

A Modified Method for Predicting Relative Permeability

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Received January 24, 2020; Accepted February 26, 2020

Abstract

Evaluating the relative permeability data is very essential as all the reservoirs contain multiple fluid phases. In absence of lab measured data, many correlations were developed to capture an accurate formula for relative permeability data prediction. The objective of this work focuses on how to calculate the value of exponents incorporated into generalized Corey's correlation instead of using pre-assumed fixed values and use the estimated values of exponents to predict relative permeability data. A giant database of experimental results for 750 plugs, covering different types of reservoir rocks and fluid systems, was involved in the methodology development to test its validity and reliability. Relative permeability data prediction was performed for 750 plugs using the generalized Corey's correlation and the proposed methodology to estimate exponents. Predicted relative permeability data were compared to the collected actual experimental results and the prediction results of other common published correlations through extensive statistical analysis. Statistical analysis showed that the proposed methodology has significant reliability to predict relative permeability data.

Keywords: Relative permeability; Corey's correlation; Exponents; Relative permeability curve end points.

1. Introduction

Various laboratory studies provided that the effective permeability for all reservoir phases relies greatly on phase saturation and reservoir wettability for that phase. The effective permeability value used into Darcy's equation depends on relative permeability value for the flowing fluids. Relative permeability is a giant effective controlling factor in history matching process for reservoir simulation and production optimization. Enhanced oil recovery methods use the relative permeability data to monitor fluid advance and movement while saturation change during flooding like for breakthrough time prediction. From relative permeability curve, we can know the residual oil saturation which affect the oil recovery factor and hence reservoir economic feasibility. In the absence of lab measured data, many correlations were developed using regression techniques to capture an accurate formula for relative permeability data prediction.

2. Literature review

Most of the proposed correlations to predict relative permeability are exploiting the effective fluid saturation as input and an exponent that determines the equation nonlinearity.

The following set of relationships defines the effective phase saturation [\[1\]](#):

$$S_o^* = \frac{S_o}{1 - S_{wc}} \quad (1)$$

$$S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc}} \quad (2)$$

$$S_g^* = \frac{S_g}{1 - S_{wc}} \quad (3)$$

2.1. Common previous published relative permeability correlations [1]

Corey provided a clear generalized mathematical model for relative permeability data generation of the gas-oil and water-oil systems [1]:

$$k_{ro} = (k_{ro})_{Swc} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^n \quad k_{rw}(k_{rw})_{Sorw} \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^m \quad (4)$$

Wyllie and Gardner [1] built his model upon his observation that, in some reservoir rocks, the correlation between the reciprocal of capillary pressure squared ($1/P_c^2$) and the water effective saturation S_w^* is linear for a high range of saturation. Torcaso and Wyllie [1] proposed a simple expression to calculate k_{ro} in a gas-oil system. The proposed model allows k_{ro} calculation from k_{rg} measurements since k_{rg} lab determination experiments are easily performed, but k_{ro} determination experiments are usually made with some difficulty. Having some petrophysical considerations, Pirson [1] made some generalized relationships for calculating wetting and nonwetting phase relative permeability for both drainage and imbibition processes. Hornarpour and Koederitz [2] suggest correlations for two sets of rock samples with varying wettabilities: Sandstones, Limestones and dolomites through stepwise linear regression analysis that would mimic the relative permeability lab measurements using some inputs as phases saturations, absolute permeability, and porosity. In 2001, Ibrahim and Koederitz [3] exploited performed linear regression analysis to develop their relative permeability prediction equations. according to reservoir rock lithology and wettability.

2.2. Comments on previous published relative permeability correlations

It was noticed that nearly all previously published relative permeability prediction correlations assume a fixed number for the exponent controlling the relation between effective fluid saturation and the values of fluids relative permeability ignoring effect of relative permeability end points on curve shape. From author’s point of view, a giant discrepancy between the predicted and the actual measured in many cases can be due to that the correlation fixed value parameters invented through matching trials to certain group of samples and it is not mandatory that it can be applied for all samples with the same lithology and wettability.

3. Objective

The objective of this work is to find a formula that can predict relative permeability data precisely through a modification applied to generalized Corey’s correlation. The modification focuses on how to calculate the value of exponents incorporated into generalized Corey’s correlation “m&n” instead of using pre-assumed fixed values.

4. Methodology hypothesis

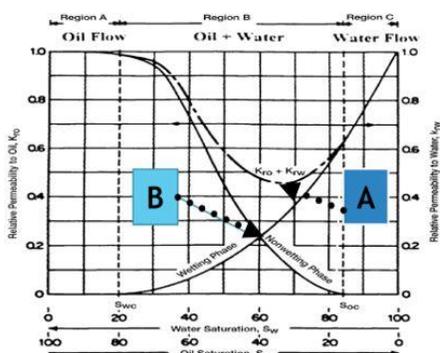


Figure 1. Typical two-phase relative permeability curves [1]

Figure 1 represents a typical two-phase flow behavior for water-oil system. It was noticed the following points:

1. For the curve representing summation of oil and water relative permeability, the curve is minimized at point (A)
2. The two curves of oil and water relative permeability are intersecting at point (B) which means that the relative permeability of the two phases are equal at this point.

Methodology hypothesis is that we can assume the water saturation at point (A) is the same as the saturation at point of intersection (B) as the difference@ X-axis (S_w) is low. This

hypothesis is just proposed for simplification and approximation but it is not strictly correct in many cases.

From the previous hypothesis, we can have two conclusions at the point of relative permeability curves intersection:

1. Oil relative permeability = Water relative permeability

Using the mathematical expression of generalized Corey's correlation for relative permeability:

$$(k_{ro})_{S_{wc}} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{n_o} = (k_{rw})_{S_{orw}} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{n_w} \quad (5)$$

2. The first and second partial differentiation for $(K_{ro} + K_{rw})$ with respect to (S_w) are equal to zero as it is "minimum value" for the curve.

The first partial differentiation:

$$\frac{\partial(k_{ro} + k_{rw})}{\partial S_w} = 0 \quad (6)$$

The second partial differentiation:

$$\frac{\partial^2(k_{ro} + k_{rw})}{\partial S_w^2} = 0 \quad (7)$$

5. Methodology mathematical derivation

Using the second conclusion (equ 6) for the first partial differentiation :

$$\frac{\partial(k_{ro} + k_{rw})}{\partial S_w} = 0$$

Substituting by mathematical expression of Corey's (equ 4) into (equ 6):

$$\begin{aligned} (k_{ro})_{S_{wc}} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{n_o - 1} * \left[\frac{-1}{1 - S_{wc} - S_{orw}} \right] * n_o \\ + \\ (k_{rw})_{S_{orw}} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{n_w - 1} * \left[\frac{1}{1 - S_{wc} - S_{orw}} \right] * n_w = 0 \end{aligned} \quad (8)$$

Using the first conclusion (equ 5): Oil relative permeability = Water relative permeability
By elimination the equal terms into (equ 8):

$$\left[\frac{1}{1 - S_w - S_{orw}} \right] * n_o = \left[\frac{1}{S_w - S_{wc}} \right] * n_w \quad (9)$$

By readjusting terms into (equ 9):

$$\left[\frac{S_w - S_{wc}}{1 - S_w - S_{orw}} \right] = \frac{n_w}{n_o} \quad (10)$$

For simplifying we can assume that:

$$\frac{n_w}{n_o} = T \quad (11)$$

Substituting by (equ 11) into (equ 10):

$$\left[\frac{S_w - S_{wc}}{1 - S_w - S_{orw}} \right] = \frac{n_w}{n_o} = T \quad (12)$$

Using the second conclusion for the second partial differentiation (equ 7):

$$\frac{\partial^2(k_{ro} + k_{rw})}{\partial S_w^2} = 0$$

Substituting by mathematical expression of Corey's correlation into (equ 7):

$$\begin{aligned} (k_{ro})_{S_{wc}} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{n_o - 2} * \left[\frac{1}{1 - S_{wc} - S_{orw}} \right]^2 * n_o * (n_o - 1) + (k_{rw})_{S_{orw}} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{n_w - 2} * \\ \left[\frac{1}{1 - S_{wc} - S_{orw}} \right]^2 * n_w * (n_w - 1) = 0 \end{aligned} \quad (12)$$

By elimination the equal terms into (equ 12):

$$\left[\frac{S_w - S_{wc}}{1 - S_w - S_{orw}} \right]^2 = \frac{n_o * (n_o - 1)}{n_w * (n_w - 1)} \quad (13)$$

But we have from (equ 10):

$$\left[\frac{S_w - S_{wc}}{1 - S_w - S_{orw}} \right] = \frac{n_w}{n_o}$$

By elimination the equal terms into (equ 13):

$$\left[\frac{S_w - S_{wc}}{1 - S_w - S_{orw}} \right] = \frac{(n_o - 1)}{(1 - n_w)} \tag{14}$$

But we have from (equ 11)

$$\left[\frac{S_w - S_{wc}}{1 - S_w - S_{orw}} \right] = \frac{n_w}{n_o} = T; n_w = T * n_o$$

By substitution into (equ 14) eliminate the equal terms:

$$\frac{(1 - T * n_o)}{(n_o - 1)} = T$$

By readjusting terms and substituting

$$n_o = \frac{(1 + T)}{2 * T} \quad n_w = \frac{(1 + T)}{2}$$

Using regression analysis available in Excel we can know value of (S_w) at intersection point that minimizes summation oil and water relative permeability

$$T = \left[\frac{S_w - S_{wc}}{1 - S_w - S_{orw}} \right]$$

$$k_{ro} + k_{rw} = (k_{ro})_{swc} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{\frac{(1+T)}{2 * T}} + (k_{rw})_{sorw} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{\frac{(1+T)}{2}}$$

By having the value of T-factor we can calculate the exponents' value and use it into calculating relative permeability values

$$n_o = \frac{(1+T)}{2 * T}; n_w = \frac{(1+T)}{2}$$

For water-oil system:

$$k_{ro} = (k_{ro})_{swc} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{\frac{(1+T)}{2 * T}} \quad k_{rw} = (k_{rw})_{sorw} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{\frac{(1+T)}{2}}$$

It is important to mention that the same derivation steps is valid for other fluid systems to get the exponents' value and use it into calculating relative permeability values.

6. Model tuning

Methodology hypothesis is that we can assume the water saturation at point (A) is the same as the saturation at point of intersection (B) as the difference@ X-axis (S_w) is low. The difference of saturation for the two points can be negligible in some cases or significant in other cases depending on system wettability Figure 1. The proposed Adjusting technique to overcome this shortage was to multiply the exponents by a (lithology/wettability) dependent factor that gets best prediction for our collected K_r dataset.

$$k_{ro} = (k_{ro})_{swc} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{A * \frac{(1+T)}{2 * T}} \quad k_{rw} = (k_{rw})_{sorw} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{B * \frac{(1+T)}{2}}$$

The adjusting factors values are chosen to achieve best match to collected actual data. Modified models with adjusting factors for different reservoir systems are found into Appendix A.

7. Database building

A giant database of experimental results for 750 plugs was involved in the methodology development to test its validity and reliability. Data was collected from available SCAL reports on the internet, SPE papers published until 2017 and reports from some petroleum companies handed without locating data. Nearly 80% of results were used for modelling and 20 % were used for model testing, validation and comparison to other common correlations. In case of water-oil system, data is sorted according to wettability using Table 1. „modified craig’s rule” [4].

In case of gas systems, it is assumed that gas is always non-wetting phase Table 2 shows the collected data after sorting and the number of results used for modelling or validation.

Table 1. Modified Craig’s wettability rule

Rock Wettability	S_{wc}	S_w at which k_{rw}^* and k_{row}^* are equal	k_{rw}^* at $S_w = 100 - S_{orw}$ (fraction)
Strongly Water-Wet:	$\geq 15\%$	$\geq 45\%$	≤ 0.07
Water-Wet:	$\geq 10\%$	$\geq 45\%$	$0.07 < k_{rw}^* \leq 0.3$
Oil-Wet:	$\leq 15\%$	$\leq 55\%$	≥ 0.5
Intermediate: (Mixed-Wet)	$\geq 10\%$	$45\% \leq S_w \leq 55\%$	> 0.3
	$\leq 15\%$	OR $45\% \leq S_w \leq 55\%$	< 0.5

Table 2. Collected data after sorting

System	Lithology	Wettability	Total no.	Modelling data	Validation Data
Water & Oil	Sandstone	Water wet	300.00	250.00	50.00
	Limestone	Water wet	35.00	28.00	7.00
	Dolomite	Water wet	17.00	14.00	3.00
	Sandstone	Oil wet	44.00	36.00	8.00
	Limestone	Oil wet	16.00	13.00	3.00
	Dolomite	Oil wet	5.00	4.00	1.00
	Sandstone	Mixed wet	59.00	48.00	11.00
	Limestone	Mixed wet	30.00	24.00	6.00
	Dolomite	Mixed wet	13.00	11.00	2.00
Gas & Oil	Sandstone		156.00	126.00	30.00
	Limestone	Oil wet	16.00	13.00	3.00
	Dolomite		16.00	13.00	3.00
Gas & Water	Sandstone	Water wet	28.00	22.00	6.00
	Limestone		1.00	1.00	0.00
Gas & Condensate	Sandstone	Condensate wet	19.00	15.00	4.00
	Limestone		3.00	3.00	0.00
Total			758	621	137

8. Statistical analysis [5]

A statistical comparison was performed between model’s results and three common correlations: Corey’s correlation, Honarpour’s correlation and Pirson’s correlation. Although there are numerous recent correlations for relative permeability prediction but the three previous mentioned correlations were selected for statistical comparison due to their common usage and proven reliability in many reservoir engineering cases for many years. Table 3 shows the

statistical comparison for different reservoir fluid systems. For gas/condensate system, previous correlations gas/oil models were used as they did not consider model for gas/condensate system specifically. Figures 2 & 3 & 4 & 5 show examples for results comparison.

Table 3. Statistical comparison results

System	Lith	Wett	R-Square				Mean Absolute Error			
			Modified Corey	Corey	Honar-pour	Pirson	Modif Corey	Corey	Honar-pour	Pirson
Water-Oil	SS	WW	0.93	0.89	0.94	0.95	0.04	0.06	0.06	0.07
	LS	WW	0.97	0.94	0.95	0.97	0.01	0.02	0.02	0.04
	Dolo	WW	0.96	0.78	0.83	0.84	0.01	0.01	0.02	0.03
	SS	OW	0.971	0.953	0.973	0.968	0.01	0.02	0.02	0.05
	LS	OW	0.94	0.92	0.97	0.98	0.01	0.02	0.03	0.02
	Dolo	OW	0.99	0.93	0.97	0.99	0.00	0.01	0.01	0.02
	SS	MW	0.965	0.920	0.948	0.951	0.01	0.02	0.01	0.01
	LS	MW	0.86	0.85	0.90	0.89	0.03	0.03	0.04	0.03
	Dolo	MW	0.97	0.90	0.96	0.95	0.01	0.02	0.01	0.02
Gas-Oil	SS	**	0.97	0.84	0.94	0.98	0.01	0.05	0.17	0.02
	LS	**	0.94	0.83	0.88	0.96	0.06	0.19	0.15	0.09
	Dolo	**	0.98	0.86	0.99	0.99	0.01	0.07	0.11	0.03
Gas - water	SS	**	0.86	0.76	N/A	0.90	0.01	0.02	N/A	0.01
	LS	**	0.99	0.92	N/A	0.98	0.01	0.20	N/A	0.13
Gas - cond	SS	**	0.92	N/A	N/A	N/A	0.06	N/A	N/A	N/A
	LS	**	0.95	N/A	N/A	N/A	0.05	N/A	N/A	N/A
Average			0.95	0.88	0.94	0.95	0.02	0.05	0.05	0.04

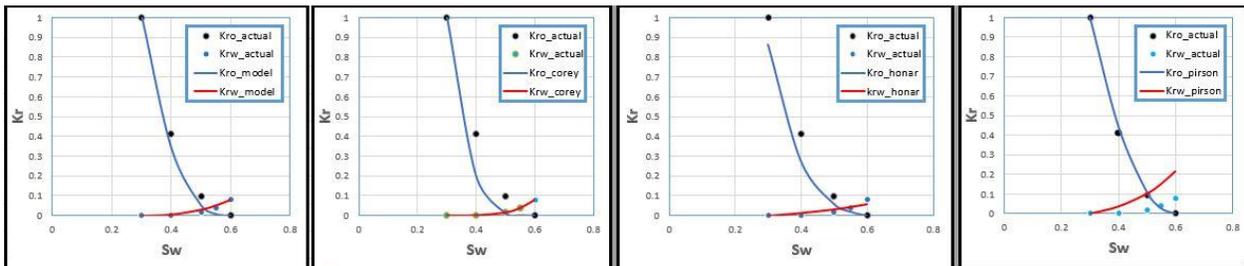


Figure 2. Results comparison for water/oil water wet sandstone system

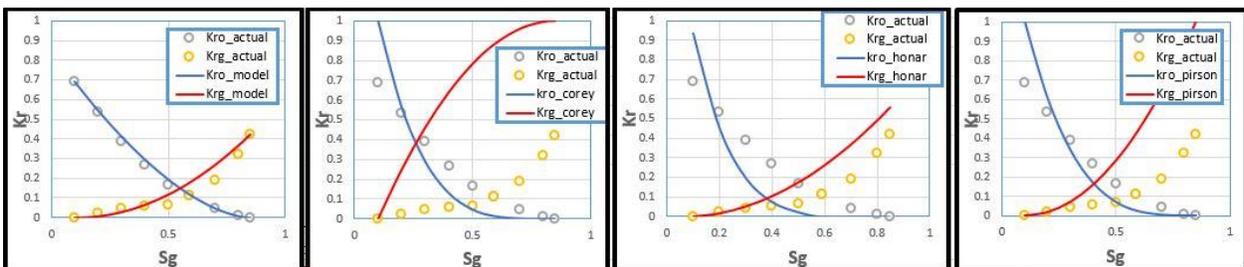


Figure 3. Results comparison for gas/oil limestone system

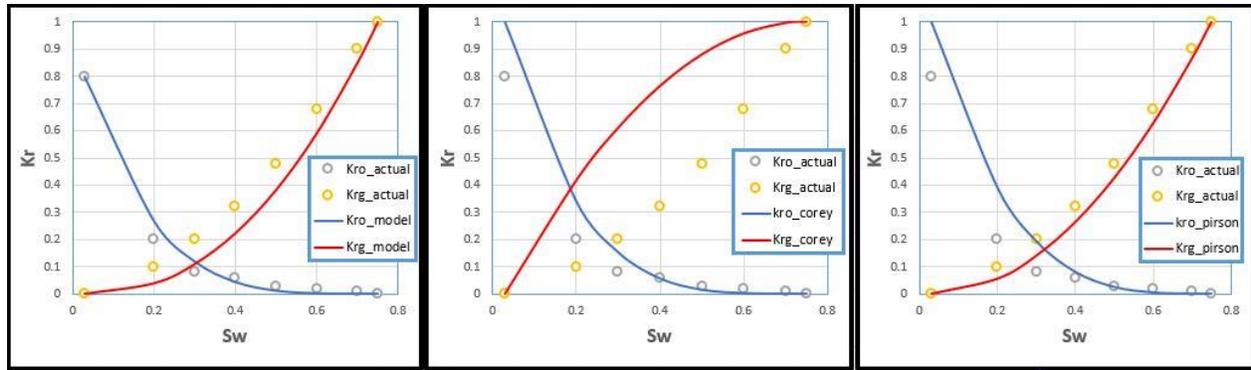


Figure 4. Results comparison for gas/water sandstone system

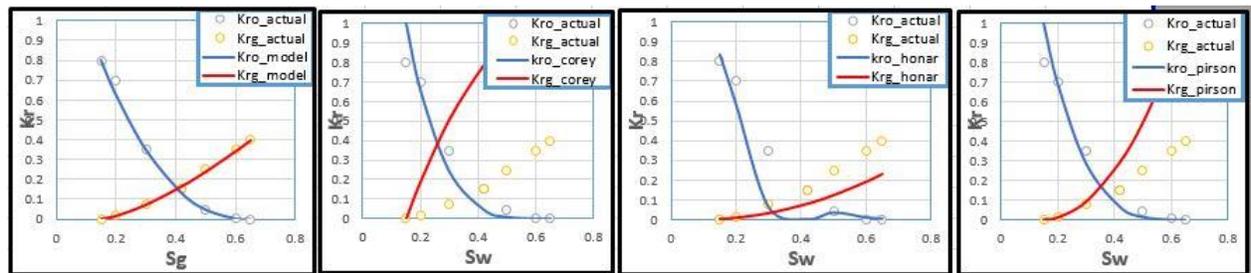


Figure 5. Results comparison for gas/condensate limestone system

9. Conclusion

1. Unlike previously published correlations, the methodology honors not only empirical background but also mathematical derivation based on experimental observations that were not exploited before.
2. A mathematical relation was invented to calculate the relative permeability relation exponents instead of assuming them.
3. The mathematical modification allows variation of relative permeability relation exponents with variation of end points that can be related to permeability and porosity distribution specially in numerical reservoir simulation studies.
4. The proposed model considers gas/condensate systems unlike previously published common relative permeability prediction correlations.
5. A giant data base of actual experimental relative permeability measurements " 750 datasets" was used to test model validity and reliability.
6. Statistical analysis was made for model results comparison to actual data and three other common correlations; Corey, Honarpour and Pirson. Statistical analysis showed that the proposed methodology has significant reliability to predict relative permeability data.
7. The proposed model is valid to be used for almost all reservoir fluid systems, lithologies and wettabilities.

Model program availability

The model macro enabled excel sheet can be downloaded through the below link:

https://drive.google.com/open?id=1tx_JjVrgrfd_5VI61uCK-p0kSYh3MdEV

Funding

This research received no external funding.

Nomenclature

k_{ro}	Oil relative permeability
k_{rg}	Gas relative permeability
k_{rw}	Water relative permeability

k	Absolute permeability by air
$Lith$	Reservoir lithology
Wet	Reservoir wettability
Φ	Porosity
S_{lc}	liquid total critical saturation
$(k_{ro})_{swc}$	Oil relative permeability at connate-water saturation
$(k_{ro})_{sgc}$	Oil relative permeability at critical gas saturation
$(k_{rw})_{sorw}$	Water relative permeability at the residual oil saturation
$(k_{rc})_{scc}$	Condensate relative permeability at critical gas saturation
S_{orw}	Residual oil saturation in the water-oil system
S_{org}	Residual oil saturation in the gas-oil system
S_{gc}	Critical gas saturation
n_o	Oil exponent on relative permeability curves
n_w	Water exponent on relative permeability curves
n_g	Gas exponent on relative permeability curves
n_c	Condensate exponent on relative permeability curves
T	T-factor in the modified model
$S_{w,x}$	Value of water saturation at the intersection point
A	Adjusting factor in the modified model
B	Adjusting factor in the modified model
R^2	Coefficient of determination
MAE	Mean absolute error

Appendix: Modified models for different reservoir rock and fluid systems

A. Water-Oil system:

1. Water-wet sandstone:

$$k_{ro} = (k_{ro})_{swc} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{3.5 * \frac{(1+T)}{2}}$$

$$k_{rw} = (k_{rw})_{sorw} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{1.6 * \frac{(1+T)}{2}}$$

2. Oil-wet sandstone:

$$k_{ro} = (k_{ro})_{swc} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{4 * \frac{(1+T)}{2}}$$

$$k_{rw} = (k_{rw})_{sorw} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{2.03 * \frac{(1+T)}{2}}$$

3. Mixed-wet sandstone:

$$k_{ro} = (k_{ro})_{swc} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{3.8 * \frac{(1+T)}{2}}$$

$$k_{rw} = (k_{rw})_{sorw} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{1.46 * \frac{(1+T)}{2}}$$

4. Water-wet limestone:

$$k_{ro} = (k_{ro})_{swc} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{2.5 * \frac{(1+T)}{2}}$$

$$k_{rw} = (k_{rw})_{sorw} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{1.13 * \frac{(1+T)}{2}}$$

5. Oil-wet limestone:

$$k_{ro} = (k_{ro})_{swc} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{4.5 * \frac{(1+T)}{2}}$$

$$k_{rw} = (k_{rw})_{sorw} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{0.96 * \frac{(1+T)}{2}}$$

6. Mixed-wet limestone:

$$k_{ro} = (k_{ro})_{swc} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{4.35 * \frac{(1+T)}{2}}$$

7. Water-wet Dolomite:

$$k_{rw} = (k_{rw})_{sorw} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{0.86 * \frac{(1+T)}{2}}$$

$$k_{ro} = (k_{ro})_{swc} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{3.5 * \frac{(1+T)}{2 * T}}$$

$$k_{rw} = (k_{rw})_{sorw} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{0.87 * \frac{(1+T)}{2}}$$

8. Oil-wet Dolomite:

$$k_{ro} = (k_{ro})_{swc} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{4 * \frac{(1+T)}{2 * T}}$$

$$k_{rw} = (k_{rw})_{sorw} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{0.92 * \frac{(1+T)}{2}}$$

9. Oil-wet Dolomite:

$$k_{ro} = (k_{ro})_{swc} * \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{3.9 * \frac{(1+T)}{2 * T}}$$

$$k_{rw} = (k_{rw})_{sorw} * \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{1.84 * \frac{(1+T)}{2}}$$

B. Gas-Oil system:

1. Sandstone:

$$k_{ro} = (k_{ro})_{sgc} * \left[\frac{1 - S_g - S_{lc}}{1 - S_{gc} - S_{lc}} \right]^{3.54 * \frac{(1+T)}{2 * T}}$$

$$k_{rg} = (k_{rg})_{sorg} * \left[\frac{S_g - S_{gc}}{1 - S_{gc} - S_{lc}} \right]^{1.76 * \frac{(1+T)}{2}}$$

$$S_{lc} = S_{wc} + S_{org}$$

2. Limestone:

$$k_{ro} = (k_{ro})_{sgc} * \left[\frac{1 - S_g - S_{lc}}{1 - S_{gc} - S_{lc}} \right]^{1.85 * \frac{(1+T)}{2 * T}}$$

$$k_{rg} = (k_{rg})_{sorg} * \left[\frac{S_g - S_{gc}}{1 - S_{gc} - S_{lc}} \right]^{1.8 * \frac{(1+T)}{2}}$$

$$S_{lc} = S_{wc} + S_{org}$$

3. Dolomite:

$$k_{ro} = (k_{ro})_{sgc} * \left[\frac{1 - S_g - S_{lc}}{1 - S_{gc} - S_{lc}} \right]^{5.4 * \frac{(1+T)}{2 * T}}$$

$$k_{rg} = (k_{rg})_{sorg} * \left[\frac{S_g - S_{gc}}{1 - S_{gc} - S_{lc}} \right]^{1.53 * \frac{(1+T)}{2}}$$

$$S_{lc} = S_{wc} + S_{org}$$

C. Gas-Water system:

1. Sandstone:

$$k_{rw} = (k_{rw})_{sgc} * \left[\frac{1 - S_g - S_{wc}}{1 - S_{gc} - S_{wc}} \right]^{3.74 * \frac{(1+T)}{2 * T}}$$

$$k_{rg} = (k_{rg})_{swc} * \left[\frac{S_g - S_{gc}}{1 - S_{gc} - S_{wc}} \right]^{2.43 * \frac{(1+T)}{2}}$$

2. Limestone:

$$k_{rw} = (k_{rw})_{sgc} * \left[\frac{1 - S_g - S_{wc}}{1 - S_{gc} - S_{wc}} \right]^{3.74 * \frac{(1+T)}{2 * T}}$$

$$k_{rg} = (k_{rg})_{swc} * \left[\frac{S_g - S_{gc}}{1 - S_{gc} - S_{wc}} \right]^{2.43 * \frac{(1+T)}{2}}$$

D. Gas-condensate system:

1. Sandstone:

$$k_{rc} = (k_{rc})_{sgc} * \left[\frac{1 - S_g - S_{lc}}{1 - S_{gc} - S_{lc}} \right]^{4.46 * \frac{(1+T)}{2}}$$

$$k_{rg} = (k_{rg})_{scc} * \left[\frac{S_g - S_{gc}}{1 - S_{gc} - S_{lc}} \right]^{1.785 * \frac{(1+T)}{2}}$$

$S_{lc} = S_{wc} + S_{cc}$

2. Limestone:

$$k_{rc} = (k_{rc})_{sgc} * \left[\frac{1 - S_g - S_{lc}}{1 - S_{gc} - S_{lc}} \right]^{2.66 * \frac{(1+T)}{2}}$$

$$k_{rg} = (k_{rg})_{scc} * \left[\frac{S_g - S_{gc}}{1 - S_{gc} - S_{lc}} \right]^{1.22 * \frac{(1+T)}{2}}$$

$S_{lc} = S_{wc} + S_{cc}$

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