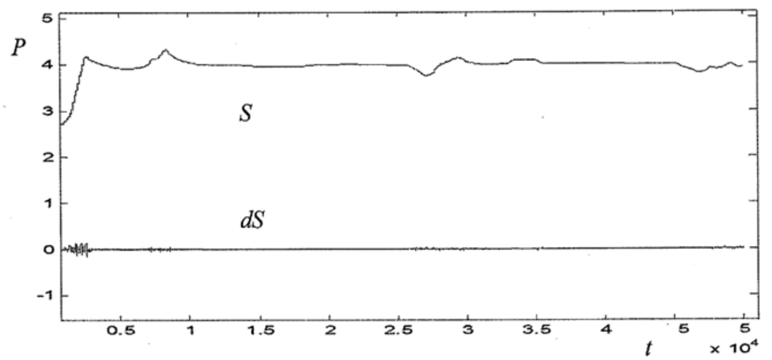

Figure 1. Signal *S* and its derivative *dS*, *m*=80

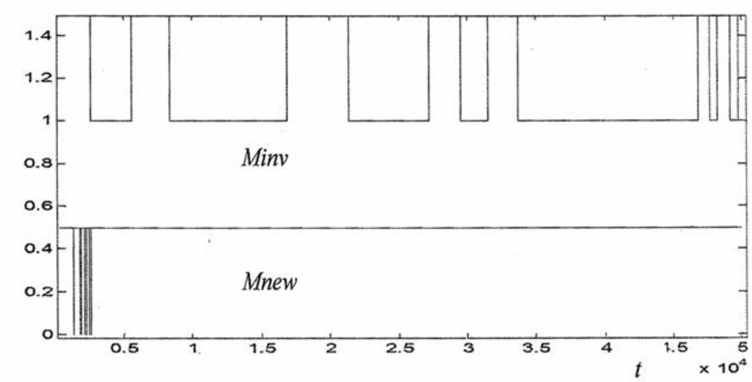

Figure 2. Determination of stationarity *m*=80

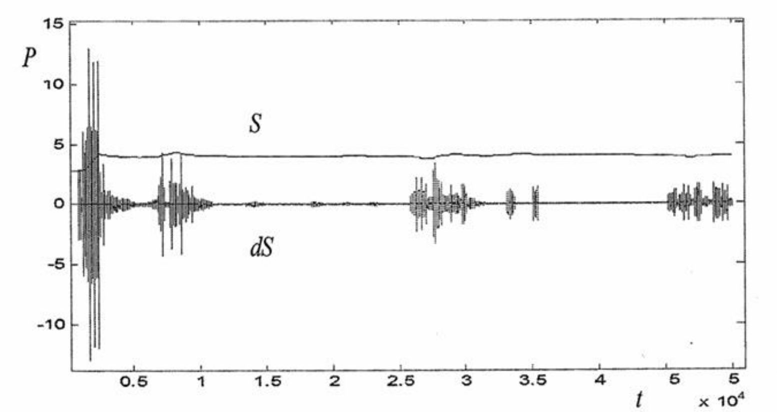

Figure 3. Signal *S* and its derivative *dS*, *m*=10

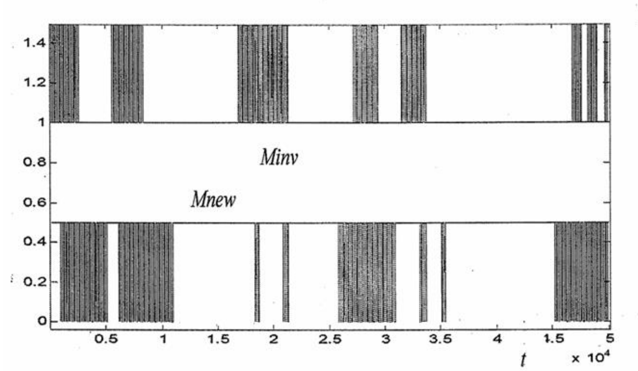

Figure 4. Determination of stationarity, *m*=10, β=0.1

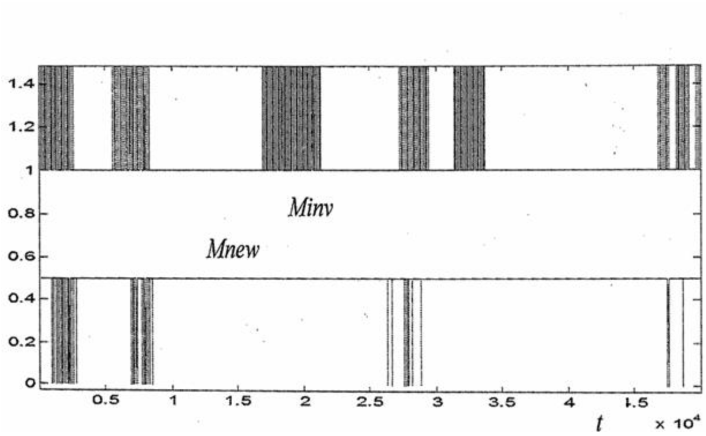

Figure 5. Determination of stationarity, *m*=10, β=0.1

The graphs show that the higher is the number of simultaneously processed measurements, the better are the results of the inversions test - the boundaries between the modes of the process appear more distinct [17]. In general, this is true for the proposed method as well, however the inversions test has a significant drawback, namely, it cannot determine the stationarity mode in strict adherence to the probability distribution table and does not allow for independent changes of the delimiting conditions of stationarity. In contrast, the proposed method allows for independent selection of derivative variation intervals marking the process as a stationary one from additional information, conditions, personal experience, technical documentations.

A significant departure of this method from the more traditional ones [2-4] lies in the fact that it allows selection of the confidence intervals in determination of the stationary modes guided by different considerations, such as accuracy of the instruments that measure the controlled parameters.

Another great feature of the method is its high resistance to effects of differentiated signal measurement error and thus is more suitable for real-time applications, in contrast to stationarity criteria used in mathematical statistics (run test, inversions test) [5].

5. Conclusions

The results given in the previous sections allow for the following conclusions.

1. The proposed method of process stationarity determination that is based on the NDS algorithm which in its turn uses sliding quadratic approximation and pseudoinverse matrices, allows for computation of derivatives with high accuracy as well as for determination of stationarity of real world processes. It has a significantly higher noise immunity than the methods based on classical statistical criteria of stationarity.
2. In contrast to the inversions test the proposed algorithm allows for varying the confidence intervals in the wide range, thus significantly simplifying adjustments of implementations to correctly determine the initial moment and duration of the stationary or nonstationary mode.
3. Selection of variation range of the derivative is made depending on conditions and features of a concrete process.
4. The algorithm implementing the proposed method is quite simple and approachable for hardware and software implementation.

References

[1] Bulinskii AV, Shiriaev AN. *Teoriia sluchainykh protsessov* [Theory of random processes]. - M.: Fizmatlit, 2003. - 400 p.

[2] Bendat Dzh, Pirsol A. *Prikladnoi analiz sluchainykh dannykh* [Applied analysis of random data]. - M.: "Mir", 1989. - 542 p.

[3] Ventcel' ES. Teorija verojatnostej [Probability theory]. Ucheb. dlja stud. vuzov 10-e izd., ster. – M.: Izdatel'skij centr «Akademija», 2005. - 576 p.

[4] Feller V. Vvedenie v teoriju verojatnostej i ee prilozhenija [Introduction to the theory of probability and its applications]. T2 – M.: Mir, 1967. - 752 p.
[5] Maistrenko AV, Svetlakov AA, Starovoitov NV. Tsifrovoe differentsirovanie signalov v real'nom masshtabe vremeni s primeneniem skol'ziashchei kvadratichnoi approksimatsii [Digital differentiation signals in real time using a quadratic fit sliding]. Omskii nauchnyi vestnik - Omsk Scientific Bulletin, 2006, 7(43), 106-108.
[6] Maistrenko AV, Svetlakov AA, Starovoitov NV. Tsifrovoe differentsirovanie signalov s primeneniem mnogotochechnykh metodov v sistemakh avtomaticheskogo regulirovaniia protsessov [Digital differentiation signals using multipoint method in the automatic control of processes]. Doklady TUSURa - Reports TUSUR, 2009, 2(20), 86-89.
[7] Maistrenko AV, Svetlakov AA, Starovoitov NV. Tsifrovoe differentsirovanie izmeriaemykh signalov s primeneniem integral'nykh uravnenii V.Vol'terra i ego reguliarizatsiia [Digital differentiation of the measured signals using integral equations V.Volterra and regularization]. Omskii nauchnyi vestnik - Omsk Scientific Bulletin, 2013, 2 (120), 308-313.
[8] Gantmaher FR. Teorija matric [Matrix theory]. – M.: Nauka, 1967. - 575 p.
[9] Il'in VA. Osnovy matematicheskogo analiza. [Fundamentals of mathematical analysis]. V 2 ch. Part.1: Uch. dlia vuzov - M.: FIZMATLIT, 2005. - 648 p.
[10] Fihtengol'c GM. Osnovy matematicheskogo analiza [Fundamentals of mathematical analysis]. Ucheb. dlja vuzov. Ch. 1. – SPb.: Lan', 2001. - 440 p.
[11] Tihonov AN, Arsenin VJa. Metody reshenija nekorrektnyh zadach [Methods of the solution of incorrect tasks]. Izd. 2-e. – M.: Nauka, 1979. - 286 p.
[12] Demidovich BP, Maron IA. Osnovy vychislitel'noj matematiki [Foundations of Computational Mathematics]. – M.: Nauka, 1963. - 660 p.
[13] Svetlakov AA. Traditsionnoe i netraditsionnoe otsenivanie neizvestnykh velichin [Traditional and non-traditional assessment unknowns]. V 2 ch. Ch 1. Prosteishie zadachi otsenivaniia neizvestnykh velichin po rezul'tatam ikh eksperimental'nykh izmerenii [The simplest problem of estimating unknown quantities according to the results of experimental measurements]: uch. posobie. – Tomsk: Tomsk. gos. un-t sistem upr. i radioelektroniki - Tomsk state University of control systems and Radioelectronics, 2007. - 550 p.
[14] Maistrenko AV, Svetlakov AA. Sintez mnogotochechnogo metoda tsifrovogo differentsirovaniia signalov [Synthesis of a multipoint method of digital differentiation of signals]. Omskii nauchnyi vestnik - Omsk Scientific Bulletin, 2009, 3 (83), 201-204.
[15] Maistrenko AV, Svetlakov AA, Starovoitov NV. Reguljarizacija prostejshego algoritma cifrovogo differencirovanija signalov [Regularization of the elementary algorithm of digital differentiation of signals]. Nauchnyj vestnik Novosibirskogo gosudarstvennogo tehnicheskogo universiteta - Scientific bulletin of Novosibirsk state technical university, 2006, 4 (25), 53-67.
[16] Dmitriev VM, Gandzha TV, Gandzha VV, Panov SA. Computer simulation of the visual interface of virtual instruments and devices. Scientific Visualization, 2016; 8(3): 111–131.
[17] Dmitriev VM, Gandzha TV, Dolganov IM, Pisarev MO, Dolganova IO, Sizova EN, Ivashkina EN. Structure of network simulator for training and retraining of operators of controlled technological objects of oil and gas industry. Pet Coal, 2015; 57(6): 691-695.

To whom correspondence should be addressed: Department of foreign languages of the Institute of Cybernetics, Tomsk Polytechnic University, 30, Lenin Avenue, Tomsk, 634050, Russia; tel.: (+7-923)4577428, E-mail: polozova15@tpu.ru