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A New Correlation to Find the Productivity Index for Horizontal Wells in Naturally Fractured Reservoirs

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Abstract

Several studies have proposed correlations of the productivity index for horizontal and vertical wells and, in some cases, hydraulically fractured vertical wells. In this paper, a new correlation is introduced from a simulation study to generate base results that allow adjusting the results to a new correlation, capable of being applied in naturally fractured reservoirs. The first simulation results get quite a difference with the results of the already existing correlation for horizontal wells. Subsequently, the parameters having a great effect on naturally fractured formations such as the dimensionless storativity ratio and the interporosity flow coefficient begin to be added. Finally, the new correlation capable of calculating the productivity index in naturally fractured deposits is obtained. This correlation was then successfully tested with examples reported in the literature.

Keywords: *Productivity index; Naturally fractured reservoirs; Dimensionless storativity ratio; Interporosity flow parameter.*

1. Introduction

When talking about naturally fractured formations (NFRs), it follows that they are those deposits that have received fracturing through geological processes that commonly occur in sedimentary rocks according to Nelson [1], without human intervention like in the case of hydraulically fractured wells that are considered artificial factors that cause fracturing. Natural fractures influence the behavior of the flow of fluids that are deposited there Berkat *et al.* [2]. Additionally, previous studies of have shown that fracture distribution and permeability depend on the anisotropy caused by stress Igbokoyi and Tiab [3].

The wells in production tend to generate changes in the pressure of the formation that generate responses along the reservoir in the fractures due to their high diffusivity; on the other hand, the matrix receives a "delayed" response of these pressure changes. These responses cause a decreasing trend in fracture pressure with respect to the matrix, which simultaneously induces a cross-flow from the matrix to fracture Streltsova [4].

It is important to keep in mind that NFRs comprise two types of porosity and permeability: one of the matrix (ϕ_m, k_m) and another of the fracture (ϕ_f, k_f) since the difference of these determines flow direction; in case the porosity and permeability of the matrix is less than that of the fracture, the flow goes from the matrix and the fractures; on the other hand, if these matrix properties are equivalent to zero, the flow will only occur from the fractures according to Escobar [5].

Warren and Root [6] presented a solution to the radial flow behavior of a slightly compressible fluid in a naturally fractured reservoir, assuming that the presence of flow is only in the fractures and that the matrix blocks, joined together as an evenly distributed source, deliver the fluid to the fracture system according to Uldrich and Ershaghi 7[13]. In other words, its radial flow solution is limited for an isotropic system where the permeability ratio is equal to the unit Igbokoyi and Tiab [3].

Productivity index is one of the most important parameters to calculate in the life of hydrocarbon deposits, specifically in naturally fractured formations, since it implies money reflected in the amount of pressure drop needed to produce a barrel of oil per day according to Escobar [5]. However, in the literature, there is no correlation that allows calculating the productivity index in NFRs; but only perform the calculation on horizontal wells. Some correlations for homogeneous reservoirs were introduced by researchers such as Giger, Merkulov, Renard and Dupuy, Joshi, Penmatcha *et al.* information collected by Escobar *et al.* [8], and Escobar and Montealegre [9]. These last researchers Escobar and Montealegre [9] are highlighted; they made a comparison between the different correlations, with the aim of concluding that the correlation introduced by Joshi is the most accurate until then and that it best fits with the results generated by models of the commercial simulator. From this analysis, Escobar and Montealegre [9] introduce Eq. (2), reducing the difference between the simulator results against the correlation of productivity index in horizontal wells, i.e., higher accuracy than the others. The correlation of Escobar and Montealegre [9] is expressed in terms of flow rate, but the same productivity index, PI, is spoken of in pseudosteady-state conditions, Eq. (3).

Evans [10] proposed a correlation to calculate the dynamic productivity index, which occurs when a horizontal well traverses fractures in an infinite reservoir, or a reservoir with defined boundaries in which a pseudosteady state has not been reached. The productivity index of such a system will depend on the time Evans [10]. Additionally, Evans [10] suggests a method to give a forecast of NFR production without relying on simulators due to the high cost and time involved, but through material balance analysis.

Tariq *et al.* [11] presents a greater approximation to the productivity index in NFRs using the nonlinear regression technique, where he developed Eq. (6) for the input performance ratio that considers the fracture parameters and adjusts to the dimensionless pressure over time for vertical wells producing under a gas solution mechanism in solution.

For a better interpretation of how the flow behaves in NFRs, Demarchos, Porcu and Economides [12] considered a horizontal well located in the vertical center of a reservoir with a thickness h, allowing to study a region of radius $h/2$ around the well. Outside the fracture, the flow is linear, while inside, there is a radial flow; these two flows combined result in a pressure drop that is known as the skin effect.

Currently, it is desired to create a correlation applicable to NFRs, starting from correlation Escobar and Montealegre [9] as a basis, with the knowledge that it is the most accurate and best fits with the results of the simulator. Subsequently, differences were found in the calculation of the productivity index in NFR with the calculation in horizontal wells without being fractured, where two parameters necessary to characterize the deviation of the behavior of a medium with "double porosity" according to Warren and Root [6] are highlighted, that is, the NFR, which are: the dimensionless storativity ratio, ω, eq. (4) describing the storage capacity of the matrix system Tariq *et al.* [11], and the interporosity flow parameter, λ, eq. (5) Escobar [5], whose function is to define the flow capacity in the fracture system Tariq *et al.* [11]. These parameters can be assessed through a proper analysis of the pressure accumulation data obtained from tests Warren and Root [6], which in this case are different tests developed in a commercial simulator.

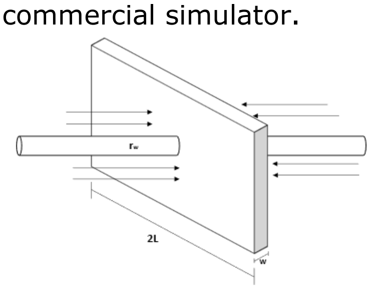

Figure 1. Diagram of fluid flow from the reservoir to the fracture

This paper seeks to calculate the productivity index for NFRs by including the addition of the parameters λ and ω in correlation Escobar and Montealegre [9], among other adjustments, eq. (7), and as an extra objective, to propose the same expression for gas fields, eq. (8). For this, 19 simulation runs were started in the commercial simulator with different values of λ and ω to generate results of time, change of pressure, and flow that are used to take out the IPR curve to which the results generated using the new correlation will be adjusted. Subsequently, empirically, an approximation of the results of the 19 simulation tests and the simulator began to be converged into a single curve such that this correlation covered the calculations necessary to find the productivity index in NFRs. Needless to say that the estimation of the average reservoir pressure is quite importance for the calculation of the productivity index. The average reservoir pressure can also be estimated from drawdown tests as given by Escobar, Palomino and Jongkittinarukorn [13] for vertical wells or Escobar, Palomino and Suescún [14] for horizontal wells.

Unlike the correlation of Escobar and Montealegre [9], eq. (2), the obstacle found was the lack of background for the calculation in YNF, which is why despite comparing the correlations generated in this article with those such as that of Joshi, eq. (1), which only covers horizontal wells, the results are not going to be so close.

2. Mathematical development

Provided below are the best existing correlations to determine the flow rate and/or productivity index in horizontal wells:

Joshi's correlation [8]:

$$q = \frac{2\pi k_h h \Delta P/(\mu_0 \beta_0)}{ln\left[\frac{a+\sqrt{a^2-(L/2)^2}}{L/2}\right]+(h/L)\,ln[h/(2r_w)]} \quad (1)$$

where $a = (L/2)\left[0.5+\sqrt{0.25+(2r_{eh}/L)^4}\right]^{0.5}$

Escobar & Montealegre's correlation, [9]:

$$J = \frac{0.007078 k_h h \Delta P/(\mu_0 \beta_0)}{cosh^{-1}\left[1.075(0.5+\sqrt{0.25+(2r_e/L)^4})^{0.5}\right]} + 0.874(h/L)\,ln[h/(2r_w)] \quad (2)$$

Escobar & Montealegre's correlation for pseudosteady state:

$$q = \frac{0.007078 k_h h \Delta P/(\mu_0 \beta_0)}{cosh^{-1}\left[1.075(0.5+\sqrt{0.25+(2r_e/L)^4})^{0.5}\right]} + 0.874(h/L)\,ln[h/(2r_w)] \quad (3)$$

Tariq's correlation [11]:

$$\frac{q_0}{q_{max}\left(\frac{Pwf}{P_R}\right)\left(\frac{Pwf}{P_R}\right)^{2.76}} \quad (4)$$

where $\alpha = \frac{0.715\lambda^{0.04}}{\omega^{0.027}}$; $\beta = \frac{0.388\omega^{0.11}}{\lambda^{0.030}}$

The characteristic parameters of double porosity systems are as follows:

$$\omega = \frac{(\phi_{Ct})_f}{(\phi_{Ct})_f+(\phi_{Ct})_m} \quad (5)$$

$$\lambda = \frac{4n(n+2)k_m r_w^2}{k_f h_m^2} \quad (6)$$

The correlation developed in this study is the following:

$$q = \frac{0.007078 kh\Delta P/\mu_o \beta_o}{cosh^{-1}\left[1.075\left(0.5+\sqrt{0.25+\left(\frac{2r_e}{L}\right)^4}\right)^{0.5}\right] \ldots + \lambda^{0.05} + 0.874\left(\frac{h}{L}\right) ln\left(\frac{h}{2r_w}\right) + 2.3\omega^{0.002}} \quad (7)$$

Since the dimensionless pressure and pseudopressure are defined by

$$P_D = \frac{kL\Delta P}{141.2 q\mu B} \quad (8)$$

$$m(P)_D = \frac{hL[m(P_i)-m(P)]}{1422.52 q_g T} \quad (9)$$

Using Equations (8) and (9), the expression for gas provided in this study is as follows:

$$q_g = \frac{0.000703 kL_w[m(P_i)-m(P)]}{T\,cosh^{-1}\left[1.075\left(0.5+\sqrt{0.25+(2r_e/L)^4}\right)^{0.5}\right] \ldots + \lambda^{0.05} + 0.874(h/L)\,ln(h/2r_w) + 2.3\omega^{0.002}} \quad (10)$$

Additionally, with the use of the Eqs. (8)–(9), the correlation of Escobar and Montealegre [9] applicable to gas wells is presented here:

$$J = \frac{0.000703 k_h L[m(P_i)-m(P)]}{T\,cosh^{-1}\left[1.075(0.5+\sqrt{0.25+(2r_e/L)^4})^{0.5}\right]} + 0.874(h/L)\,ln[h/(2r_w)] \quad (11)$$

3. Proposed formulation

3.1. Base case generated by simulation

As mentioned, 19 simulation tests were performed, 10 of which contained a set value of λ, while the values of ω changed with each test; on the other hand, the remaining 9 simulated tests contained a fixed value of ω, while the values of λ varied. This for the correlation to be more accurate and cover a greater range of values that are entered when calculating the productivity index in NFR. Table 1 presents the petrophysical properties that remained constant for the 19 tests.

Table 1. Petrophysical properties of simulated cases

Parameter	Value	Parameter	Value
k, md	100	re, ft	1053
h, ft	100	q, STB/día	1000
μo, cP	2	Pi, psia	6000
Bo, rb/STB	1.05	Reservoir geometry	Rectangular
L, ft	1000	ct, psi^{-1}	3e-6
rw, ft	0.35	s	0

3.2. Variation of the dimensionless storarivity ratio ω

For the correlation of productivity index to be more accurate, the first case was studied, where the parameter ω varies between the minimum and maximum range that the parameter can understand in an NFR, while the parameter λ is set with a value of $1x10^{-6}$.

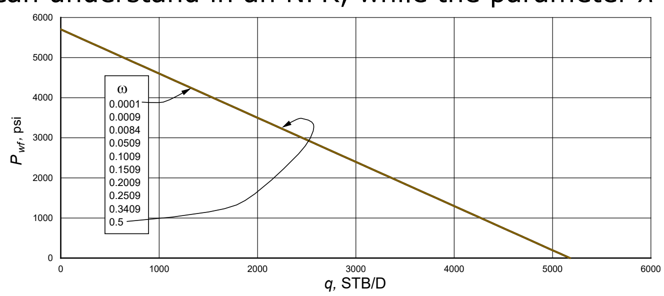

Figure 2. Simulated IPR curve, q vs P_{wf}, using different values of ω and a fixed value of λ and equal to $1x10^{-6}$

The initial value of ω for the simulation tests is 0.0001, and it goes until a maximum value of 0.5 with a total of 10 simulation runs. This ensures a good range of application of the correlation. Fig. 1 shows the results of the tests representing pressure against flow rate during the simulation runs.

As can be seen, the difference is not significant in the curves since the change is minimal when applying different values of ω. After analyzing the decreasing tendency of the curve, it follows that the best way to include the parameter ω to the new correlation is to add it for a better adjustment and because it has a positive effect on the productivity index; however, these changes do not have great repercussions, so it is decided to leave it in the denominator being multiplied by a constant created for a better adjustment and understanding the range it manages. Finally, parameter ω is raised to a power to increase the accuracy of eq. (7).

As can be seen in Fig. 3, the results obtained after applying the proposed correlation are shown, which at first glance does not show a greater difference compared with the results of Fig. 2 generated using the simulator, but the best adjustment is reflected in Section 3.4 of this article, more specifically in Fig. 7.

3.3. Variation of the interporosity flow parameter λ

The next stage is to obtain approximate data on the real behavior of a well with the characteristics mentioned in the base case; for this, a simulation was used. In this case, 9 simulation runs were generated, maintaining the same petrophysical properties of the reservoir, see Table 1, including a constant λ value of 0.0084, but varying the value of λ between $1x10^{-5}$ and $5x10^{-7}$. The results of these simulations are shown in Fig. 4.

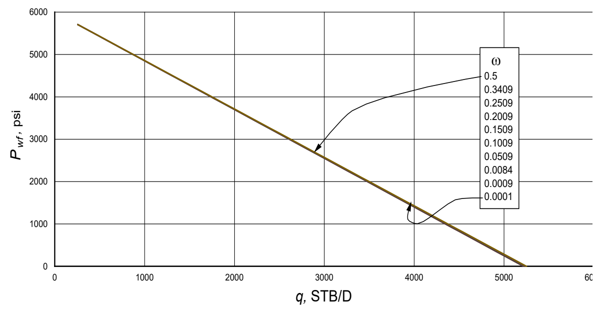

Figure 3. IPR curve, q vs P_{wf} , from the correlation of this study using different values of ω and a set λ value of 1x10^{-6}

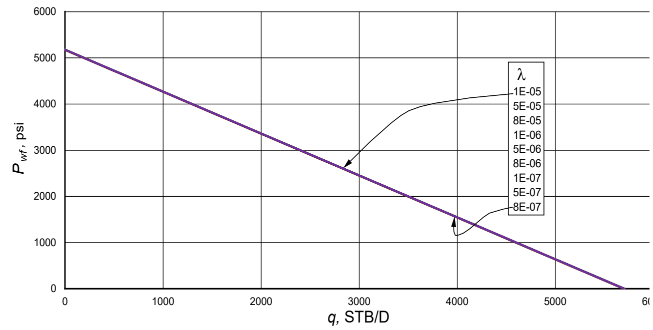

Figure 4. Simulated IPR curve, q vs Pwf, using different values of λ and a fixed ω value of 0.0084

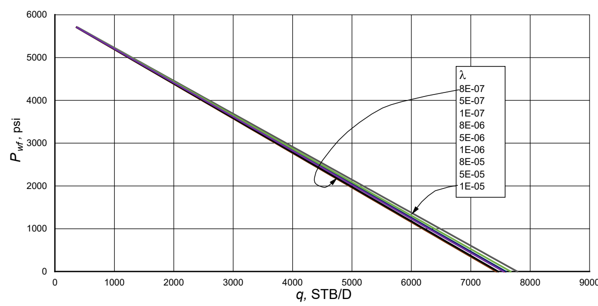

Figure 5. Simulated IPR curve q vs P_{wf} , with the new correlation, Eq. 7, using different values of λ and a fixed ω value of 0.0084

Afterward, that same parameter λ is raised to a power of lesser magnitude to increase the accuracy of the solution. With the illustrated results, it was decided to add the effect of λ since it positively affects the productivity index. But being in the denominator because of its low effect on the results; its direct relationship in the values of the numerator is avoided since the values in the numerator generally affect the productivity index values more. Thereafter, the same parameter λ is raised to a power of lesser magnitude to increase the accuracy of the solution.

When plotting the results of the different simulations obtained in the same graph, at first glance, it is observed that, despite including values of λ in a wide range, there is no great difference in the results. However, when looking at the data in detail, one can notice variations of ±100 barrels of crude oil per day. In the same way that was done with the tests varying the dimensionless storativity ratio, results were generated by applying the correlation with 9 tests, as reported in Fig. 5.

3.4. Comparison with other correlations

The next step was to compare the IPR curve using the base correlations that were taken for horizontal wells, namely the Joshi correlations and that of Escobar and Montealegre. The results obtained were plotted together with those simulated and provided in Fig. 6.

With the obtained result, it was decided to plot again, Fig. 7, the comparative results, excluding those of Joshi, due to their great difference compared with the actual results, and those calculated using the correlation proposed in this study and that proposed by Escobar and Montealegre.

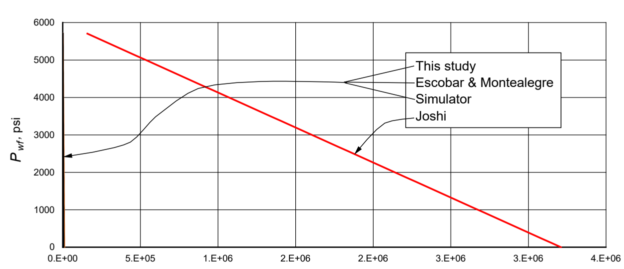

Figure 6. IPR Curve, q vs $P_{wf,}$ obtained by simulation versus flow calculated with different correlations of the literature and the one from this study

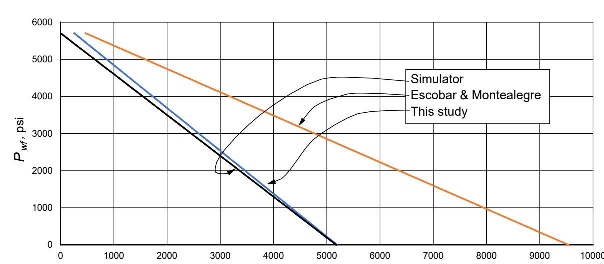

Figure 7. IPR curve, q vs $P_{wf,}$ obtained by simulation versus flow calculated with different correlations excluding Joshi´s correlation

4. Examples

The expressions developed in this study were evaluated applying them to three exercises taking as reference data reported in the literature such as Escobar and Montealegre [4-5]. Using the petrophysical characteristics of the well and reservoir, different pressure tests and models were created, with which the flow rates from the different correlations would later be calculated, in addition to the respective P_{wf} to be plotted, and then these data were entered the simulator to generate the IPR plot to obtain the pressure differentials with respect to the flow rate.

To illustrate the performance of the developed correlation in predicting the productivity index of a horizontal well in an NFR, a wide range of storativity ratios and interporosity flow parameters were used, essential characteristics in an NFR, in addition to fracture network permeability and fluid viscosity, as well as reservoir thickness. The data used in Examples 1-3 are presented in Table 2.

Figs. 8 and 9 are respectively given for Examples 1 and 2. These show the comparative plot of the IPR curves calculated by simulation, the proposed correlation in this study and the correlation of Escobar and Montealegre. It was decided not to plot the curve obtained using the Joshi correlation due to the large margin of difference in the results presented in the 19 tests where the new correlation was generated, which prevents seeing in detail the curve behavior with the other results.

Table 2. Relevant data for examples 1-3

Parameter	Value example 1	Value example 2	Value example 3
k_m, md	14	10	8
B_o, rb/STB	1.1	1.32	1.18
h, ft	90	100	70
μ_o, cp	1.5	1.1	1.3
r_e, ft	4132	4132	4132
r_w, ft	0.29	0.35	0.3
L, ft	1000	1000	1000
λ	$5\text{x}10^{-5}$	$1\text{x}10^{-6}$	$1\text{x}10^{-7}$
ω	0.0105	0.0084	0.23

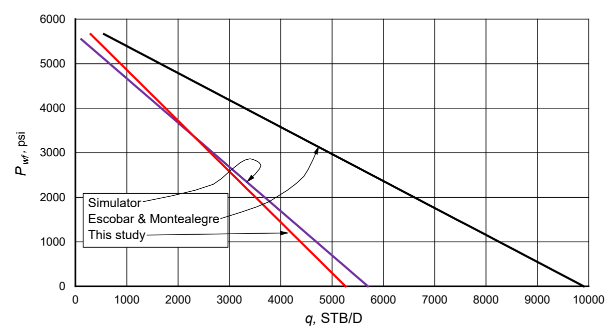

Figure 8. IPR curve for Example 1

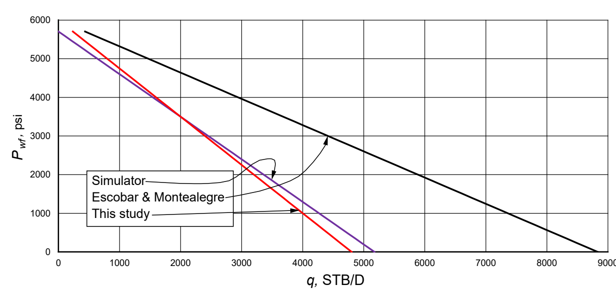

Figure 9. IPR curve for Example 2

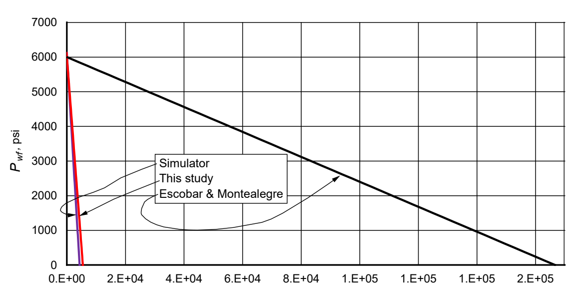

Figure 10. IPR curve for Example 3

By the same token, Table 2 show the values taken for Example 3, from which the flow rates calculated by simulation and reported in Fig. 10, which also contains the flow rates estimated using the correlation developed in this study, eq. (7).

5. Comments on the results

In the three examples, the results obtained using the adjusted correlation are much closer to those calculated using the base correlation of Escobar and Montealegre [9].

As expected, the base correlation utilized for the adjustment, purpose of this study, is not completely accurate to the results obtained according to the results reported in the study; therefore, this difference on results when varying the petrophysical properties of the deposit could have been maintained.

Even if the correlation is perfectly adjusted to the parameters ω and λ, a difference resulting from the base equation will be maintained. Additionally, by varying the values of the test properties not only the new adjustment but also the adjustment taken from the base correlation.

6. Conclusions

An expression is presented to determine the productivity index in horizontal oil and gas wells that drain naturally fractured formations. The results, despite not being the same as the base values, show a result much closer than the values given by the correlations found in the literature. At smaller values of λ, a lower IPR is obtained.

Nomenclature

B_o	*Oil volumetric factor, rb/STB.*
ω	*Dimensionless storativity ratio.*
λ	*Flow capacity ratio.*
ct	*Total compressibility, 1/psi.*
h	*Formation thickness, ft.*
k	*Permeability, md.*
k_m	*Matrix permeability, md.*
P	*Pressure, psi.*
Pi	*Initial Pressure, psi.*
Pwf	*Well flowing pressure, psi.*
q	*Liquid flow rate, STB/day.*
q_g	*Gas flow rate, Mscf/day*
r	*Radius, ft.*
r_e	*Drainage radius or reservoir external radius, ft.*
r_w	*Wellbore radius, ft.*
	Time, hr.
μ_o	*Oil viscosity, cp*
S	*Skin factor.*
J	*Productivity index, bbl/day/psi.*
L	*Horizontal wellbore length, ft.*
NFR	*Naturally Fractured Reservoir*
IPR	*Inflow Performance Relationship*

Greek

ω	*Dimensionless storativity ratio.*
Δ	*Change.*
ϕ	*Porosity, fraction.*
λ	*Flow capacity ratio.*
M	*Viscosity, cP*

Suffices

w	*Well*
f	*Fracture*
m	*Matrix*
e	*External*

References

[1] Nelson R A. Evaluating Fractured Reservoirs, in Geologic Analysis of Naturally Fractured Reservoirs, 2001, pp. 1–100. doi: 10.1016/b978-088415317-7/50004-x.

[2] Berkat A. et al. Identification and characterization of producing fractures in naturally fractured reservoirs using PIWD, Society of Petroleum Engineers - International Petroleum Technology Conference 2009, IPTC 2009, 1, pp. 1–8. doi: 10.3997/2214-4609-pdb.151.spe120687.

[3] Igbokoyi AO, and Tiab D. New method of well test analysis in naturally fractured reservoirs based on elliptical flow, Canadian International Petroleum Conference 2008, (2): 15.

[4] Streltsova TD. Well Pressure Behavior of a Naturally Fractured Reservoir. Society of Petroleum Engineers journal, 1983;23(5): 769–780.

[5] Escobar FH. Novel, Integrated and Revolutionary Well Test Interpretation and Analysis, Intech |Open Mind, England, 2019. 278p. doi: http://dx.doi.org/10.5772/intechopen.81078.

[6] Warren JE, and Root PJ. The Behavior of Naturally Fractured Reservoirs, Society of Petroleum Engineers Journal, 1963; 3(03): 245–255.

[7] Uldrich DO, and Ershaghi I. A Method for Estimating the Interporosity Flow Parameter in Naturally Fractured Reservoirs. Society of Petroleum Engineers of AIME Journal, 1979; 19(5). 324–332.

[8] Escobar FH, Saavedra NF, Aranda RF, and Herrera JF. An Improved Correlation to Estimate Productivity Index in Horizontal Wells. Paper SPE 88540, Proceedings, SPE Asia Pacific Oil and Gas Conference and Exhibition (APOGCE), 2004, to be held to be held 18-20, in Perth, Australia.

[9] Escobar FH, and Montealegre M. A More Accurate Correlation for the Productivity Index of Horizontal Wells, Journal of Engineering and Applied Sciences, 2008; 3(1): 70–78.

[10] Evans RD. Production forecasting and economic evaluation of horizontal wells completed in natural fractured reservoirs, Annual Technical Meeting, ATM 1996. doi: 10.2118/96-21.

[11] Tariq Z. et al. New methodology to quantify productivity of vertical wells in naturally fractured solution gas drive reservoirs with dual porosity and dual permeability, Society of Petroleum Engineers - PAPG/SPE Pakistan Section Annual Technical Conference and Exhibition 2016, pp. 1–17. doi: 10.2118/185314-ms.

[12] Demarchos AS, Porcu MM, and Economides MJ. Transversely multifractured horizontal wells: A recipe for success, Proceedings - SPE Annual Technical Conference and Exhibition, 2006; 3(4), 2006: 1–9.

[13] Escobar FH, Palomino AM, and Jongkittinarukorn K. Average Reservoir Pressure Determination from Drawdown Tests by the TDS Technique. Petroleum and Coal, 2019;61(6): 1341-1351.

[14] Escobar FH, Palomino AM, and Suescún D. Determination of the Average Reservoir Pressure from Horizontal Well Flow Tests Using the Pressure and Pressure Derivative Plot. ARPN Journal of Engineering and Applied Sciences, 2021; 16(22): 2440-2444.

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