

## IRREDUCIBLE WATER SATURATION AND POROSITY MATHEMATICAL MODELS FOR KWALE SANDS, NIGER DELTA

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### Abstract

Irreducible water saturation value is an indication of total volume of oil and water producible from a reservoir. It also influences the production rate from that reservoir. The irreducible water saturation for some Kwale sands were analysed in this work using Timur, Tixier and Coates-Dumanoir models. It was discovered that the Timur and Tixier models gave reasonable irreducible water saturation values with the Timur values being closer to the water saturation values and hence of more accurate values. The Coates-Dunamior model gave almost 100%  $S_{wirr}$  values but this is unrealistic value for any oil reservoir, except for rocks in water zone with initial water saturation of almost 100%. Moreover, porosity for the different sands were analyzed to obtain a relationship such that porosity is function of water saturation only for each sand. It was also observed that porosity-water saturation relationship is a polynomial equation of degree 2 or 3 for the Kwale sands. Permeability was also analyzed as a function of porosity. For Kwale sand C that has average porosity closest to the most common porosity in Kwale reservoir, the model equations obtained are  $\phi = 0.0054S_w^2 - 0.6538S_w + 36.331$  for porosity-water saturation;  $K=95000\phi^2-34100\phi+3058$  for permeability-porosity relationship and  $S_{wirr}=-446.5\phi^2+162.4\phi-14.09$  for the irreducible water saturation- porosity relationship.

**Keywords:** Porosity model; Kwale sandstone; irreducible water saturation; porosity variation; water saturation, permeability model.

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### 1. Introduction

The profitability of a reservoir is directly related to the amount of hydrocarbon that is producible from the reservoir and how fast and at what cost of production. The irreducible water saturation determines the maximum water producible with oil from a reservoir and influenced the process and cost of separation, or treatment, required for the production from a well. The higher the irreducible water saturation, the higher the relative permeability to oil and the higher the production rate of oil from a reservoir. Hence, this work computes the irreducible water saturation for some Kwale, Niger-Delta reservoir based on the log analysis of permeability, porosity and water saturation carried out by Ekine and Iyabe [1].

Ezatollah *et al.* [2] estimated hydraulic flow units required in the estimation of irreducible water saturation using capillary pressure data and hydraulic flow units. They used measured porosity and permeability data on the samples of a carbonate reservoir. Xiao *et al.* [3] combined Timur and Schlumberger—Doll Research (SDR) models to obtain a new model for calculating  $S_{wirr}$  using 36 core samples from Xujiahe Formation in Bao-jie region of Triassic, Sichuan basin, southwest China. They stated that this new method has the advantage of obtaining all required data for  $S_{wirr}$  from NMR logs accurately.

Timur [4] suggested that relationship for estimating permeability of sandstones from in situ measurements of porosity and also residual fluid saturation obtained from nuclear magnetism log will eliminate the expensive coring process. He stated that the porosity and the saturations can be obtained from logging while drilling and also from drill cuttings. During the research the measurements of permeability (K) was obtained in the laboratory while the porosity ( $\Phi$ ), and residual (irreducible) water saturation ( $S_{wirr}$ ) were also determined using 155 sandstone samples from three different oil fields in North America. He obtained an empirical equation to calculate the permeabilities and also obtained another



The irreducible water saturation,  $S_{wirr}$ , can be computed from the above models (equations 1 to 3) such that:

$$\text{Timur } S_{wirr} = \sqrt{\frac{0.135\phi^{4.4}}{K}} \tag{5}$$

$$\text{Tixier } S_{wirr} = 250 \cdot \frac{\phi^3}{K^{1/2}} \tag{6}$$

$$\text{Coates-Dumanoir } S_{wirr} = \frac{100 \cdot \phi^2}{K^{1/2} + 100 \cdot \phi^2} \tag{7}$$

For Kwale sands, available permeability, porosity and water saturation data was applied to compute the irreducible water saturation as proposed in equations 5-7 above using the three models of Timur, Tixier and Coates-Dumanoir.

### 2. Methodology

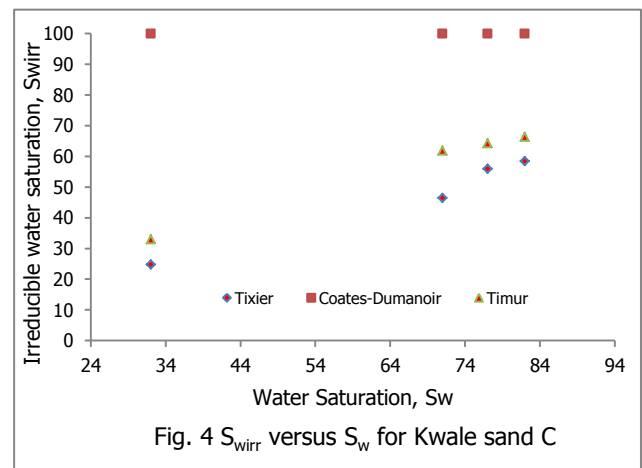
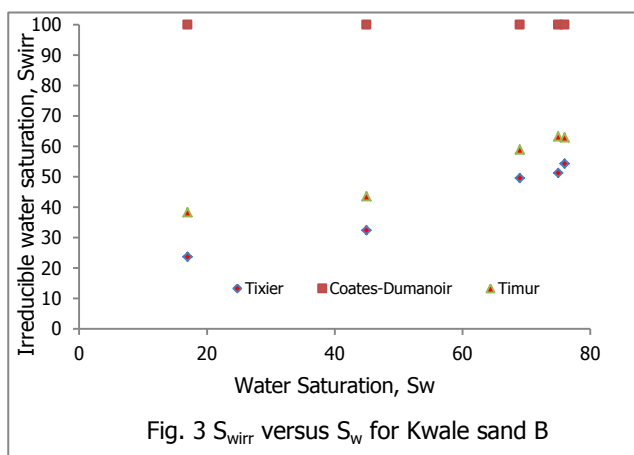
This paper utilized the published log analysis data of Ekine and Iyabe [1] for Kwale sand to analyze irreducible water saturations for the sand using established permeability models of Timur, Tixier and Coates-Dumanoir. This irreducible water saturation is a measure of total producible water from a reservoir and is an indication of the extent of production facility required to separate hydrocarbon and water production from the reservoir. Moreover, it is also an indirect measurement of maximum recoverable hydrocarbon from a reservoir. The data was also utilized to obtain equations of porosity as a function of water saturation only for the different sands.

The Porosity-Water Saturation Plots For Kwale Sands is presented with the appropriate equation in Appendix B. The lines of best-fit were also determined. In Appendix C, the models of Timur, Tixier and Coates-Dumanoir were used to compute the irreducible water saturation for the Kwale sands.

Irreducible Water saturation,  $S_{wirr}$  – porosity analysis was carried out for Sand C in Appendix D. This was because Sand C has an average porosity nearest to the most common average value for the whole of Kwale reservoir and is therefore used for the  $S_{wirr}$  – porosity model.

### 3. Results

The computed irreducible water saturation for the Kwale sands is as stated in the figures 3-13 below. While the results for the porosity-water saturation, irreducible water saturations, irreducible water saturation versus porosity for sand C are stated in Appendix B, C and D respectively.



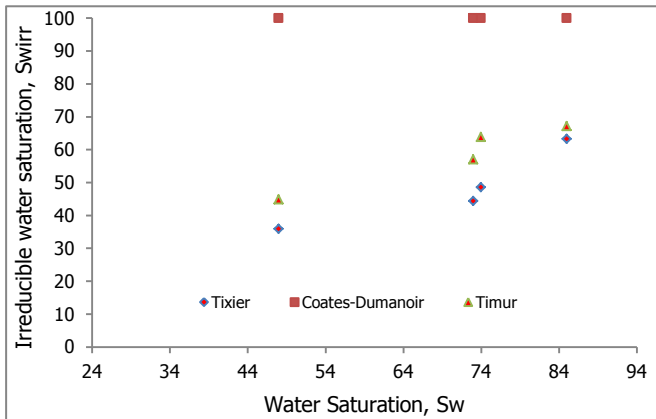


Fig. 5  $S_{wirr}$  versus  $S_w$  for Kwale sand D

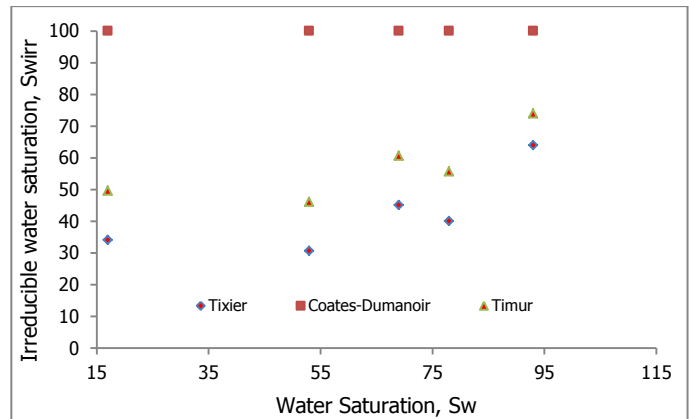


Fig. 6  $S_{wirr}$  versus  $S_w$  for Kwale sand F

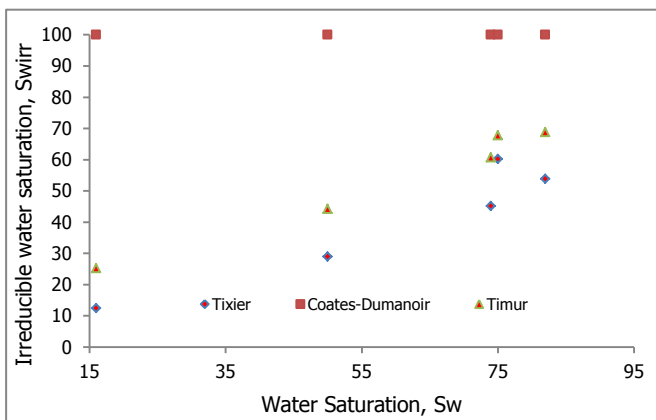


Fig. 7  $S_{wirr}$  versus  $S_w$  for Kwale sand G

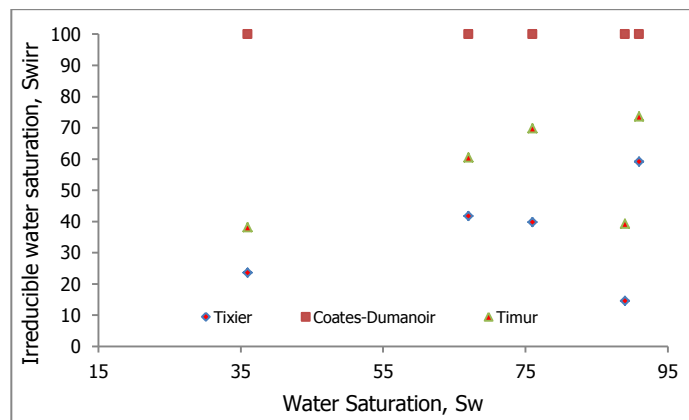


Fig. 8  $S_{wirr}$  versus  $S_w$  for Kwale sand H

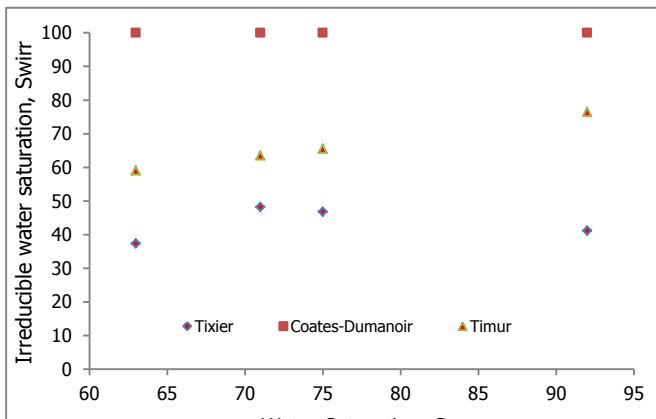


Fig. 9  $S_{wirr}$  versus  $S_w$  for Kwale sand I

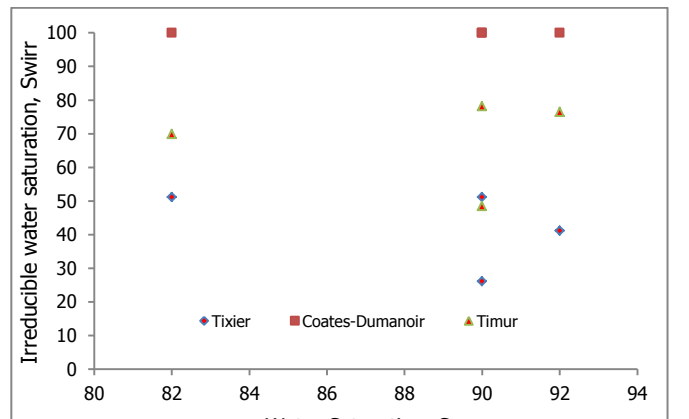


Fig. 10  $S_{wirr}$  versus  $S_w$  for Kwale sand J

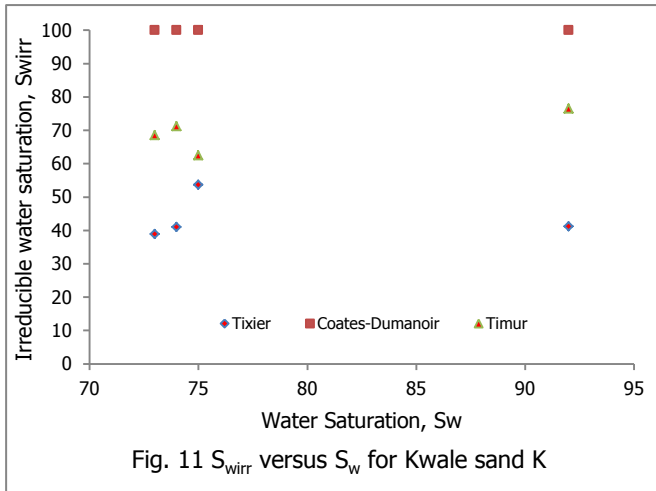


Fig. 11  $S_{wirr}$  versus  $S_w$  for Kwale sand K

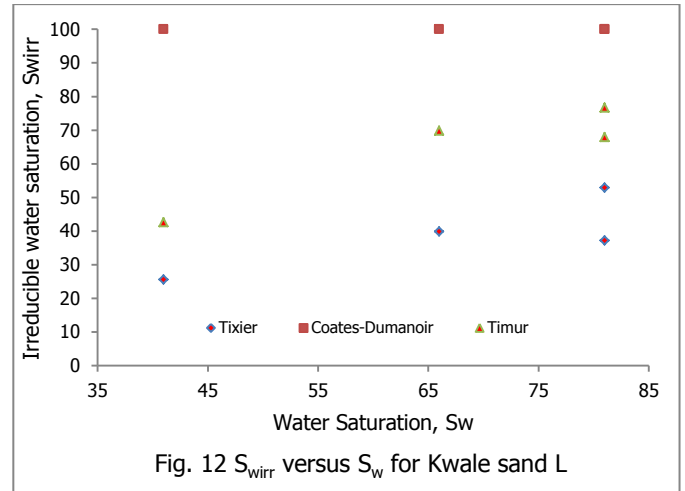


Fig. 12  $S_{wirr}$  versus  $S_w$  for Kwale sand L

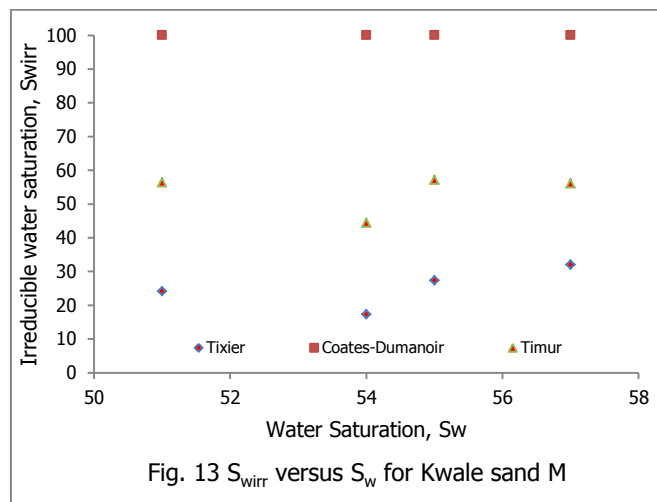


Fig. 13  $S_{wirr}$  versus  $S_w$  for Kwale sand M

Analysis of the available data for some well in Kwale field shows that Kwale Sands have unique relationship between the porosity and water saturations. Model equations were proposed from the analysis of the available measured data and was based on the porosity-water saturation plots of appendix B as stated below.

For Kwale Sand B, the porosity variation with water saturation is best described by a proposed model equation

$$\phi = -0.0015S_w^2 + 0.1638S_w + 14.643 \tag{8}$$

For Kwale sand C, the porosity reduces with increasing water saturation until a minimum at  $S_w$  of 0.6 and thereafter increases. Proposed model equation is as follows

$$\phi = 0.0054S_w^2 - 0.6538S_w + 36.331 \tag{9}$$

For Kwale sand D, the analysis shows that the porosity reduces with increasing water saturation until a minimum at  $S_w$  of 0.7 and thereafter increases

$$\phi = 0.0002S_w^3 - 0.0268S_w^2 + 1.2395S_w + 3.3739 \tag{10}$$

For Kwale sand F, porosity increases gradually with water saturation. The proposed model equation is as follows:

$$\phi = 0.0004S_w^2 - 0.0382S_w + 17.566 \tag{11}$$

For Kwale sand G, porosity also increases gradually with water saturation. The proposed model equation is as follows:

$$\phi = -0.0004S_w^2 + 0.069S_w + 14.953 \tag{12}$$

Kwale Sand H has porosity reducing sharply with increasing water saturation until a minimum at  $S_w=0.8$  after which it increases again. The proposed model equation is:

$$\phi = 0.0008S_w^3 - 0.1501S_w^2 + 9.1334S_w - 153.25 \tag{13}$$

Kwale Sand I has high water saturation and the porosity was observed to increase with water saturation between  $S_w$  of 0.6 and 0.7 and then drops. The proposed model equation is

$$\phi = -0.0129S_w^2 + 1.8974S_w - 53.012 \quad (14)$$

Like Sand I, Kwale Sand J has high water saturation but the porosity was observed to decrease sharply with water saturation and the model equation for the Sand is

$$\phi = -0.075S_w^2 + 12.65S_w - 517 \quad (15)$$

Kwale Sand K has almost constant porosity of 13% for water saturation range except an observed abnormality at  $S_w$  of 0.75. The sand was observed to have very high  $S_w$  with a minimum of 75%. The proposed model equation is

$$\phi = -0.0551S_w + 17.076 \quad (16)$$

where  $\phi$  is porosity and  $S_w$  is water saturation.

#### 4. Conclusion

It was observed that irreducible water saturation increases with increasing water saturation. Timur and Tixier models gave  $S_{wirr}$  values that are reasonable when compared with the water saturation values but Coates-Dumanoir model gave a practically impossible  $S_{wirr}$  values. Timur and Tixier models are therefore applicable in computation of  $S_{wirr}$  or permeability for Kwale sands.

It was also observed that porosity is of second degree polynomial in  $S_w$  for B, C, F, G, I and J; third degree polynomial in  $S_w$  for sands D and H; and a linear relationship for sand K.

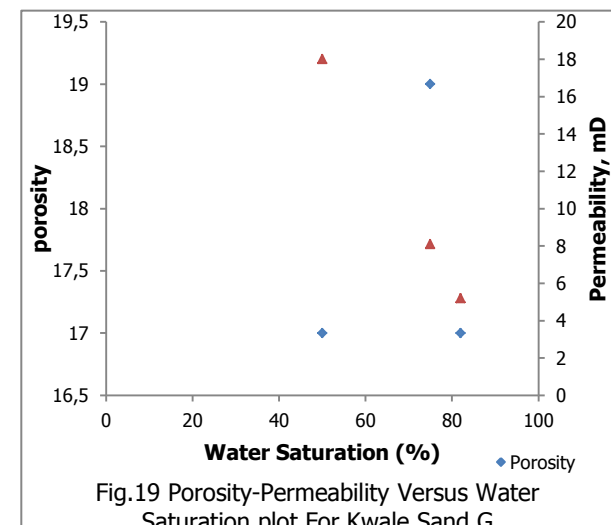
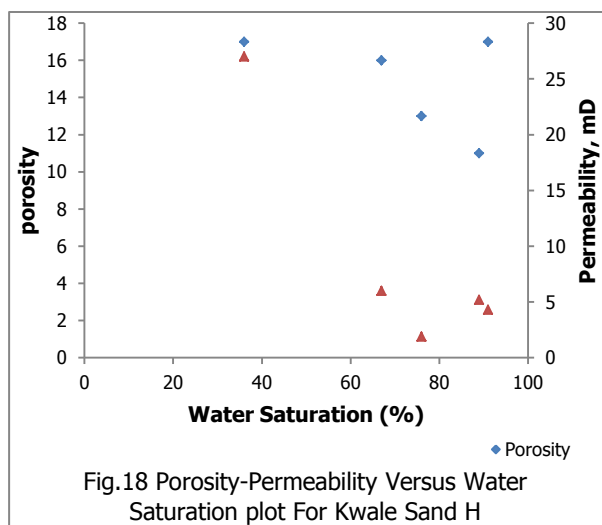
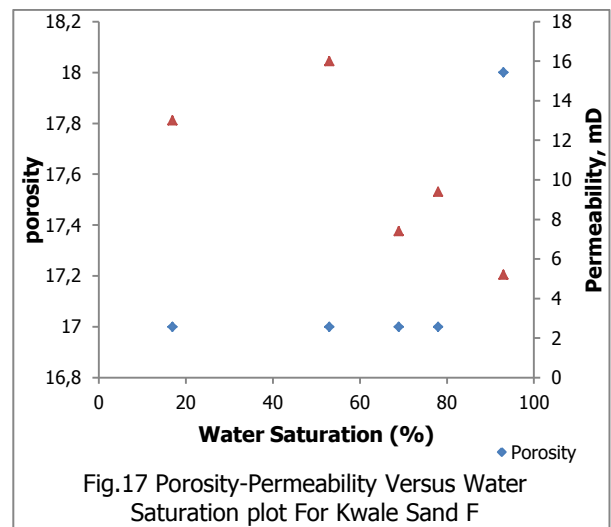
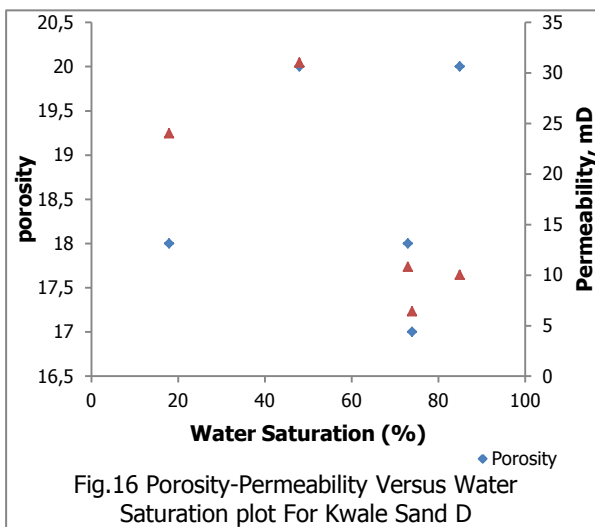
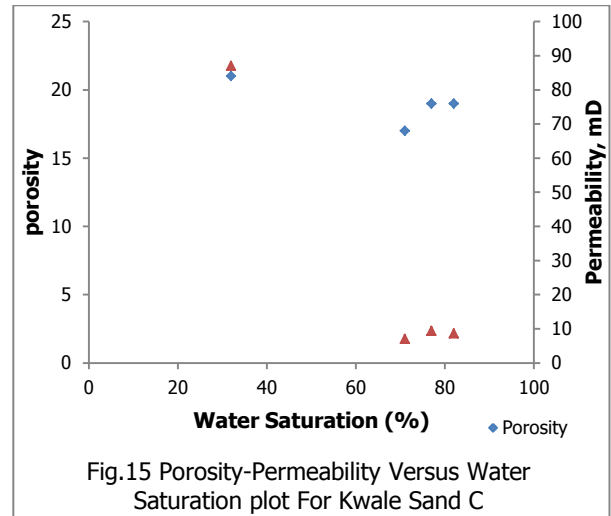
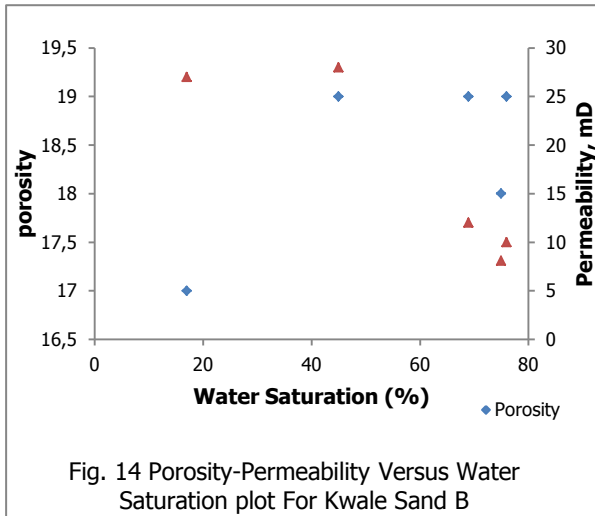
Timur model gave higher values than the Tixier model and the model to use will then be dependent on the nature of the wetting phase in the rock. For a water-wet rock,  $S_{wirr}$  will be higher than for an oil-wet rock since much of the water will be immobile due to its preferential sticking to the reservoir rock grains and Timur model may then be a better estimate while for oleophilic rock, the Tixier model will be the preferred model.

#### References

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- [4] Timur A. An Investigation Of Permeability, Porosity And Residual Water Saturation Relationships For Sandstone Reservoirs. *The Log Analyst* 1968; 9(4): 3 – 5.
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- [6] Coates GR, Dumanoir JL. A New Approach To Improve Log-derived Permeability. 1973, Proceedings by SPWLA, Lafayette, 16<sup>th</sup> Annual Logging Symposium Paper R.
- [7] Tixier MP. Evaluation of permeability from electric log resistivity gradient. *Earth Sci. J.* 1949; 2: 113-113.

### Appendix A

The Porosity-Permeability-Water saturation relationship For Kwale Sands is presented in this section and this is based on the published measured data of some sands in Kwale reservoir (Ekine and Iyabe [1]).



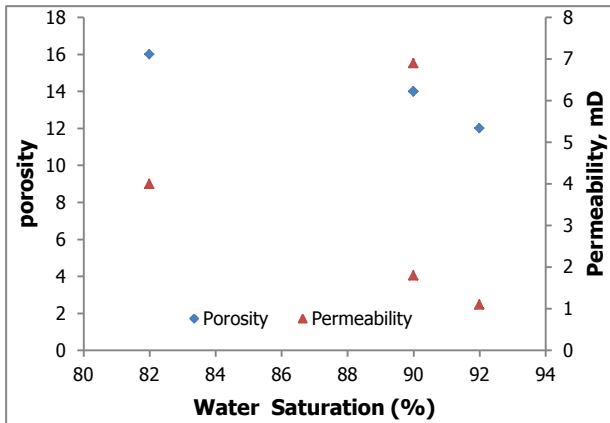


Fig.20 Porosity-Permeability Versus Water Saturation plot For Kwale Sand J

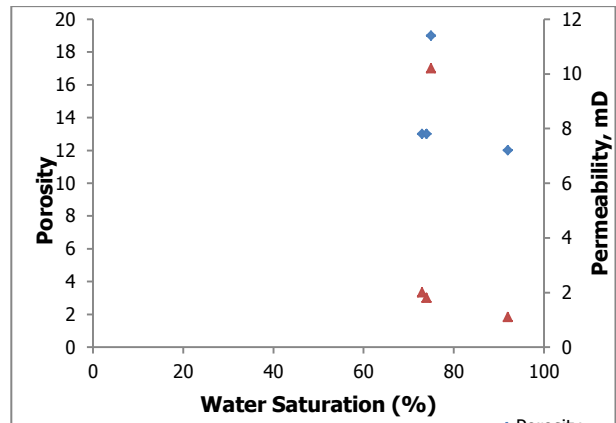


Fig.21 Porosity-Permeability Versus Water Saturation plot For Kwale Sand K

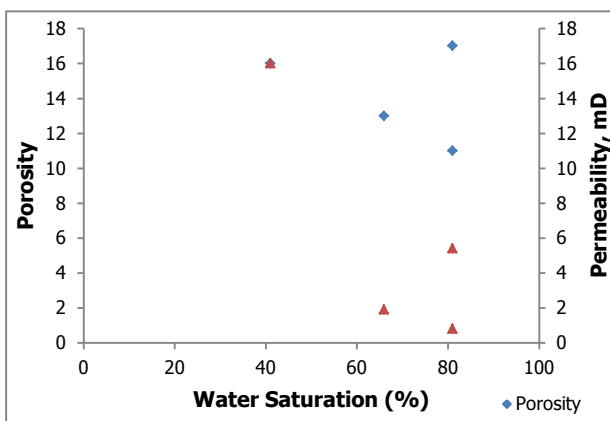


Fig.22 Porosity-Permeability Versus Water Saturation plot For Kwale Sand L

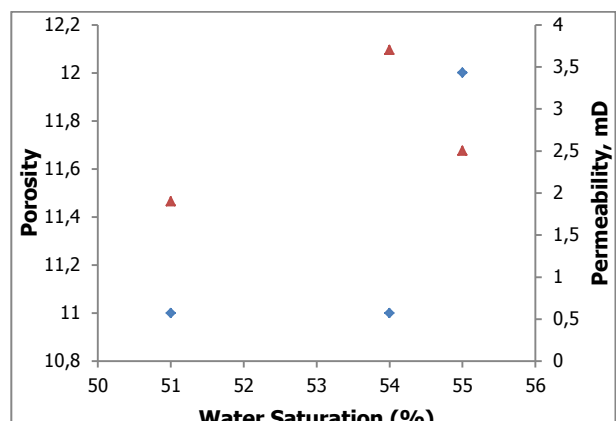


Fig.23 Porosity-Permeability Versus Water Saturation plot For Kwale Sand M

## Appendix B

### Porosity-Water Saturation Models For Kwale Sands

The Porosity-Water Saturation Plots For Kwale Sands is presented in this section with the appropriate equation. The lines of best-fit were also determined.

The best-fit curves for the porosity-water saturation for the Kwale sands are as stated in Figs. 24-34. This was done for only Kwale sands with sufficient data, the relationship between the porosity and permeability is established graphically below with the equation that best described the relationship on each plot.

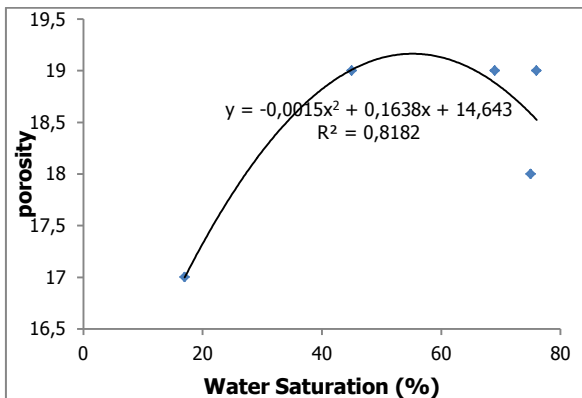


Fig.24 Best-fit Polynomial Curve For Porosity Variation With Water Saturation For Kwale Sand B

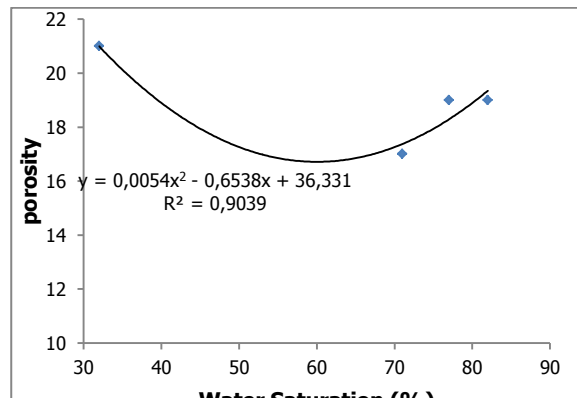
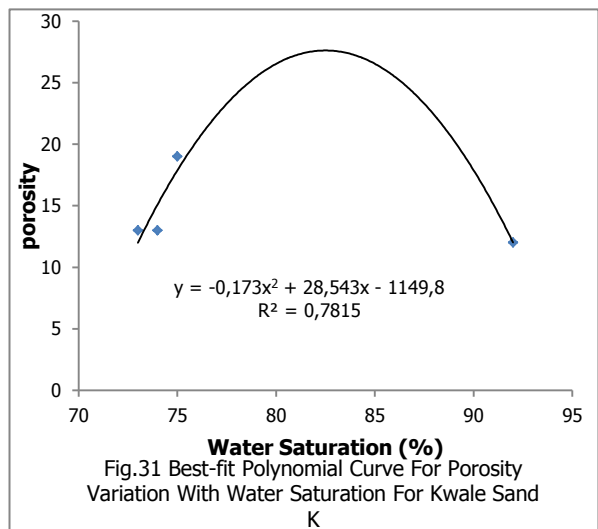
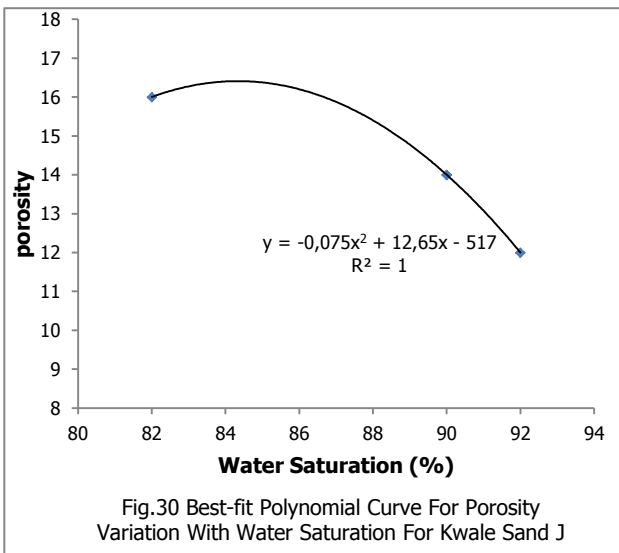
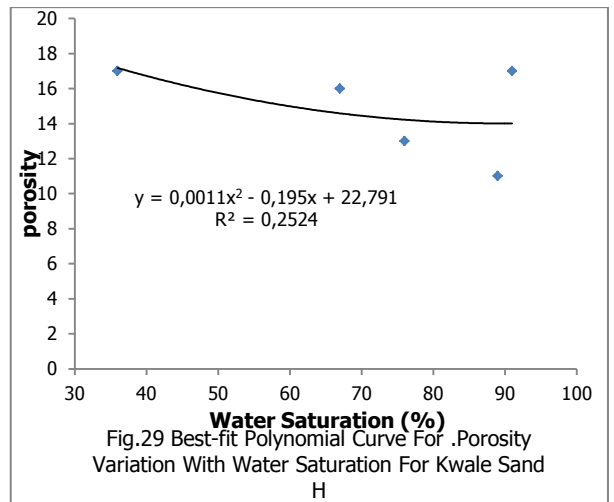
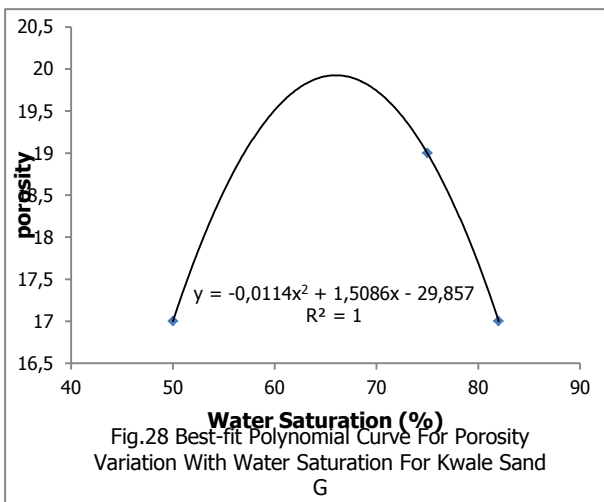
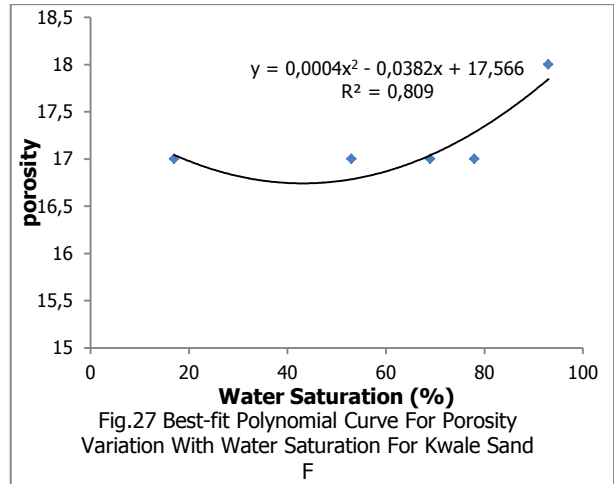
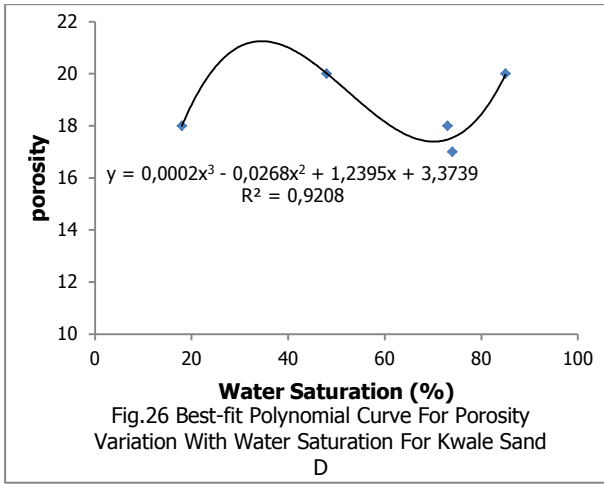
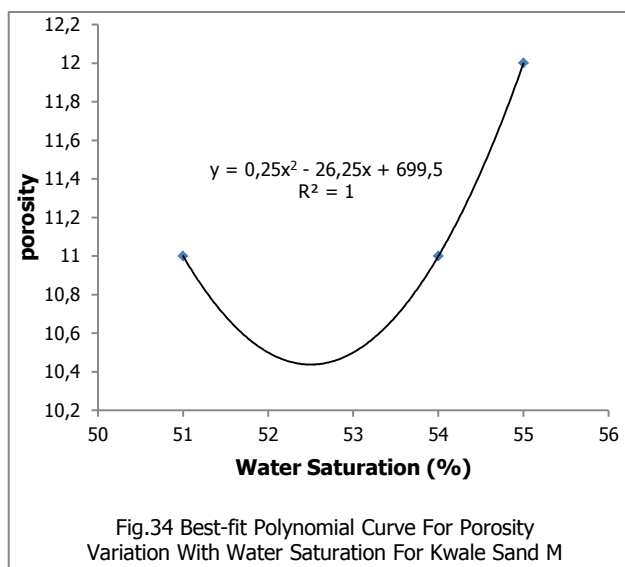
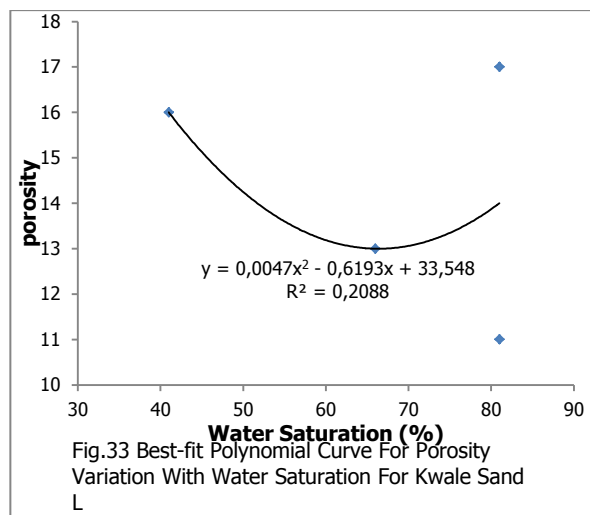
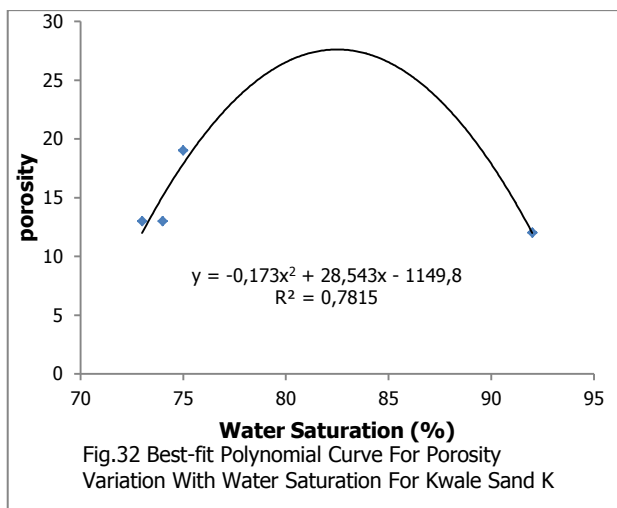


Fig.25 Best-fit Polynomial Curve For Porosity Variation With Water Saturation For Kwale Sand C







### Appendix C

#### Irreducible Water Saturation Computation For Kwale Sands Using Timur, Tixier and Coates-Dumanoir Models

In this section, the models of Timur, Tixier and Coates-Dumanoir were used to compute the irreducible water saturation for the Kwale sands. From Tables 1 to 9 below, it was discovered that Timur and Tixier gave reasonable irreducible water saturation values for a hydrocarbon-water reservoir for all the sands. Coates-Dumanoir gave impracticable irreducible water saturation. With consideration to the values of measured water saturation, on the average, Timur gave more acceptable irreducible water saturation than that of Tixier. Timur model is therefore adopted for the irreducible water saturation computation for the Kwale reservoir.

Table 1 Computed Irreducible Water Saturation For Kwale For Kwale Sand A

Permeability	Porosity	Irreducible Water Saturation			Sw
		Timur Model	Tixier Model	Coates - Dumanior Model	
16	21	60.4642	57.8813	99.9909	74
37	17	34.1207	20.1923	99.979	17

Table 2 Computed Irreducible Water Saturation For Kwale Sand B

Permeability	Porosity	Irreducible Water Saturation			Sw
		Timur Model	Tixier Model	Coates - Dumanior Model	
8.1	18	63.2306	51.2289	99.9912	75
10	19	62.8701	54.2252	99.9912	76
12	19	58.9217	49.5006	99.9904	69
27	17	38.2154	23.6377	99.982	17
8.1	18	63.2306	51.2289	99.9912	75

Table 9 Computed Irreducible Water Saturation For Kwale Sand J

Permeability	Porosity	Irreducible Water Saturation			Sw
		Timur Model	Tixier Model	Coates - Dumanior Model	
4	16	69.8824	51.2	99.9922	82
6.9	14	48.4985	26.1156	99.9866	90
1.8	14	78.2217	51.1314	99.9932	90
1.1	12	76.5084	41.1896	99.9927	92

The petro-physical geometric average values for the Kwale sand is calculated as follows.

Table 10 Calculated Geometric Average Properties For Kwale Sands

Sand	Porosity $\emptyset$	Permeability K	Water Saturation	Timur Swirr
B	18.270	16.900	57.054	53.538
C	20.429	30.105	64.474	51.283
D	18.920	16.723	59.742	58.121
F	17.102	10.142	62.360	59.760
G	16.515	20.490	60.267	38.932
H	15.991	9.595	70.243	53.001
J	14.420	3.554	88.143	69.371
K	16.980	4.498	77.895	88.332
L	15.822	6.689	66.351	62.004
M	11.309	2.694	53.382	46.673

## Appendix D

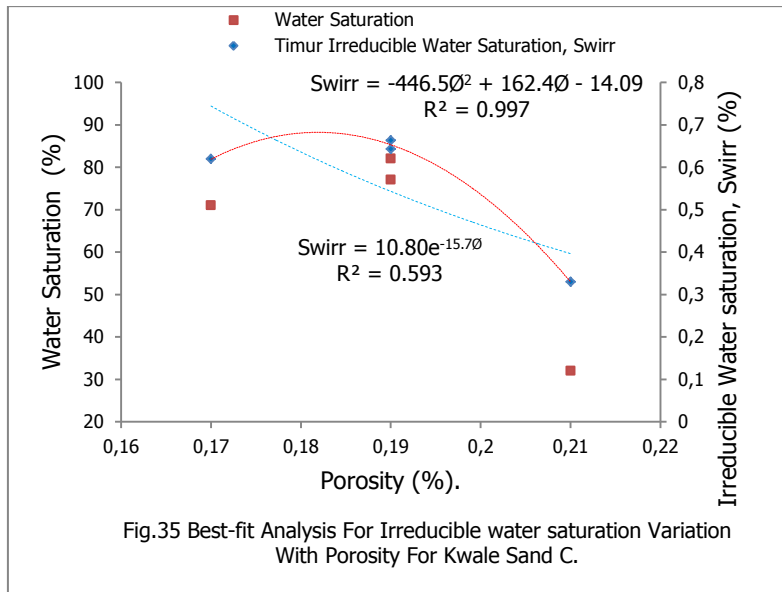
### Irreducible water saturation – porosity analysis for Sand C

Sand C gave average porosity nearest to the most common average value for the Kwale reservoir and is therefore used for the analysis of  $S_{wirr}$  – porosity model.

Table B8 Kwale Sand C Permeability, Porosity &amp; Irreducible water Saturation

Water Saturation, $S_w$	Permeability K	Porosity $\emptyset$	Timur Irreducible Water Saturation, $S_{wirr}$
32	87	0.21	0.32981
82	8.6	0.19	0.66332
77	9.4	0.19	0.64268
71	7	0.17	0.619

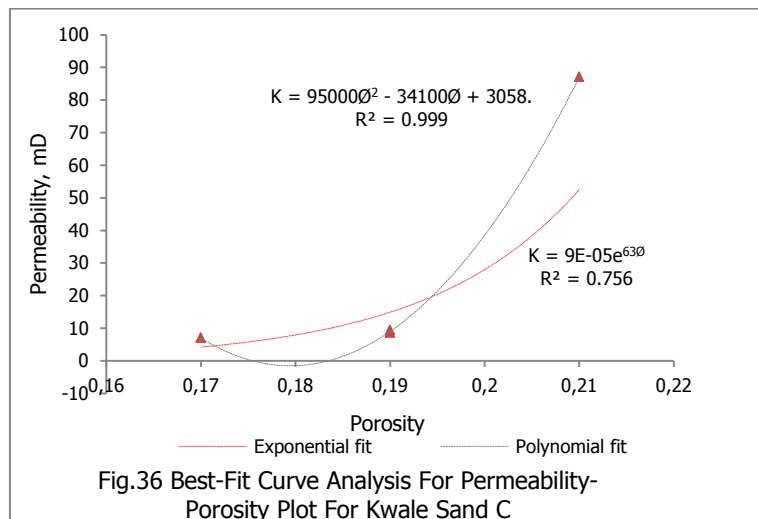
The best-fit curve and equation for the Kwale sand C is as shown in Fig.35.



From above figure, the best irreducible water saturation value is obtained with the equation

$$S_{wirr} = -446.5\phi^2 + 162.4\phi - 14.09$$

The best two trend-line fit is the exponential and polynomial fits. But from the above, exponential fit is more applicable practically, though it gave less root mean square than that of polynomial fit, but it shows the expected decrease in irreducible water saturation as porosity increases in non- producing storage reservoir. Volume of water is expected to either remain constant or reduce while the volume of injected fluid increases in that reservoir. This will result into lower water saturation and corresponding lower irreducible water saturation with increasing formation of secondary porosity.



From above, the best fit based on the root mean square is the polynomial relationship given as

$$K = 95000\phi^2 - 34100\phi + 3058$$

where K = permeability (mD) and  $\phi$  = porosity in fraction.