GENERALIZED REGRESSION NEURAL NETWORK – A BETTER PREDICTIVE TOOL FOR ESTIMATING PVT PROPERTIES OF NIGER DELTA CRUDE OILS

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Abstract
PVT properties such as Bubble-point Pressure, Viscosity and Oil Formation Volume Factor, are very important in reservoir engineering computations and analyses. This makes their accurate determination necessary. Ideally, these properties are obtained from Laboratory analysis on crude samples. However, often these measurements are either not available or very costly to obtain. Standard PVT Experiments could cost hundreds of thousands of dollars and there are bigger costs in terms of facility sizing and performance predictions in the face of inaccurate PVT measurements. As an alternative, different correlations have been established from a range of PVT data to predict these important properties however the uncertainty in the predicted property depends on the accuracy of the correlations used. Different correlations have been developed for estimating the PVT properties of Crude Oil samples using linear, non-linear or graphical means. However each correlation was developed for a certain range of reservoir fluid characteristics and geological setting with similar fluid compositions. Thus, these correlations would not be accurate when applied in different fluid and geological settings. Artificial Neural Network (ANN) models once successfully trained offer a better alternative to obtain reliable results for the determination of crude oil PVT properties. Also, its ability to continuously improve its prediction through the enrichment of its training dataset thus reducing its average percentage error has made this tool widely applicable even in the Oil and Gas industry. The objective of this work is to develop a Generalized Regression Neural Network (GRNN), training the network with PVT properties from crude samples in Niger Delta to predict the Oil Formation Volume Factor ($B_o$) and Bubble-point Pressure ($P_b$). A detailed comparative study between the performance of the GRNN against a basic ANN model and other published correlations are presented for these Niger Delta Crude Oil Samples. Also trend analyses were performed to check the behavior of the predicted $B_o$ and $P_b$ for any change in reservoir temperature, gas-oil ratio, gas gravity and oil gravity.

Keywords: PVT correlation; Generalized Regression Neural Network; Niger Delta; PVT properties; Trend Analysis.

1. Introduction

Oil PVT properties are invaluable parameters often used in field development and reservoir engineering computations such as: reserve estimation, material balance calculations, design and optimization of surface facilities. The reliability of the analyses and studies done on the field and reservoir relies on the accuracy of the PVT properties used. Among such PVT properties include; Bubble-point pressure and Oil Formation Volume Factor.

The bubble-point pressure, $P_b$, of a hydrocarbon system is the highest pressure at which gas is first liberated from oil [1]. It can be measured experimentally by conducting a constant-composition test. Oil formation volume factor, $B_{ob}$, is the ratio of the volume of oil (plus the gas in solution) at the prevailing reservoir temperature and pressure to the volume of oil at standard conditions.

These properties are usually obtained from laboratory analysis on crude samples either from surface or subsurface sampling method. However, these experiments are very costly to obtain and are often times not readily available or reliable. Therefore several empirical correlations have been proposed over the decades for estimating these properties. These correlations were usually developed using linear or non-linear multiple regression or
graphical techniques often using crude oil samples from a particular geological setting with similar fluid compositions. Thus, bound to have a higher degree of uncertainty when used applied for crude oil samples from a different geological setting i.e. the correlations are more accurate within the range of data that were used to develop them[2].

These correlations are essentially based on the assumption that \( P_b \) and \( B_{ob} \) are strong functions of the solution gas-oil ratio \( (R_s) \), the reservoir temperature \( (T) \), the gas specific gravity \( (\gamma_g) \), and the oil specific gravity \( (\gamma_o) \) that is:

\[
P_b = f_1(R_s, T, \gamma_g, \gamma_o)
\]

\[
B_{ob} = f_1(R_s, T, \gamma_g, \gamma_o)
\]

The objective of this study is to develop new predictive models for \( P_b \) and \( B_{ob} \) based on Generalized Regression Neural Network (GRNN) a form of Artificial Neural Network (ANN) using field data collected from Niger Delta reservoirs and compare it with existing empirical correlations and a Back Propagation Neural Network (BPNN) model.

ANNs are biologically inspired computer models which are adaptive, distribute and possess massive parallel processing system, have been used in many disciplines as an effective tool for pattern recognition.

This study presents two new models developed using GRNN to predict \( P_b \) and \( B_{ob} \). The developed models are based on 269 field data sets collected from different Niger Delta reservoirs. These data were divided into three groups. The first one (196 sets) was used to train the GRNN models, the second group (49 sets) was used to cross-validate the training, and the last group (24 sets) was used to test the models to evaluate their accuracy and trend stability. Results show that the developed models outperformed published correlations and ANN models developed using the feed-forward back-propagation (BPNN) algorithm.

Trend tests performed to check the behavior of the predicted values of \( P_b \) and \( B_{ob} \) for any change in reservoir temperature, gas-oil ratio, oil gravity and gas gravity, showed that the developed models obey physical laws.

2. Objective of Study

The objective of this study is to develop new predictive models for \( P_b \) and \( B_{ob} \) based on Generalized Regression Neural Network (GRNN) a form of Artificial Neural Network (ANN) using field data collected from Niger Delta reservoirs and compare it with existing empirical correlations and a Back Propagation Neural Network (BPNN) model.

2.1 PVT Empirical Correlations

Standing [3] in 1947, presented graphical correlations for estimating bubble-point pressure \( (P_b) \) and the oil formation volume factor \( (B_{ob}) \). It was based on laboratory experiment carried out on 105 samples from 22 different crude oils in California. Average relative errors of 4.8% and 1.17% were reported for \( P_b \) and \( B_{ob} \) respectively. In 1958, Lasater [4] developed an empirical correlation based for estimating bubble point pressure based on Henry’s law. He used a total of 137 crude oil and gas mixture from North and South America to develop the correlation. An average error of 3.8% was reported. Vazquez and Beggs [5] in 1980 presented correlations for oil formation volume factor. He used more than 6000 data points from 600 laboratory measurements in developing the correlations. He presented two correlations, one for crude with \(^{aAPI} > 30\) and for crude with \(^{aAPI} < 30\). An average error of 4.7% was reported for their correlation of \( B_{ob} \).


Labeledi [10] also developed correlations for \( B_{ob} \) using African Crude Oil samples comprising of 129 data sets. He used 97 data sets from Libya, 28 sets from Nigeria, and 4 sets from Angola to develop the correlation. Dokla and Osman [11] published set of correlations for estimating \( P_b \) and \( B_{ob} \) for UAE crudes. They used 51 data sets to calculate new coefficients.


In general, lots of correlations have been developed and most of these correlations show strong regional trends, and such correlations have low confidence level in other areas apart from where the data used in its development was obtained. As such, there is need for increased use of regional correlations to ensure accuracy and uncertainty reduction.

2.2 PVT Neural Network Models

Artificial Neural Network has recently been used extensively in the oil and gas industry for petroleum engineering calculations. Its adaptive and robust nature, and ability to capture non-linearity of systems makes it well suited for modeling PVT properties (Numbere et al. [17]). Gharbi and Elsharkawy [18] in 1997, published neural network models for estimating P_b and B_{ob} for Middle East crude oils. Both models were trained using 498 data sets collected from the literature and unpublished sources. The models were tested by other 22 data points from the Middle East. The ANN models showed a reduction in the average error for P_b and B_{ob} when compared with conventional empirical correlations.

All previous studies done by Elsharkawy were done using back propagation neural network. Elsharkawy [19] presented a new technique to model the behavior of crude oil and natural gas systems using a radial basis function neural network model (RBFNM). The model can predict B_{ob}, solution gas-oil ratio, oil viscosity, saturated oil density, undersaturated oil compressibility, and evolved gas gravity. He used differential PVT data of 90 samples for training and another 10 novel samples for testing the model. When the predictions made by the model were compared with all published correlations, his model showed much more accuracy. A physical trend analysis was also checked for the model and it seemed consistent. He concluded that although, the model was developed for specific crude oil and gas system, the idea of using neural network to model behavior of reservoir fluid can be extended to other crude oil and gas systems as a substitute to PVT correlations that were developed by conventional regression techniques.

Varotsis et al. [20] in 1999 introduced a novel approach for predicting the complete PVT behavior of reservoir oils and gas condensates using artificial Neural Network (ANN). The ANN model was trained with over 650 PVT data sets from all parts of the world. Tests of the trained ANN architecture utilizing a validation set of PVT studies indicates that, for all fluid types, most PVT property estimates can be obtained with a very low mean relative error of 0.5 – 2.5%. This level of error showed that it outperformed the tuned Equation of State (EOS) models. In addition to improved accuracy, the proposed ANN model avoids the ambiguity and numerical difficulties inherent to EOS models and provides for continuous improvements by the enrichment of the ANN training database with additional data.

Osman et al. [2] developed a new model to predict B_{ob} based on 803 published data sets from the Middle East, Malaysia, Colombia, and Gulf of Mexico fields. They trained a back propagation three-layer neural network model using 403 data sets, cross-validated the results using 200 data sets, and tested their model using the remaining 200 data sets. They used a new training algorithm developed by one of the authors in training the network. Results showed that their model provides better predictions and higher accuracy than the published empirical correlations (average absolute percent relative error of 1.789% and correlation coefficient of 0.998). Trend analysis performed to check the behavior of the predicted values of B_{ob} for any change in reservoir temperature, solution gas-oil ratio, gas gravity, and oil gravity. Their model was found to be physically correct.

Omole, Falode and Deng [21] developed a model to predict crude oil viscosity, based on 32 data sets collected from the Niger Delta region. 17 data sets were used to train the
model, 10 sets were used to test the accuracy of the model, and the remaining 5 sets were used to validate the relationships established during the training process. The results showed that the back propagation neural network model was better than the empirical correlations in terms of average absolute relative error and correlation coefficient.

Numbere, Azubuike and Ikiensikimama [17] in 2013 developed a neural network model for Pb prediction using 1248 data sets collected from Niger Delta region. 748 sets were used to train the ANN model, 250 were used to cross-validate the relationships established during the training process and 250 were used to test the network. They compared the predicted values with a statistical parameter called Rank, proposed by Ikiensikimama [22] in 2009. The model showed best results when compared with other empirical correlations with a rank of 17.3137 and correlation coefficient of 0.9698. Ikiensikimama and Azubuike [23] also developed a Back propagation Neural Network using Levenberg-Marquardt algorithm for predicting B_{ob} using data sets from Niger Delta.

3. Data Acquisition and Analysis

The 269 PVT data sets used for this work were collected from different Niger Delta fields. Each data set contains reservoir temperature, oil gravity, total solution gas-oil ratio, gas gravity, separator pressure and temperature, bubble-point pressure and oil formation volume factor. From the 269 data sets, 196 sets were used to train the ANN models, 49 were used to cross-validate the relationships established during the training process and 24 to test the model to evaluate its accuracy and trend stability. Table 1 and Table 2 contain statistical description of the training and testing data.

Table 1 Statistical Description of the 245 Training and Cross Validation Data

<table>
<thead>
<tr>
<th>Property</th>
<th>Min</th>
<th>Max</th>
<th>Average</th>
<th>St. Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubble Point Pressure</td>
<td>367.000</td>
<td>6560.000</td>
<td>3110.371</td>
<td>1099.329</td>
</tr>
<tr>
<td>Oil FVF at Pb</td>
<td>1.035</td>
<td>5.035</td>
<td>1.461</td>
<td>0.495</td>
</tr>
<tr>
<td>Temperature</td>
<td>100.000</td>
<td>256.000</td>
<td>173.654</td>
<td>29.547</td>
</tr>
<tr>
<td>Gas-Oil Ratio</td>
<td>39.000</td>
<td>8118.000</td>
<td>841.781</td>
<td>835.422</td>
</tr>
<tr>
<td>Gas Gravity</td>
<td>0.564</td>
<td>1.293</td>
<td>0.694</td>
<td>0.091</td>
</tr>
<tr>
<td>API Oil Gravity</td>
<td>16.204</td>
<td>55.843</td>
<td>34.397</td>
<td>8.466</td>
</tr>
</tbody>
</table>

3.1 Neural Networks

Many Science disciplines have been working towards the common goal of building an intelligent system. The theory behind the artificial neural network originated from biology. ANN is a computer model that attempts to mimic simple biological learning processes and simulate specific functions of the human nervous system. Neural Networks have the capacity to learn, memorize and create relationships amongst data. It is an adaptive, parallel information processing system and the most popular intelligent technique for pattern recognition to date. Once trained, these network models can predict output values for novel sets of input. Several papers about the application of neural network in the oil and gas industry have been presented. Almost all these papers have used back propagation neural network model.

The advantages of an artificial neural network include:
- An ANN learns the behavior of a database population by self-tuning its parameters in such a way that the trained ANN matches the desired output accurately.
- If the data used are sufficiently descriptive, the ANN provides a rapid and confident prediction as soon as a new case, which has not been seen by the model during the training phase, is applied.
- Its important aspect is their ability to discover patterns in data that are so obscure as to be imperceptible to normal observation and standard statistical methods.

An ANN model can accept more information as input to the model when the user wishes to increase its modeling range, thereby improving the accuracy of the predictions i.e. it can be retrained with a larger database increasing its expertise.

This work is aimed at presenting a new algorithm that is Generalized Regression Neural Network (GRNN) and to show that once it has been successfully trained, it can be reliable to predict PVT properties of crude oil. The performance of this new network will be compared...
with the back propagation network and other empirical correlations.

3.2 Description of the Generalized Regression Neural Network

A GRNN is a variation of the radial basis neural networks, which is based on kernel regression networks. A GRNN does not require an iterative training procedure as back propagation networks. It approximates any arbitrary function between input and output vectors, drawing the function estimate directly from the training data. In addition, it is consistent that as the training set size becomes large, the estimation error approaches zero, with only mild restrictions on the function.

A GRNN consists of four layers: input layer, pattern layer, summation layer and output layer as shown above. Two models were developed for predicting the bubble point pressure and oil formation volume factor. The input layer for the two models contain four neurons, each presenting the input data such as reservoir temperature, oil gravity, solution gas-oil ratio, and gas gravity. The input layer is connected to the pattern layer and in this layer each neuron presents a training pattern and its output. The pattern layer is connected to the summation layer. The summation layer has two different types of summation, which are a single division unit and summation units. The summation and output layer together perform a normalization of output set. In training of network, radial basis and linear activation functions are used in hidden and output layers. Each pattern layer unit is connected to the two neurons in the summation layer, S and D summation neurons. S summation neuron computes the sum of weighted responses of the pattern layer. On the other hand, D summation neuron is used to calculate un-weighted outputs of pattern neurons. The output layer merely divides the output of each S-summation neuron by that of each D-summation neuron, yielding the predicted values of $P_b$ and $B_{ob}$ for both models.

The distance of the point being evaluated to each of the other points is computed and a radial basis function (kernel function) is applied to the distance to compute the weight (influence) for each point. Therefore, the further some other point is from the new point, the less influence it has. The spread constant value of the model determines how quickly the function declines as the distance is increased from the point. With higher spread values, distant points have a greater influence. The primary work in training the GRNN is to select the optimal spread value to control the spread of the radial basis function.

Table 2 Statistical Description of the 24 Testing PVT Data

<table>
<thead>
<tr>
<th>Property</th>
<th>Min</th>
<th>Max</th>
<th>Average</th>
<th>St. Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubble Point Pressure</td>
<td>488.000</td>
<td>4845.000</td>
<td>2897.000</td>
<td>1215.914</td>
</tr>
<tr>
<td>Oil FVf at Pb</td>
<td>1.067</td>
<td>2.678</td>
<td>1.467</td>
<td>0.364</td>
</tr>
<tr>
<td>Temperature</td>
<td>149.500</td>
<td>242.100</td>
<td>173.796</td>
<td>20.822</td>
</tr>
<tr>
<td>Gas-Oil Ratio</td>
<td>56.100</td>
<td>2786.000</td>
<td>855.396</td>
<td>658.471</td>
</tr>
<tr>
<td>Gas Gravity</td>
<td>0.567</td>
<td>1.015</td>
<td>0.724</td>
<td>0.107</td>
</tr>
<tr>
<td>API Oil Gravity</td>
<td>21.473</td>
<td>47.795</td>
<td>37.184</td>
<td>7.820</td>
</tr>
</tbody>
</table>

Over-training the network must however be avoided, therefore it is important to monitor
the error as training progresses. When over-training occurs, the network tends to memorize results rather than generalize. This results in the model perfectly predicting data similar to the training data and performing badly if new cases are presented to it. The cross-validation technique was used in this study to avoid over-training.

3.3. Statistical Error Analysis

Statistical error analysis was performed to compare the performance and accuracy of the new model to a back propagation model and other empirical correlations. Average absolute percent relative error, minimum and maximum absolute percent error, standard deviation and correlation coefficient were used as comparison criteria. Figure 2 -5 shows the plot of the average absolute error and correlation coefficient for the Pb and Bob models.

3.4 Trend Analysis

A trend test was conducted in the model to make sure that it is physically correct and that overfitting did not occur. The trend test involves using hypothetical intermediate data points, and the dependence of Pb and Bbob on Temperature, solution gas-oil ratio, oil API gravity and gas gravity were studied. Figure 6 – 9 shows the Pb variation trend analysis on reservoir temperature, solution gas-oil ratio, oil gravity, and gas specific gravity. While Figure 10 - 13 shows the same trend analysis for Bbob. The results show that the proposed GRNN model obeys the physical law and follow the same trend as other correlations.

4. Results and Discussion

After training the network, the last data group (24 data sets) which was not seen by the network during training, was used to test the network. The performance and accuracy of the model was compared with a BPNN model and 4 other empirical correlations. For Pb model, the correlations are Glaso [6], Standing [3], Al-Marhoun [8], and Petrosky [7]. For Bbob model, the correlations are Standing [3], Glaso [6], Al-Marhoun [8], and Petrosky [7].

4.1 Bubble Point Pressure Model

The GRNN model outperformed the BPNN model (with 2 neurons in the first hidden layer and 3 neurons in the second hidden layer) and other empirical correlations. The proposed model had a correlation coefficient of 0.977, lowest average absolute percentage error, and lowest standard deviation. Table 3 contains statistical comparison of the proposed GRNN model with a BPNN model and other correlations. Figure 14 - 19 illustrates the scatter diagrams of the predicted values of Pb versus the measured values. Figure 14 showed the tightest cloud of points around the 45° line when compared with others, indicating a good match between the predicted values and the measured values.

Table 3 Statistical Analysis of different Pb Correlations

<table>
<thead>
<tr>
<th></th>
<th>Average Absolute Percent Error</th>
<th>Standard Deviation</th>
<th>Minimum Error</th>
<th>Maximum Error</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRNN</td>
<td>7.635</td>
<td>5.494</td>
<td>0.397</td>
<td>22.650</td>
<td>0.977</td>
</tr>
<tr>
<td>BPNN (2-3)</td>
<td>10.887</td>
<td>11.910</td>
<td>0.280</td>
<td>45.422</td>
<td>0.954</td>
</tr>
<tr>
<td>Standing</td>
<td>13.564</td>
<td>8.563</td>
<td>2.544</td>
<td>32.852</td>
<td>0.929</td>
</tr>
<tr>
<td>Glaso</td>
<td>14.945</td>
<td>12.790</td>
<td>0.523</td>
<td>41.508</td>
<td>0.952</td>
</tr>
<tr>
<td>Al-Marhoun</td>
<td>20.490</td>
<td>16.736</td>
<td>0.225</td>
<td>63.069</td>
<td>0.944</td>
</tr>
<tr>
<td>Petrosky</td>
<td>20.612</td>
<td>22.852</td>
<td>0.664</td>
<td>85.310</td>
<td>0.961</td>
</tr>
</tbody>
</table>

4.2 Oil Formation Volume Factor

The proposed GRNN model outperformed the best BPNN model (with 3 neurons in the first hidden layer and 5 neurons in the second hidden layer) and other empirical correlations. The GRNN model had a correlation coefficient of 0.9899 with the lowest standard deviation. However, Petrosky [7] performed quite well also with a correlation coefficient of 0.9891 and lower average error. Table 4 contains statistical comparison of the proposed GRNN model with
the BPNN model and other empirical correlations. Figure 20 - 25 shows the cross plot of the predicted values of Bob versus the measured values. All the plots showed quite a good cluster around the 45° line but Figure 20 still outperformed them. The reason for obtaining such a close match between all the models could be ascribed to the fact that the distribution range of the tested data sets was quite close as opposed to the $P_b$ data sets. Al-Marhoun for both cases performed the least.

Table 4 Statistical Analysis of different Bob Correlations

<table>
<thead>
<tr>
<th>Model</th>
<th>Average Absolute Percent Error</th>
<th>Standard Deviation</th>
<th>Minimum Error</th>
<th>Maximum Error</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRNN</td>
<td>2.8335</td>
<td>2.2027</td>
<td>0.0469</td>
<td>7.3693</td>
<td>0.9899</td>
</tr>
<tr>
<td>BPNN (3-5)</td>
<td>3.0760</td>
<td>2.3238</td>
<td>0.2032</td>
<td>7.8234</td>
<td>0.9891</td>
</tr>
<tr>
<td>Standing</td>
<td>3.1806</td>
<td>4.0719</td>
<td>0.0542</td>
<td>15.6233</td>
<td>0.9897</td>
</tr>
<tr>
<td>Glaso</td>
<td>3.5306</td>
<td>2.7919</td>
<td>0.3642</td>
<td>10.8894</td>
<td>0.9875</td>
</tr>
<tr>
<td>Al-Marhoun</td>
<td>4.2488</td>
<td>2.8974</td>
<td>0.0454</td>
<td>12.6277</td>
<td>0.9884</td>
</tr>
<tr>
<td>Petrosky</td>
<td>2.4486</td>
<td>2.9439</td>
<td>0.0123</td>
<td>12.3117</td>
<td>0.9891</td>
</tr>
</tbody>
</table>

4.3 Trend Analysis

The GRNN model obeyed physical law as it follows the same trend as the other correlations. From the trend analysis, it showed $P_b$ as an increasing function of $T$ and $R_s$, and a decreasing function of $\gamma_o$ and $\gamma_g$. The same deduction can also be made for the Bob model. It showed Bob as an increasing function of $T$, $R_s$ and $\gamma_g$, and a decreasing function of $\gamma_o$.

5. Conclusion

A generalized regression neural network has been developed to predict PVT properties of crude oil samples in Niger Delta. The model has been trained using 196 data sets, cross-validated with 49 PVT data sets and a novel 24 data sets were used for testing. Input data to the GRNN model are reservoir temperature, solution gas-oil ratio, stock tank oil gravity, and gas specific gravity. The newly developed GRNN model was found to perform better than a BPNN model and other existing empirical correlations. Unlike other empirical correlations, developed using regression techniques, the GRNN model was developed using artificial neural network. The best BPNN model to be used for comparison for both the $P_b$ and $B_{ob}$ modeling was selected after various network design architecture were tested. But the GRNN model still out-performed them all from the statistical error analysis. It showed to have the best correlation coefficients of 0.977 and 0.9899 for $P_b$ and $B_{ob}$ respectively. The trend analysis performed also suggested that the model obeys the physical laws. The neural network can be further refined and its accuracy further increased by incorporating new additional experimental data.

**Nomenclatures**

- $B_{ob}$: Oil Formation Volume Factor (rb/STB)
- $P_b$: Bubble point pressure (psia)
- $R_s$: Solution gas-oil ratio (scf/STB)
- $T$: Reservoir Temperature (°F)
- $\gamma_g$: Gas Gravity (dimensionless)
- $\gamma_o$: Oil Gravity (dimensionless)
- ANN: Artificial Neural Network
- BPNN: Back Propagation Neural Network
- GRNN: Generalized Regression Neural Network
References


Amendment - Figures

Figure 2 \( P_b \) Average Absolute Percent Error

Figure 3 \( P_b \) Correlation Coefficient

Figure 4 \( B_{ob} \) Average Absolute Percent Error

Figure 5 \( B_{ob} \) Correlation Coefficient

Figure 6 Temperature Trend on Bubble Point Pressure

Figure 7 Solution Gas-Oil Ratio Trend on Bubble Point Pressure

Figure 8 Oil Gravity Trend on Bubble Point Pressure

Figure 9 Gas Gravity Trend on Bubble Point Pressure
Figure 18 Cross Plot of Al-Marhoun (1988) correlation for $P_b$

Figure 19 Cross Plot of Petrosky Correlation for $P_b$

Figure 20 Cross Plot of GRNN model for $B_{ob}$

Figure 21 Cross Plot of BPNN model for $B_{ob}$

Figure 22 Cross Plot of Standing Correlation for $B_{ob}$

Figure 23 Cross Plot of Glaso Correlation for $B_{ob}$

Figure 24 Cross Plot of Al-Marhoun (1988) correlation for $B_{ob}$

Figure 25 Cross Plot of Petrosky Correlation for $B_{ob}$