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APPRAISAL OF THE INTERWELL CONNECTIVITY FROM PRODUCTION DATA

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Abstract

The understanding of geological characteristics and heterogeneity of a reservoir enables better decisions for reservoir development. Statistical methods were used to infer communication between injector-producer well pairs in a waterflood reservoir using production and injection rate data. The multivariate linear regression (MLR) technique's weighting coefficients relates the fraction of the flow in a producer that comes from each of the injectors. Diffusivity filters were applied to model the time lag and attenuation between the stimulus (injection) and the response (production), and further modify the model by successive elimination of negative weighting coefficients (SEN) and successive elimination of positive coefficients larger than 1 (SEP). Diffusivity filters do not improve the results for the case considered. The statistical implications of the SEN and SEP procedures were compared with a less complex simple linear model (SLM) which eliminates the need to make ad hoc assumptions.

A statistical hypothesis test (P-Value test) was carried out to determine the significance of each injector-producer well pair relationship. This avoids making statistically questionable assumptions to eliminate injector-producer well pairs with connection strengths outside the range [0, 1]. SLM model with statistical significance test gives a better approach to improve sweep among injector-producer well pairs.

Keyword: interwell connectivity; weighting coefficients; oil displacement; diffusivity filters; production performance.

1. Introduction

Predicting the amount of oil and gas that will be recovered from a reservoir is an important task to solve as a petroleum engineer. The process of reservoir characterization is critical to this process and requires information from various data sources such as core analysis, construction of reservoir models, 3D seismic interpretation, geology, well logging, well and fluid testing, identification of reserve growth potential and many others to improve strategies for the development of early and mature fields.

For an effective reservoir development plan, a sound reservoir description and a better knowledge and understanding of how the initial field conditions and progressive performance of the field is essential. The acquisition and processing of some of this information throughout the life of the reservoir is expensive and, in many cases the information required is unavailable.

The resources for building and using various modeling methods such as numerical simulation and the lack of important information make the process of reservoir characterization difficult.

Methods using production data have been developed to determine the recovery of a field undergoing waterflooding. Recorded monthly production and injection rates are the most accessible data in any field. The analysis of production data is being used to determine reservoir characteristics, completion effectiveness and hydrocarbons- in-place. More frequently, injection and production rates along with reservoir description and characterization are used to qualitatively determine the influence of injectors on producers in a field. Plots showing cumulative injection rates, cumulative production rates, water-oil ratios, gas-oil ratios, total production and oil recoveries, all usually as a function of time are a few of the plots used to better understand waterflood flood performance.

In an enhanced oil recovery or secondary recovery system where the production rates of individual wells are affected by injection rates, an understanding of the interwell communication would maximize the performance of an existing flood.

Several methods have been developed to evaluate the rate performance of an existing well with that of the surrounding injectors. The Spearman rank correlation method tests the relationship between variables regardless of the size of the populations from which the samples are drawn has been used in recent years to infer relationships between injector-producer well pairs. Through the spearman rank correlations, Heffer [8] infer injector-producer pairs and related their association with geomechanics. Heffer [8] suggested that the geomechanical-fluid flow through numerical modeling could simulate reservoir behavior. Refunjol [12] utilized the Spearman rank correlation analysis to determine preferential flow trends in a reservoir. Refunjol [12] correlated the ranks of time series of injection and production rates from pairs composed of each injection well and all adjacent producers. Panda and Chopra [11] extended the work on inter well connectivity through integrated approach by applying artificial neural networks (ANN). DeSant' Anna Pizarro [2] confirmed the Spearman rank approach through reservoir numerical simulation and as a results suggested some merits and demerits of the technique.

Soeriawinata and Kelkar [15] utilized the Spearman rank to relate injectors to their adjacent producers and pointed out that existence of good communication between two injectors and producer is seen if the cross correlation of the water injection rate to liquid production rate is higher than the cross correlation of single injector. Soeriawinata and Kelkar [15] also accounted for the superposition effect in the reservoir caused by the influence of multiple injection wells on a producing well.

A method that calculates the fraction of the flow in a producer was developed by Albertoni [1]. Albertoni [1] evaluated the connectivity between injectors and producers in a waterflood through Multivariate Linear Regression (MLR) and the Balanced Multivariate Linear Regression (BMLR) methods. Albertoni [1] views the reservoir as a system that processes a stimulus (injection) and returns a response (production). The methods presented by Albertoni use liquid (water and oil) production and injection rates in reservoir volumes of every well in a waterflood system. The reservoir effect on the input signal (injection) and the output signal (production) is dependent on the location and the orientation of each injector-producer pair in that system. Diffusivity filters are employed to account for the time lag and attenuation that occurs between the injector and producer.

Dinh [3] used the Albertoni method to quantify communication between well pairs in a reservoir using injection and production rate data. The method was tested on a synthetic reservoir model using the BOAST98 numerical simulator and then was applied to a waterflood field and pointed out that approach without diffusivity filters yielded better results than the reverse. Dinh and Tiab [4] used the MLR approach introduced by Albertoni and Lake [1] to determine the interwell connectivity between injector and producer well-pairs in a waterflood system using bottom-hole pressures of injectors and producers. They suggested that the use of bottom-hole pressures eliminated the need to apply diffusivity filters to flow rate data to account for the time lag and attenuation that occurred between injector-producer well pairs, making this approach a much simpler method to infer injector-producer connectivity.

Dinh and Tiab [5] conducted an extended study of the new approach they introduced in 2007 using bottom hole pressure data of injectors and producers to determine interwell connectivity. The new method introduced by Dinh and Tiab [4] included constraints such as constant production rates and constant injection rates. They noted that the constraints placed on the new approach made it difficult to apply the technique to a real field study where production rates vary and are hardly kept constant.

Sayarpour [14] used a capacitance resistive model (CRM) to provide further knowledge about waterflood performance. According to Sayarpour [14], "unlike conventional analytical tools, the CRM can rapidly attain a performance match without having to build an independent geologic model". The "estimation of the fraction of injected water directed from an injector to various producers and the time taken for an injection signal to reach a producer are the key elements in performance assessment" Sayarpour [14]. Sayarpour [14] conducted four case studies using CRMs in complex reservoirs to show CRMs ability to "determine connectivity between injector-producer well pairs and to understand flood efficiencies for the entire or a portion of a field." Sayarpour [14] concluded that "the rapid history matching capability of the capacitance resistive model would serve as a great tool for any grid based modeling study."

The goal of this study is to determine the connectivity between injector-producer well pairs in a conceptual waterflood. The field data provided limited the study to injection and production rates. The application of statistical approaches to the Field data would enable the quantitative determination of the connectivity between injector-producer well pairs in the system without having to use simulation models which require information from other data sources which are unavailable. As discussed earlier, the Spearman rank correlation method has been used to relate pairs of wells, each pair consisting of an injector and a producer. In comparison to the Spearman rank correlation, the MLR and BMLR statistical approaches introduced by Albertoni [1] allow for the quantitative determination of the communication between wells in a waterflood in a single step. This should be a faster way to determine interwell connectivity especially in a large field with numerous injectors and producers such as the studied field in comparison to the Spearman rank model which would require analyzing the data for each well pair. Also, the application of diffusivity filters may be able to account for any time lag and attenuation that occurs between injector and producer well pairs in the system. With that being said, the statistical approach introduced by Albertoni [1] will be applied to the study field.

2. Materials and Methods

2.1. Procedure

Knowledge of the relationship between injector-producer well pairs and an estimate of the fraction of flow caused by each injector in a producer would enable a better understanding of the sweep efficiency of a field undergoing waterflood. This information could allow for suggestions to be made for operational changes that might be made to optimize oil recovery such as flood pattern changes, recompletion of wells and drilling of infill wells. Various statistical approaches have been used to infer a physical relationship between injector-producer

2.2. Multivariate Linear Regression (MLR)

According to Sachs [13] multivariate analysis is the development of general mathematical models for analyzing multiple dependent variables. The MLR method relates the variations in a response variable to the variations of hypothesized predictors.

Parameters in the model are estimated and relations among the variables are determined. An important assumption of the MLR method is that the predictor variables are linearly independent, i.e. no linear relationship exists between the predictor variables. If the predictor variables covary, the model may produce spurious results.

A field undergoing a flood has multiple injectors and producers acting at the same time. Applying the multivariate linear regression model to a flood, the liquid production rate of a well will be the dependent variable while the injection rates for every injector in the field are modeled as independent variables. Because the model assumes that no linear relationship exists between active injectors in the system, injector rate variations should only influence the production rates values in the system and not the other injection rates.

For fields where the injection and production rates are balanced (total injection and production rates are equal), this assumption seems reasonable. For unbalanced systems, this assumption is questionable. Once all the parameters are determined, the model quantifies how each injector influences each producer.

2.3. Mathematical Development

The main assumption for the analytical model is that the reservoir is flowing under a steady-state condition; thus, the pressures are under pressure control and total injection rate is equal to total production rate at the end of every rate interval.

Similar to the pressure model, the estimated flow rate j at a producer (observation) is given by Eq. 1

$$y = \beta_o + \sum_{K=1}^{K} \beta_k x_k + \varepsilon \tag{1}$$

where y is the dependent variable, x_k are the independent variables, and ε is a random error term used to account for imbalances, measurement and fitting errors in the model.

The error term is assumed to be normally distributed with a zero mean $(E(\varepsilon)) = 0$ (Lake *et al.* (1997). The β_o and the β_k terms are the coefficients to be determined by regression. To use the MLR technique to evaluate the production rate of a producer well j, Eq. 1 can be written as Eq. 2

$$\hat{q}_j(t) = \beta_{oj} + \sum_{i=1}^{I} \beta_{ij} i_i(t) \qquad (j = 1,2 \ldots . N) \tag{2}$$

where N is the total number of producers and I is the total number of injectors.

Equation 2 states that for any given time period, the total production rate of well j $\hat{q}_j(t)$ is linear combination of the rates of every injector in the field $\hat{\imath}_i(t)$ plus a constant β_{oj} term. The β_{oj} term is a constant that tries to account for the unbalance in the field. This unbalance will include liquid production not associated with injected fluid (primary production), as well as injection losses (injection not affecting producers).

If $\sum_{j=1}^{N} \beta_{oj} = 0$, then the total field is balanced. Eq. 2 suggests that injection rate changes in the model cause instantaneous production rate changes which would imply steady state flow in the reservoir. The $\beta_{ij}'s$ are the weighting coefficient terms. The β_{ij} terms represent the effect each injector *i* has on each producer j. Thus, the larger β_{ij}, the greater the effect. If the injection rate and production rates are given, the constant term β_{oj} and the weighting factors β_{ij} can be estimated. Using the given production and injection rates, the variance in the production rate can be determined as the difference between the observed production rate and the modeled production rate calculated from

$$\sigma^2{}_{MLR} = Var(\hat{q}_j - q_j) = E\left[(\hat{q}_j - q_j)^2\right] \tag{3}$$

The constant term β_{oj} and the weighting parameter β_{ij} can be determined by minimizing this variance. This will lead to the following set of equations

$$\begin{pmatrix} \sigma_{11}{}^2 & \sigma_{12}{}^2 & \ldots & \sigma_{1I}{}^2 \\ \sigma_{21}{}^2 & \sigma_{22}{}^2 & \ldots & \sigma_{2I}{}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1,I}{}^2 & \sigma_{I2}{}^2 & \cdots & \sigma_{II}{}^2 \end{pmatrix} \times \begin{pmatrix} \beta_{1j} \\ \beta_{2j} \\ \vdots \\ \beta_{1j} \end{pmatrix} = \begin{pmatrix} \sigma_{1j}{}^2 \\ \sigma_{2j}{}^2 \\ \vdots \\ \sigma_{1j}{}^2 \end{pmatrix} \tag{4}$$

The left hand side square matrix in Eq. 4, is the injector-injector covariance matrix and the right hand side vector terms are the covariance values between the injectors and each producer. There are then N equations of the form in Eqn. 4; one for each producer in the system. The weighting coefficients β_{ij} can be determined by standard linear solution method. After the matrix is solved for the β_{ij} terms, the constant term β_{oj} can be determined by

$$\beta_{oj} = \bar{q}_j - \sum_{i=1}^{I} \beta_{ij} \bar{\imath}_i \tag{5}$$

The over bar symbol in Eq. 5 represent mean values. A set of i+1 equations and i+1 unknowns must be solved for each producer in the system. After the parameters in the model are estimated and relations among the variables are determined, the modeled production rate can be compared to the actual production rate using a coefficient of determination value (R^2). The coefficient of determination represents how accurately two data populations are correlated Albertoni [1].

Therefore, this measures the quality of the correlation between modeled and observed production for the case under study. It is defined as in Eq. 6

$$R^2 = 1 - \frac{\sum_{m=1}^{M} \left(\hat{q}_j{}^{(m)} - q_j{}^{(m)}\right)^2}{\sum_{m=1}^{M} \left(q_j{}^{(m)} - \bar{q}_j{}^{(m)}\right)^2} \tag{6}$$

where m is the number of data values.

2.4. Balanced Multivariate Linear Regression (BMLR)

If the sum of the injection rates in a field is approximately equal to the sum of the production rates in the field, the flood is said to be balanced. In this case, Albertoni [1] suggests that the BMLR approach should be used. Therefore the constant term β_{oj} in Eq. 2 will be zero. The production rate for a well j in this model can be defined as

$$\hat{q}_j(t) = \sum_{i=1}^{I} \lambda_{ij} i_i(t) \qquad (j = 1,2 \ldots . N) \tag{7}$$

This equation states that the production rate can be modeled as a linear combination of the injection rate values. The weighting coefficient terms λ_{ij} account for the imbalance within the system. In the BMLR model, the coefficient terms λ_{ij} replace the coefficient terms β_{ij} used in the MLR model to differentiate between the balanced and the unbalanced models. The λ_{ij}terms and the β_{ij} terms mean the same thing with the exception of the β_{oj} term.

The balanced condition is given by

$$\sum_{j=1}^{J} q_j = \sum_{j=1}^{j} \sum_{i=1}^{I} \lambda_{ij} \bar{\iota}_i = \sum_{i=1}^{I} i_i \tag{8}$$

Eq. 8 suggests that the average liquid production rate is a linear combination of the average injection rates. Therefore liquid production rates are a result of the average fluid rates injected at the injectors. If the β_{oj} term is zero, Eq. 5 is identical to Eq. 8.

To determine the weighting coefficients the BMLR model, the squared error of the predicted value must be minimized.

$$\sigma^2{}_{MLR} = E\left[\left(\hat{q}_j - q_j\right)^2\right] \tag{9}$$

The combination of Eqs. 7 and 8 with the minimization of the variance in Eq. 9 leads to the matrix system of linear equations for the BMLR method shown below

$$\begin{pmatrix} \sigma_{11}{}^2 & \sigma_{12}{}^2 & \ldots & \sigma_{1I}{}^2 & \bar{\iota}_1 \\ \sigma_{21}{}^2 & \sigma_{22}{}^2 & \ldots & \sigma_{2I}{}^2 & \bar{\iota}_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{1,I}{}^2 & \sigma_{I2}{}^2 & \cdots & \sigma_{II}{}^2 & \bar{\iota}_1 \\ \bar{\iota}_1 & \bar{\iota}_2 & \cdots & \bar{\iota}_1 & 0 \end{pmatrix} \times \begin{pmatrix} \lambda_{1j} \\ \lambda_{2j} \\ \vdots \\ \lambda_{Ij} \\ \mu_j \end{pmatrix} = \begin{pmatrix} \sigma_{1j}{}^2 \\ \sigma_{2j}{}^2 \\ \vdots \\ \sigma_{Ij}{}^2 \\ \bar{q}_j \end{pmatrix} \tag{10}$$

The μ_j term in the matrix is the Lagrange multiplier used in the derivation process to account for the predicted rate for each producer which must be equal to the average production rate. The λ_{ij} and μ_j can be determined from the set of I +1 linear equations and I+1 unknowns in Equation (10) using any matrix solving method. The weighting coefficients λ_{ij} obtained from Eq. 10 account for the effect of each injector *i* on each producer *j*.

2.5. Diffusivity Filters

In the case of a field undergoing a flood, the flow regimes change continuously from transient flow to flow patterns that approximate steady state flow. So, the assumed steady state flow model from the application of the MLR statistical approach to the provided production and injection rate data is used for simplicity and diffusivity filters were proposed to account for the effect of transient flow.

Diffusivity filters are used to account for the time lag and attenuation that occurs between the stimulus (injection) and the response (production) Albertoni [1]. The filters transform the injection rates of an injector *i* affecting a producer *j* for an injector-producer pair so that they take the form of an equivalent injection rate acting in an incompressible medium, which results in an effective injection rate at a certain time. In cases where there are large distances between injector and producer pairs and large dissipation in the medium, the use of diffusivity filters becomes very important.

The diffusivity constant is defined by Eq. 11.

$$\eta = \frac{k}{\emptyset \mu C_t} \tag{11}$$

Dissipation is the reciprocal of the diffusivity constant

$$d = \frac{1}{\eta} = \frac{\emptyset \mu C_t}{k} \tag{12}$$

Eq. 12 suggests that a large dissipation would exist in a system with a small permeability, a large porosity, viscosity, and total compressibility. If dissipation did not exist in the reservoir, a change in the rate of injection for an injector *i*, would cause a corresponding and immediate change in the rate of production for a producer *j*. That change would be independent of the distances between injectors and producers. A time lag and attenuation of the signal between the injector and producer exists in most reservoirs. In comparison to a less dissipated reservoir, a dissipated reservoir should experience a more significant time lag between the time at which a change in injection rate occurs and the time at which a corresponding change in the production rate is observed. According to Albertoni [1], when diffusivity filters are applied to the MLR and BMLR procedures, the diffusivity filters are applied only to injection rates.

3. Results and Discussions

3.1. Field Application

The technique was applied to a portion of (12 injectors and 19 producers) of a conceptual petroleum production reservoir "Agaga" which is undergoing waterflooding. One of the assumptions of the MLR models is that the injection and production conditions are constant. Periods where the producers are shut-in should be excluded from the analysis according to Albertoni [1]. The distribution of the producers and injectors in "Agaga" field is shown in Figure 1 and their rates are shown in Table 1.

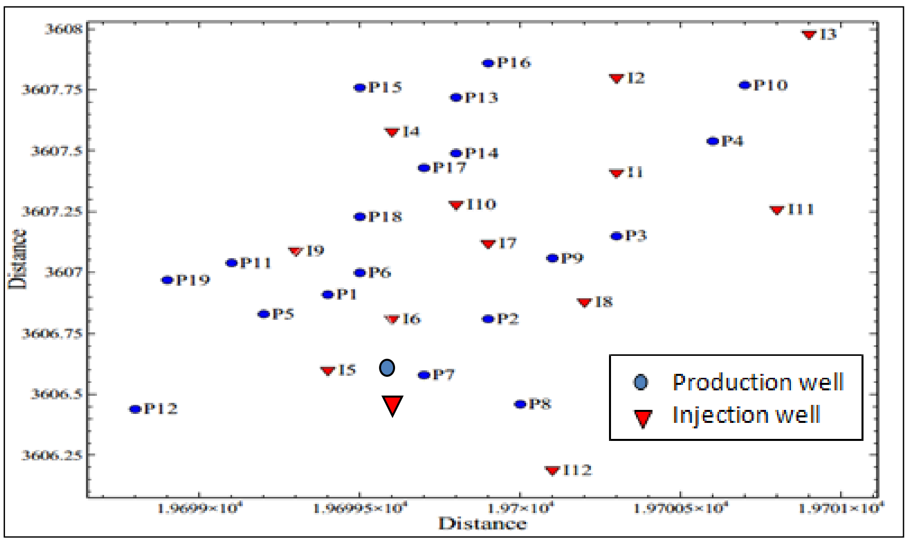

Figure 1 Location of wells

Table 1 Rate for producers and injectors

Date	Producer Well	Well Name	Production rate (m^3/day)	Injection Well	Well Name	Injection Rate (m^3/day)
	P1	Ann 6	30236.80	I1	Ann 4	42637.00
	P2	Ann 7	86090.00	I2	Ann 5	47305.00
	P3	Ann 8	31767.20	I3	Ann 25	10611.21
	P4	Ann 10	17804.00	I4	Ann 31	19486.00
	P5	Ann 39	30058.47	I5	Ann 41	47023.35
	P6	Ann 42	15864.90	I6	Ann 44	66988.00
	P7	Ann 45	24262.20	I7	Ann 47	60155.00
2006/12-2008/03	P8	Ann 49	5455.29	I8	Ann 51	49630.00
	P9	Ann 78	16590.20	I9	Ann 57	34959.00
	P10	Ann 79	15136.00	I10	B 76	25174.38
	P11	Ann 86	1190.55	I11	B 98	35618.00
	P12	Ann 55	36004.50	I12	B 70	25902.29
	P13	Ann 91	26198.30			
	P14	Ann 95	12898.41			
	P15	Ann 96	21742.30			
	P16	Ann 97	14435.60			
	P17	Ann 99	39531.27			
	P18	Ann 100	31107.60			
	P19	Ann 101	15089.33			

The selected time period for the analysis is from December, 2006 to March, 2008. For this period, there are injection and production rate data for all the selected injector and producer wells. The total monthly injection rates for injector wells for the selected time period (Figure 2) shows that all of the wells have an initial period where the monthly injection increases. The injection rates of the wells plateau leading to decline.

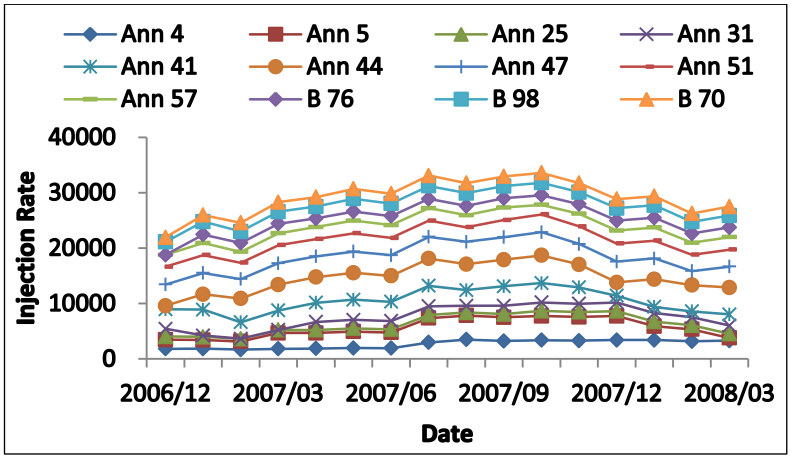

Figure 2 Monthly injection rates for injector wells for the selected time period.

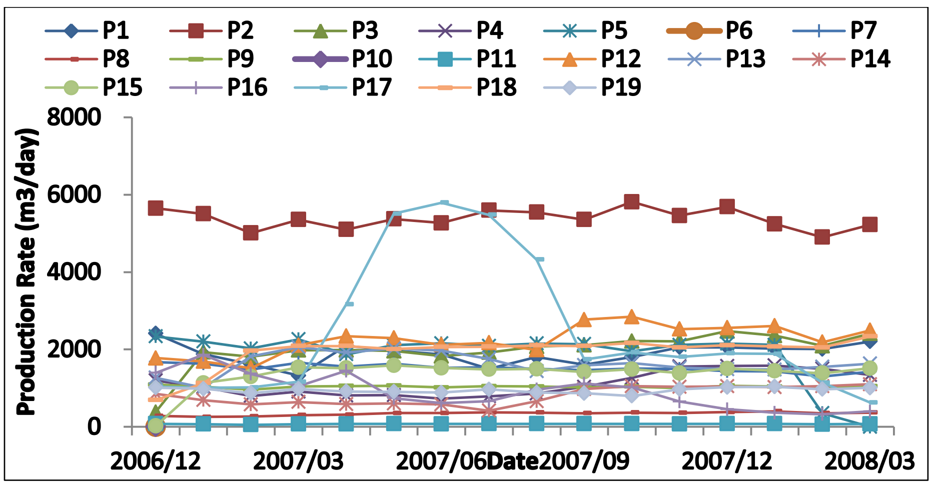

Figure 3 Monthly production rates for producer wells for the selected time period.

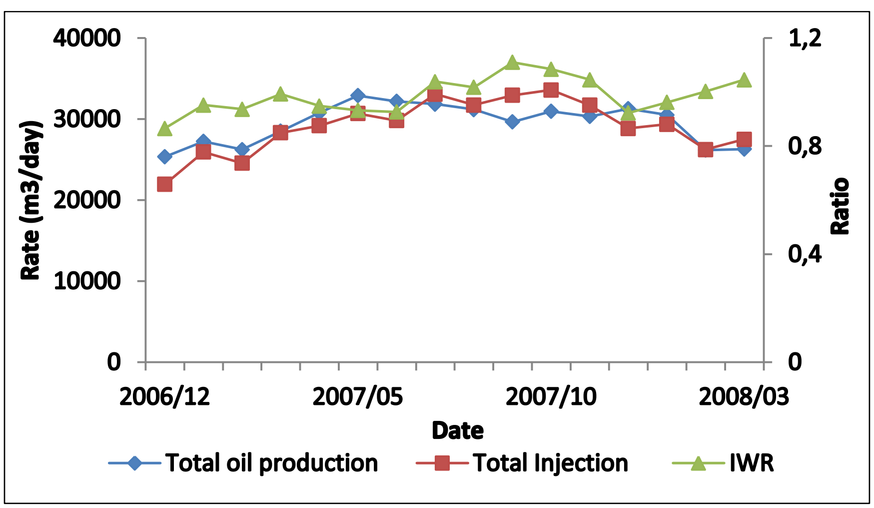

Figure 4 Total injection and production rates for the selected time period

Figure 3 shows the total monthly production rates for the production wells for the selected time period. The total injection and production values for the system during the period of analysis are 13.97×10^6 m^3 and 14.45×10^6 m^3 respectively. The cumulative injection to withdrawal ratio (IWR) is then 0.97. Figure 4 shows the total injection and production rates for the selected time period being analyzed.

3.2. Application of MLR Method

Since only a portion of the field is being analyzed, the boundaries are open and the injection and production rates for the selected period of analysis are not balanced. The multivariate linear regression (MLR) method is then the suggested method for this analysis.

The application of the MLR approach gives the weighting coefficients expressed as rose diagram as shown in (Figure 5). The weighting coefficients β_{ij} are represented by streak of lines.

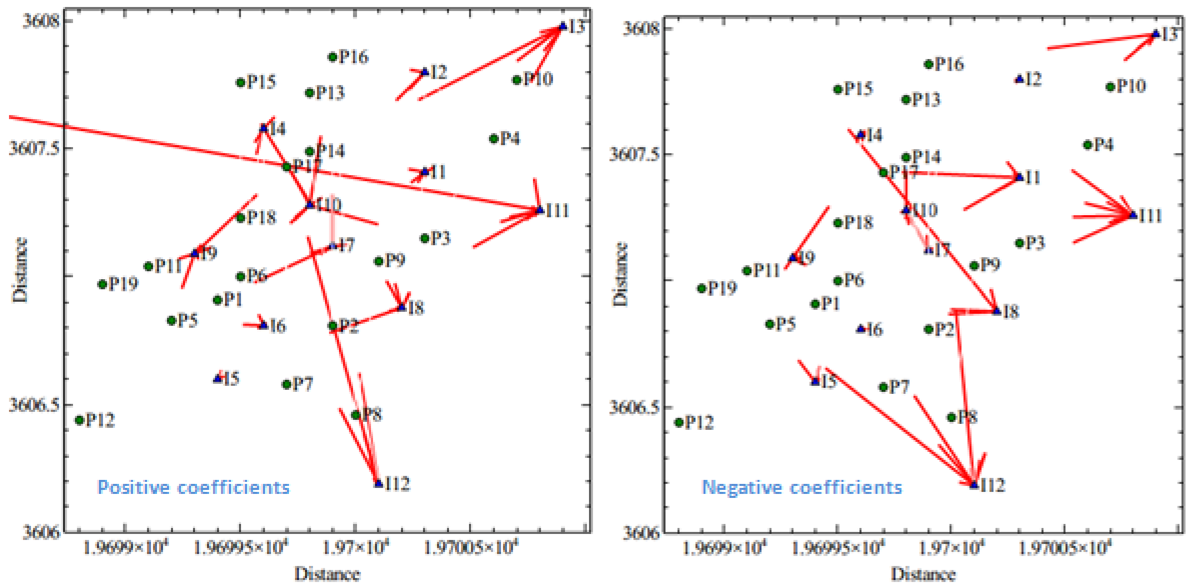

Figure 5 Representation of the weighting coefficients β_{ij}.

If it is assumed that the negative weighting coefficients have no meaning, then what the positive graph in Figure 5 shows is that, injectors I11 and I12 have the strongest influence on producers P17 as opposed to the other producers. Injector well I11 and I12 have a larger positive effect on producer wells P17 in comparison to the other producer wells in the system. Thus, as the injection rate for injector well I11 and I12 increase, a larger effect from these injectors is mostly seen in producer P17.

Injector wells I1, I2, I4, I5 I6 and I7 show no connectivity with producer wells P7, P8, P9, P10, P11, P14, P18 and P19. Producer P11 appears to register the weakest connectivity from the set of these non connecting injectors. There is the possibility that some of the injected fluid from these non connecting injectors is being lost to non-productive layers in the field; thereby reducing the effect of the injectors on the producers. In terms of the degree of connectivity, I8 connects the highest number of the producer wells about 68% with I5 showing very little connectivity of about 21% of the producer wells. Generally the average producer-injector connectivity is about 48%. Figure 6 and 7 show the degree of connectivity of each injector and productivity of each producer respectively. From the analysis of degree of producer-injector connectivity shown in Figure 7, producer P17 shows better connectivity than all the other producers, followed by P5 with P8 showing the least this contradicts Table 1. The contradiction portrays the lack of the true meaning of the weighting coefficient under these circumstances, raising the question whether the MLR model resolves the true connectivity between producers and injectors in the system. The injector-producer well pair with positive weighting coefficient that is greater than one implies the injectors have greater influence on their producers in comparison to the other producers in the model and suggest more effective sweep than the other injectors. The performance of the injector-producer well pairs is largely related to the distance connecting them, the well density and the geological properties.

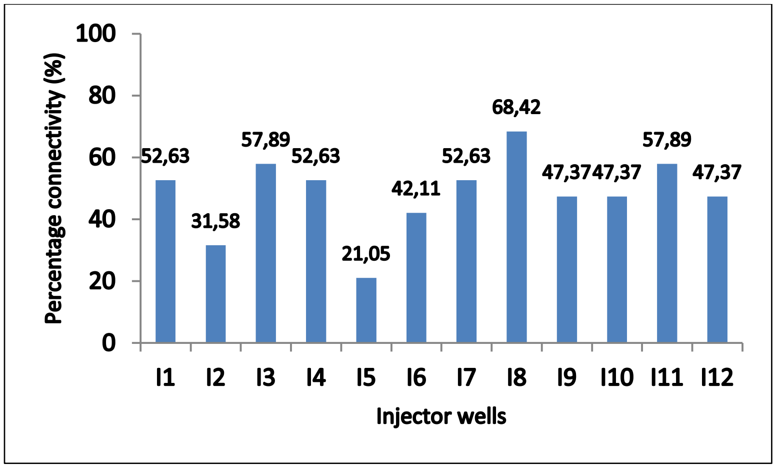

Figure 6 Representation of injector connectivity.

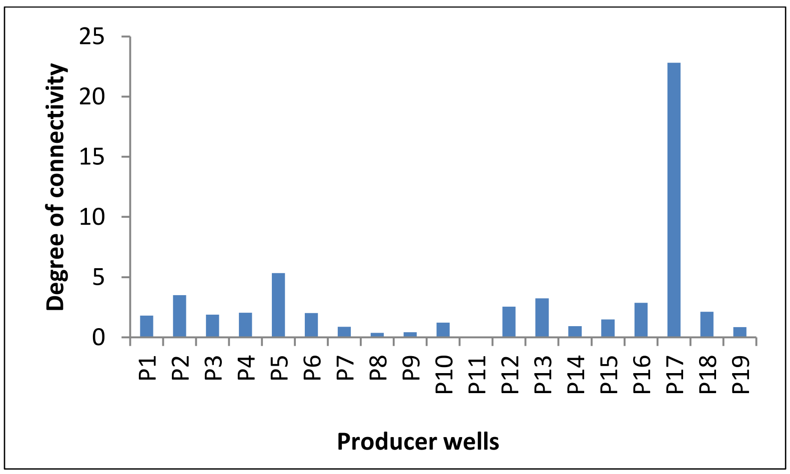

Figure 7 Representation of producer degree connectivity.

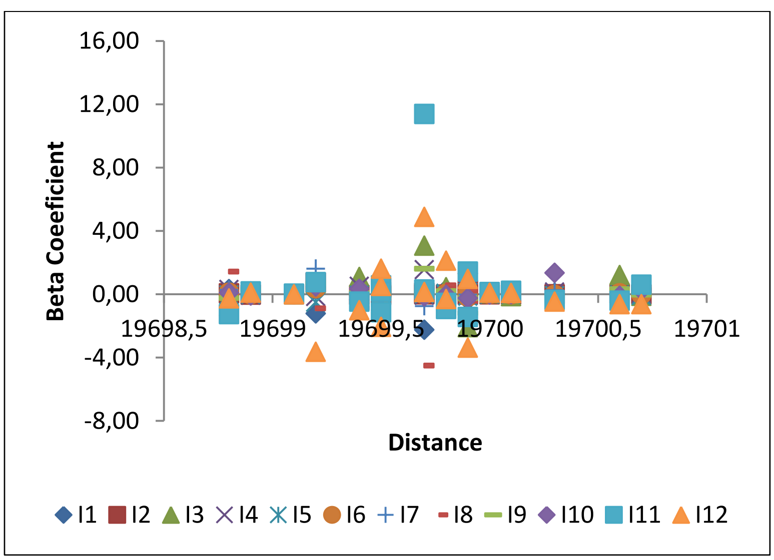

Figure 8 Beta Coefficients (β_{ij}) against injector-producer distance with SEP and SEN

Generally, closer well pairs would be expected to have larger coefficients than well pairs that are further apart. As shown in Figure 8, several of the largest values for the beta coefficients are at the lower values for the separation distance. However, there are also a number of very low beta coefficient values at these smaller well pair distances as well and most of the negative values are in the middle to large distance values. This shows that the general assumption made relating the high connectivity of an injector-producer pair to the short distance between them alone is not necessarily always true.

Figure 9 shows a comparison between the total modeled liquid production rate and the total observed liquid production rate. The coefficient of determination R^2 value which represents how accurate the MLR model and the real production rate data are correlated is 0.99.

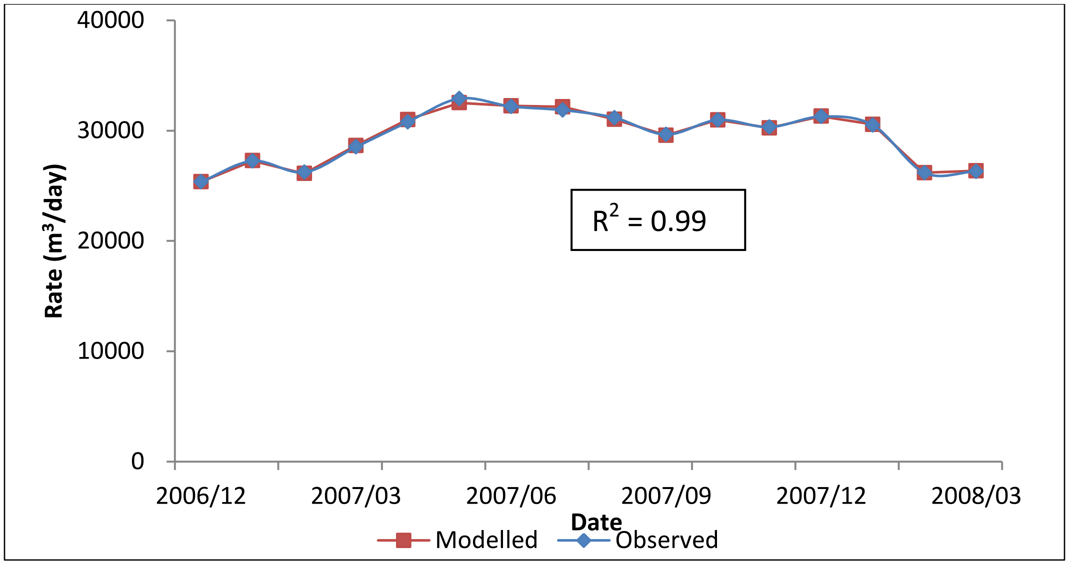

Figure 9: Comparison between total modeled liquid production rate and total observed liquid production rate, MLR with diffusivity filters.

An R^2 value of one would suggest that there is a perfect correlation between the total modeled and total actual production rates for the system which would indicate that the MLR model has been able to accurately capture rate fluctuations in the reservoir.

3.3. Application of the SEN and SEP

The successive elimination of negative weighting coefficients (SEN) and the successive elimination of positive weighting coefficients greater than one (SEP) procedure was applied to improve the results obtained from the implementation of the MLR model to field data. Excluding these injector-producer well pairs implies that there is no relationship between those injector-producer well pairs. As stated earlier, there is no statistical justification for eliminating well pairs with negative coefficients or positive weighting coefficients that are greater than one but, the SEN and SEP procedure will be applied to the MLR results to evaluate the impact of the application of this procedure.

After the application of the SEN and SEP procedure to the MLR results, there has been an overall reduction in the evaluated connectivity between injector-producer pairs. In general, the minimum weighting coefficient converges to zero as more weighting coefficients are eliminated. Results show that after the application of the SEN and SEP procedure all the negative weighting coefficients and higher positive coefficient greater one were eliminated. Out of 218 injector-producer well pairs coefficients in the original model, only 67 of them will be accounted for with this new model. There are marginal rise and fall in the remaining magnitude.

The coefficient of determination R^2 values increased marginally for some producers and reduced significantly for others. For producers P8 and P11 the R^2 values were unchanged after the application of the SEN and SEP. In Figure 10, the rose diagram of the weighting coefficients β_{ij} determined after the application of the SEN and SEP procedure indicate that injector well I8 still remains injector with the strongest degree of connectivity of about 58%, with the injector I10 having the least influence on producers accounting for about 16% degree connectivity. Unlike the previous case, producer P5 shows the best performing producer in terms of connectivity from system of injectors whiles the least performing producer is P11.

This is consistent with Table 1, as P5 is one of the best performing producers with P11 the poorest performing producer. I11 and I12 are though not the strongest injectors due to connectivity yet their performance are ranked among the best injectors. Using the SEN and SEP results to evaluate sweep improvement options would likely be only slightly different than using the unmodified MLR results. For instance, the SEN and SEP results would suggest that increasing the influence of injector-producer connection should increase recovery while the MLR results suggest that this connection has already been established.

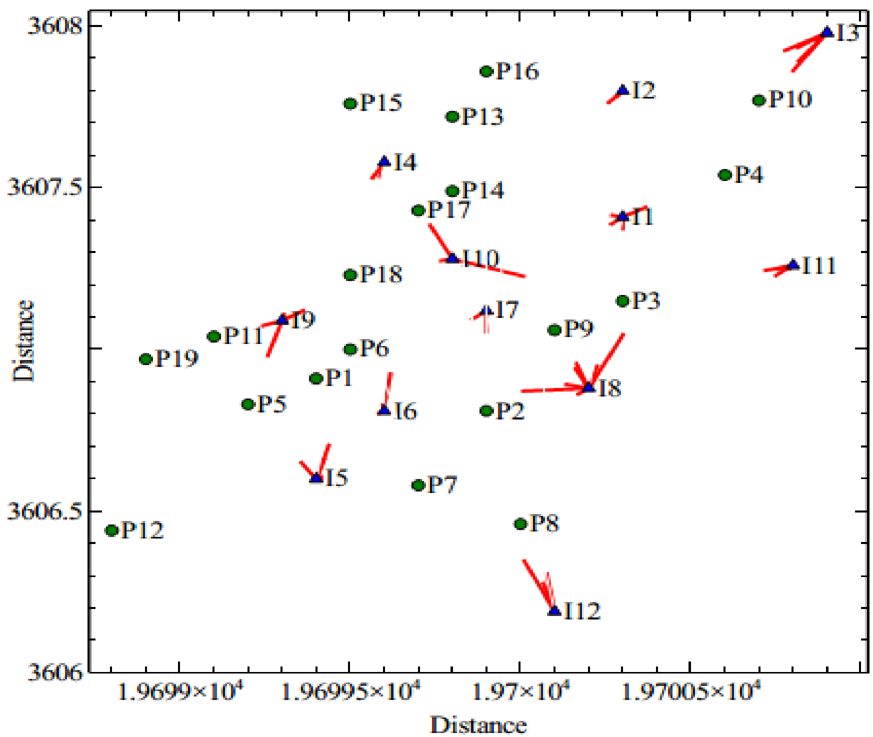

Figure 10 Weighting coefficients β_{ij} after the effect of SEN and SEP.

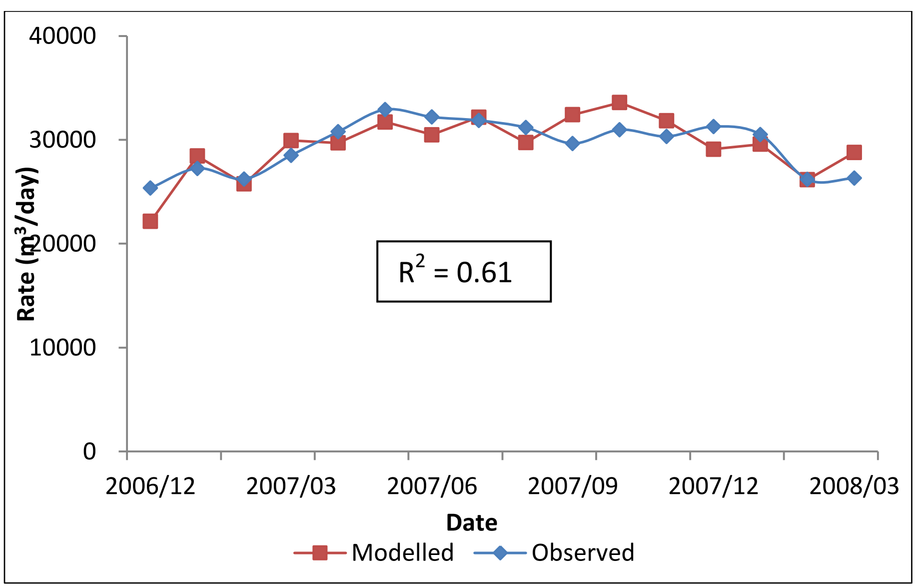

Figure 11 Comparison between total modeled liquid production rate and the total observed liquid production rate after the SE-N and SE-P procedure

Figure 11 shows a comparison between total modeled liquid production rate and the total observed liquid production rate after the application of the SEN and SEP procedure. The effect of the application of the SEN and SEP procedure can be seen by comparing Figures 9 and 10.

The plots suggest that there is a noticeable decrease in the total modeled production rate in comparison to the total observed liquid production rate after the application of the SEN and SEP procedures to the system. As shown, the R^2 value decreased from 0.99 to 0.61 after the SEN and SEP procedure was applied. This implies that the application of the SEN and SEP procedures did not improve the prediction of production well flow rates as would be expected with fewer degrees of freedom.

3.4. Application of MLR Method with Diffusivity Filters

The production rate value at a particular time is the response to injection rate changes over the time of the filter. The actual time it takes for the production rate value of a producer j to respond to an injection rate change of an injector *i* will be applied to the MLR model. A quick calculation using a radius-of-investigation-type time of the form [16-19] can be used to calculate an approximate time it takes for the production rate value of a producer j to respond to an injection rate change of an injector *i*. A total compressibility value (C_t) value of 2×10^{-4} mpa^{-1} and water viscosity (μ) value of 0.268 were used in Equation 13 to calculate time. The r_t^2 value in Equation 13 represents the distance between the injector-producer well pair. The k value represents the permeability and ø represents the porosity of the well.

$$t = \frac{948\emptyset\mu C_t r_t^2}{k} \tag{13}$$

Table 2 shows the average porosity and permeability values. Table 3 shows the time it takes in days for the production rate value of a producer j to respond to an injection rate change of an injector *i*. The result of this analysis suggests that it takes as long as 648 days and a minimum of 0.7 days for the producers to respond to the injection rate of change. On the average majority of the producers take less a 30 days to respond to an injection rate of change with the rest taking a month and more in the system. This suggests that this is a low to high dissipation system so a 1 month to 6month diffusivity filter would be the most effective to the MLR model to examine the effect of diffusivity.

Table 2 Average porosity and permeability values

Well Type	Well Name	Average Porosity	Average Permeability	Well Type	Well Name	Average Porosity	Average Permeability
P1	Ann 6	0.15	185.0	I1	Ann 4	0.14	260.0
P2	Ann 7	0.13	260.0	I2	Ann 5	0.15	313.3
P3	Ann 8	0.16	368.0	I3	Ann 25	0.06	11.0
P4	Ann 10	0.14	64.3	I4	Ann 31	0.14	105.0
P5	Ann 39	0.10	65.0	I5	Ann 41	0.13	75.0
P6	Ann 42	0.14	126.7	I6	Ann 44	0.16	245.0
P7	Ann 45	0.14	107.2	I7	Ann 47	0.14	115.6
P8	Ann 49	0.15	120.7	I8	Ann 51	0.14	80.2
P9	Ann 78	0.11	106.2	I9	Ann 57	0.14	173.8
P10	Ann 79	0.08	217.5	I10	B 76	0.16	179.6
P11	Ann 86	0.13	192.0	I11	B 98	0.14	173.8
P12	Ann 55	0.06	34.0	I12	B 70	0.11	221.7
P13	Ann 91	0.11	192.0				
P14	Ann 95	0.10	250.0				
P15	Ann 96	0.15	164.3				
P16	Ann 97	0.13	252.5				
P17	Ann 99	0.12	272.5				
P18	Ann 100	0.13	170.3				
P19	Ann 101	0.12	84.0				

Table 3a Time (days) for injector-producer well pairs

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
I1	43.1	12.7	1.5	11.8	114.8	46.5	68.8	60.6	8.2	5.4
I2	65.1	27.9	9.4	17.4	159.7	73.6	121.2	115.1	29.5	3.0
I3	138.0	57.9	23.4	31.4	312.8	167.9	223.0	190.5	74.7	1.6
I4	19.9	16.7	15.1	110.8	53.7	19.9	66.2	86.4	26.2	23.3
I5	3.9	7.2	24.8	256.9	6.9	9.8	5.9	23.2	35.3	57.2
I6	2.0	2.2	13.5	169.5	11.9	2.7	4.1	17.2	15.7	39.8
I7	12.0	2.3	3.6	73.7	42.6	10.0	21.7	27.2	2.2	19.9
I8	26.1	2.3	1.9	65.9	74.5	29.0	22.3	13.2	2.1	19.5
I9	1.7	10.7	22.4	209.3	5.8	2.8	27.6	54.2	32.2	45.3
I10	12.1	5.6	6.0	78.2	41.8	9.7	32.8	43.5	7.0	19.6
I11	84.7	24.7	5.9	13.1	203.8	101.1	109.7	78.2	26.6	5.0
I12	41.0	10.4	21.5	229.2	90.6	58.4	20.5	5.1	38.0	53.4

Table 3b Time (days) for injector-producer well pairs continue

	P11	P12	P13	P14	P15	P16	P17	P18	P19
I1	53.4	305.2	10.4	5.1	35.4	9.3	7.9	25.6	153.1
I2	68.3	392.1	7.7	6.9	29.8	4.2	10.9	36.7	188.3
I3	139.7	648.6	38.5	28.7	93.2	26.0	38.2	96.0	356.8
I4	18.3	185.5	1.8	1.0	2.0	4.3	0.7	5.0	61.3
I5	9.6	36.9	42.7	18.9	62.9	47.1	17.1	15.5	27.5
I6	10.3	74.3	26.2	10.0	42.3	30.6	8.6	7.1	36.6
I7	21.9	160.0	11.2	2.9	26.4	14.1	3.0	6.5	72.7
I8	41.9	206.0	26.1	10.5	58.6	27.0	12.1	23.3	120.7
I9	1.4	64.3	19.5	8.1	22.7	24.4	6.0	2.3	12.4
I10	18.6	163.1	5.8	0.9	14.9	8.9	0.7	3.5	64.4
I11	99.5	446.9	36.6	20.9	90.0	30.0	27.1	64.3	262.6
I12	58.4	167.6	73.3	35.3	131.0	72.6	37.2	54.9	145.6

3.4.1. MLR with 1 Month Diffusivity Filter

Similar to the case without diffusivity filters, the results shown in Figure 12 have both positive and negative linear relationships between the injector-producer well pairs.

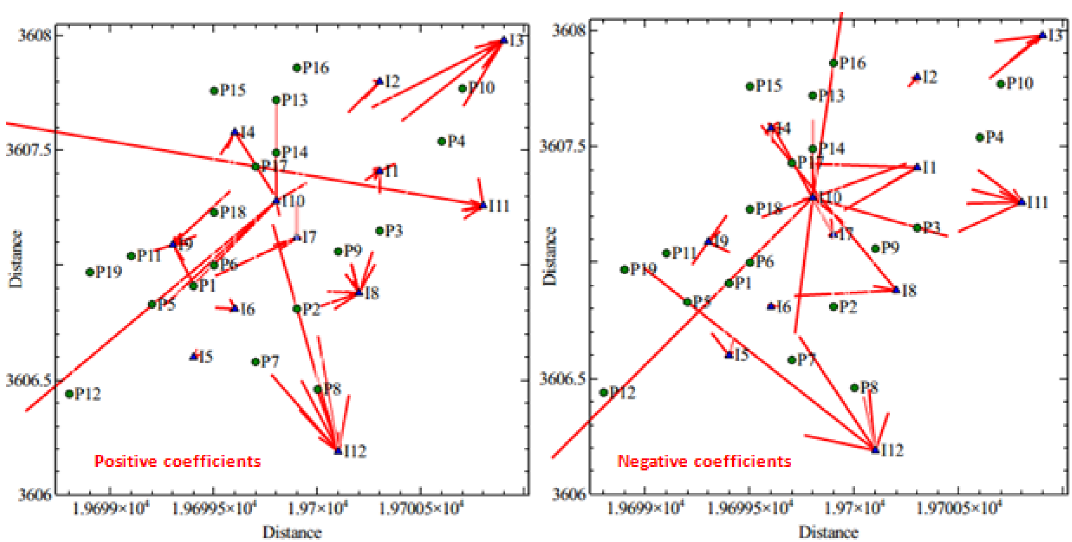

Figure 12 Weighting coefficients β_{ij} for MLR with 1-month diffusivity filters

Similar to the MLR case without Diffusivity Filters, Figure 12 shows that injector I8 has the strongest influence on the system of producers with I5 still as weakest injector connectivity. Producer P17 still shows the best performing producer within the system of injectors with P11 still performing poorly. In general, application of the 1 month diffusivity filters results in a slight decrease in the number of negative weighting coefficients between injector producer well pairs from 101 (for the case without diffusivity filters) to 96. Also, the magnitude of connectivity between injector producer well pairs increased as well as the R^2 values of all producers.

Figure 13 shows a comparison between the total modeled liquid production rate and the total observed liquid production rate. In comparison to the MLR case without diffusivity filters, Figure 13 shows that the coefficient of determination R^2 value increased from 0.99 to 0.996 after the application of the 1-month diffusivity filter. This suggests that a closer correlation between the total modeled liquid production rate and total observed liquid production rate has been achieved with the application of the 1- month diffusivity filter to the MLR model. This R^2 value is also higher than the R^2 value of 0.61 for the MLR case after the application of the SEN and SEP procedure.

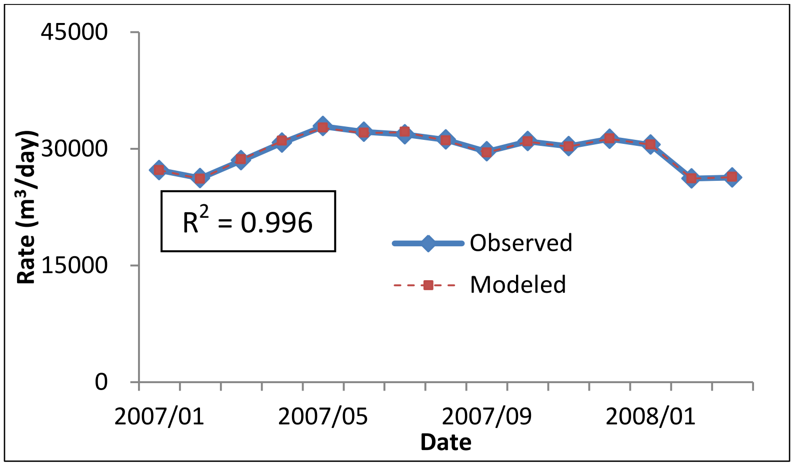

Figure 13 Comparison between total modeled liquid production rate and the total observed liquid production rate in MLR with 1-month diffusivity filters,

3.4.2 Application of the SEN and SEP procedure to MLR with 1-month Diffusivity Filters

After the successive elimination of both negative weighting coefficients and positive weighting coefficients greater than one the R^2 values for all producers decreased except P11 which remained the same. This implies that the SEN and SEP procedure did not improve the prediction of flow rates for almost all the wells. The weighting coefficients β_{ij} from the application of the SEN and SEP procedure are shown in Figure 14. It can be seen that the application of the SEP and SEN procedure to MLR reduces the strength of the connectivity between the injector-producer well pairs.

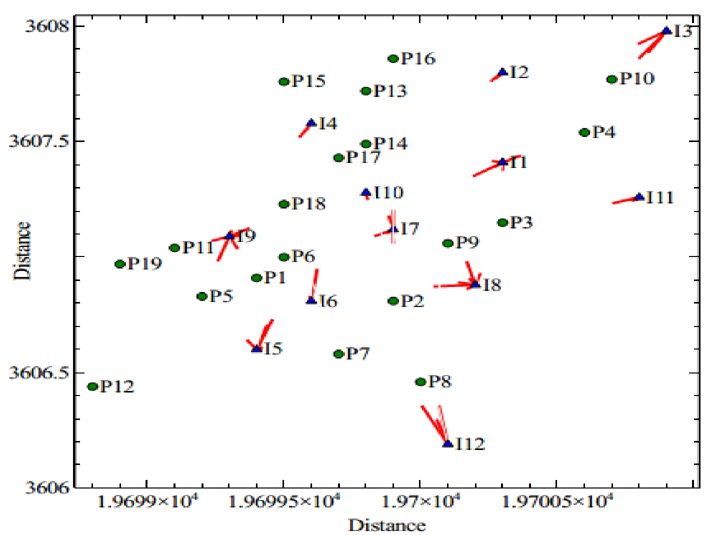

Figure 14 Positive weighting coefficients β_{ij} after the SE-N and SE-P procedure is applied to MLR with 1-month diffusivity filters

Figure 14 suggests that the sweep of fluid in the reservoir is not that efficient because the individual injectors are not influencing the producers in the area surrounding them much. Figure 15 shows a comparison between total modeled liquid production rate and the total observed liquid production rate after the application of the SEN and SEP procedure. The R^2 value decreased from 0.99 to 0.55 after the SEN and SEP procedure was applied.

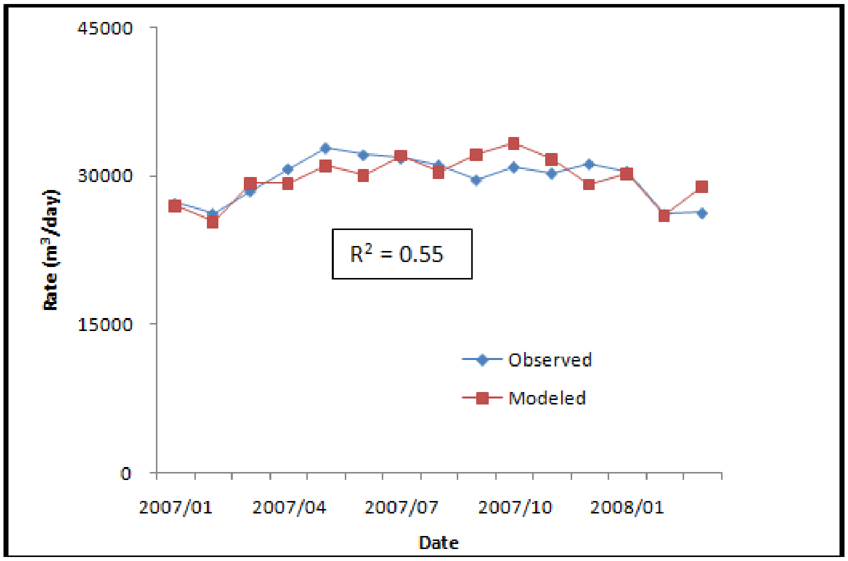

Figure 15 Comparison between total modeled liquid production rate and the total observed liquid production rate after the SE-N and SE-P procedure is applied to MLR with 1-month diffusivity filters

3.4.3. MLR with 6 Month Diffusivity Filter

The application of MLR with 6-month diffusivity filters gives the weighting coefficients shown in Figure 16. Results show both positive and negative linear relationships between the injector-producer well pairs. Contrary to the previous case of 1 month diffusivity, the degree of injector connectivity drops in some injectors and increases in others. The injector with the highest degree of connectivity changes to I7 whiles I8 still performs creditably well. In terms of producer performance, producer P17 still remains the best performing producer with producer P11 still registering lowest rate; this is consistent to a greater extent with the observed production rate as shown in Table 1. Figure 17 shows a comparison between the total modeled liquid production rate and the total observed liquid production rate.

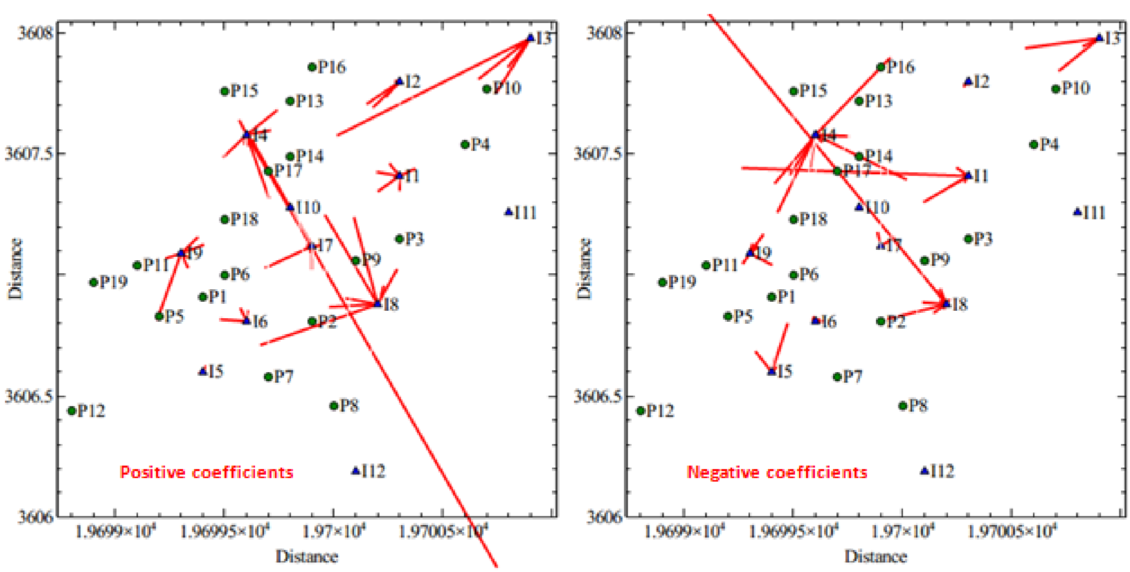

Figure 16 Weighting coefficients β_{ij} for MLR with 6-month diffusivity filters

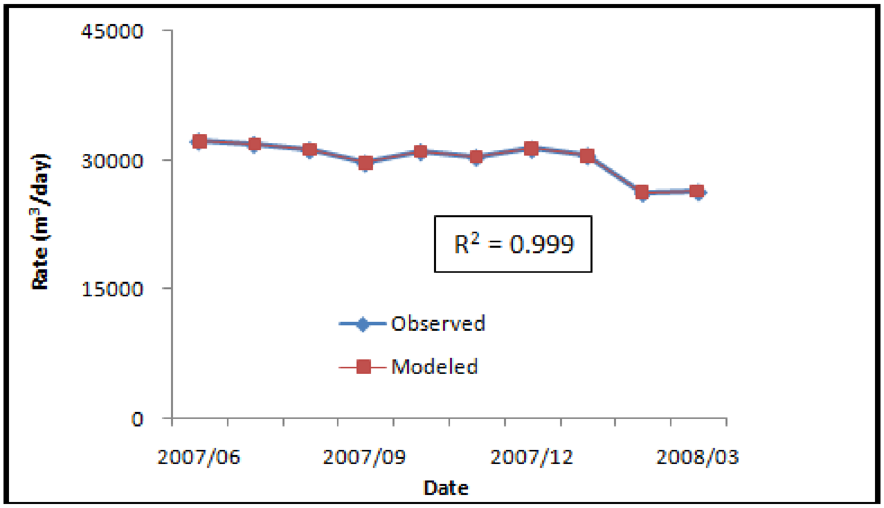

Figure 17 Comparison between total modeled liquid production rate and the total observed liquid production rate for MLR with 6-month diffusivity filters

3.4.4 Application of the SEN and SEP procedure to MLR with 6-month Diffusivity Filter

The SEN and SEP procedure did not help improve the prediction of flow rates. This was expected because; a less saturated model will have higher prediction errors. The weighting coefficients β_{ij} determined from the application of the SEN and SEP procedure is shown in Figure 18.

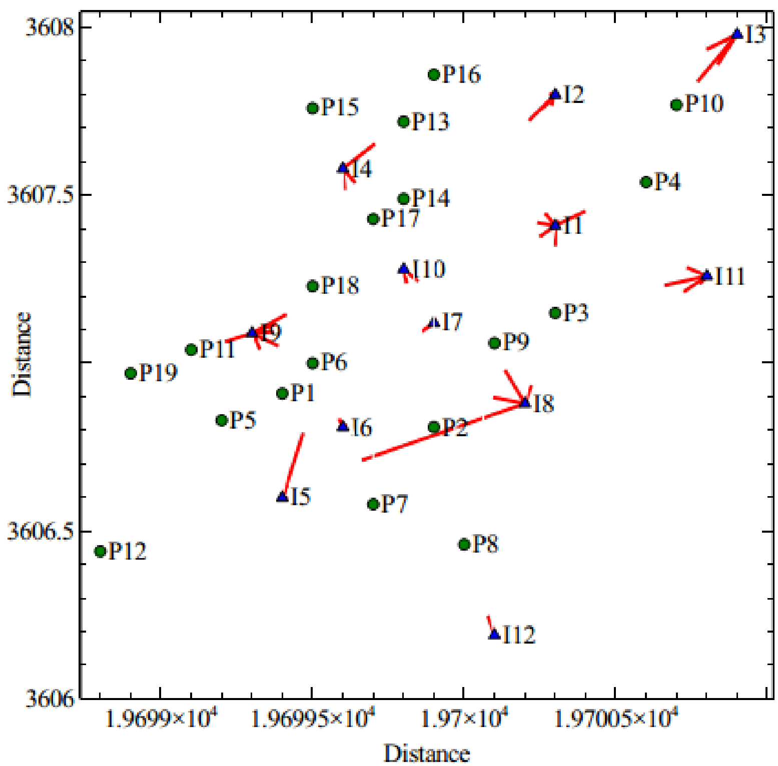

Figure 18 Positive weighting coefficients β_{ij} after the SE-N and SE-P procedure MLR with 6-month diffusivity filters

In general, Figure 18 shows that the sweep of fluid in the reservoir is not that efficient because the individual injectors are not influencing the producers in their surrounding area much. Figure 19 shows a comparison between total modeled liquid production rate and the total observed liquid production rate after the application of the SEN and SEP procedure. The R^2 value decreased from 0.99 to 0.45 after the SEN and SEP procedure was applied.

The application of the multiple linear regression techniques yielded much more scattered results.

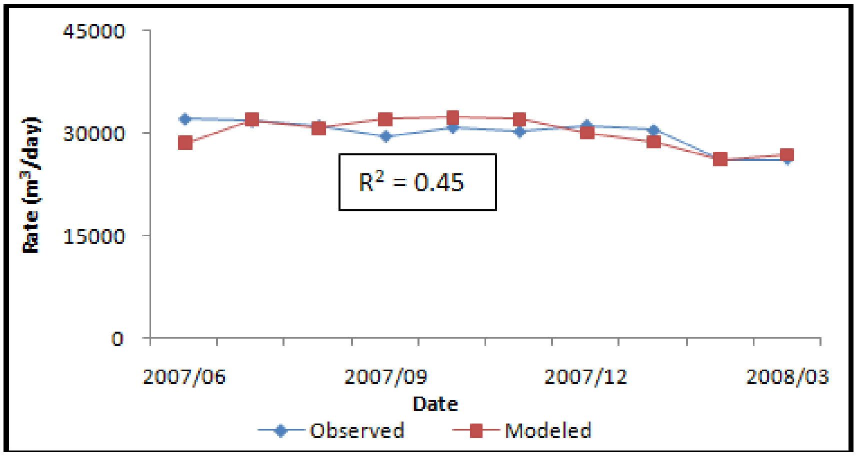

Figure 19 Comparison between total modeled liquid production rate and the total observed liquid production rate after the SE-N and SE-P procedure MLR with 6-month diffusivity filters

For this particular case there is a significant decrease in correlation coefficients for the longer diffusivity coefficients. An important assumption of the MLR method is that the predictor variables are linearly independent, that is, no linear relationship exists between the predictor variables. If the predictor variables carry common information, problems could occur in the model causing spurious results. A statistical phenomenon in which two or more predictor variables in a multiple regression model are highly correlated is called multicollinearity which could cause the negative weighting coefficients seen in the results. If multicolinearity does exist in these models, then the results obtained from the MLR model with and without diffusivity filters may be providing spurious results which should not be used for further interpretation of the sweep of fluid in the reservoir. As discussed earlier, Albertoni [1] suggests that these negative weighting coefficients should be set to zero thereby eliminating those injector-producer well pairs from the analysis. Since there is no statistical justification behind Albertoni's recommendation, to avoid the need for making ad hoc assumptions, an alternative method called the Simple Linear Model was used to evaluate injector-producer well pair connectivity.

3.5. Simple Linear Model (SLM)

The simple linear model for a production well *j* is

$$qj = a + bi_i \tag{14}$$

where the liquid production rate of a well *j*, is q_j, i_i is the injection rate of an injector *i*. The constant term *b* is simply the rate at which q_j changes with a change in i_i Edwards [7] and *a* is a constant that is added to the product between the constant *b* and i_i. When the value of *b* is positive, the relationship between the two variables is positive; that is, an increase in i_i is accompanied by an increase in q_j and a decrease in i_i is accompanied by a decrease in q_j (Edwards [7]).

A negative relationship means that an increase in i_i is accompanied by a decrease in q_j, and a decrease in i_i is accompanied by an increase in q_j (Edwards [7]). The application of the simple linear model to the system of producers and injectors gives the *b* values interestingly, similar to the MLR case without diffusivity filters with results indicating negative weighting coefficients. The system still registers negative weighting coefficients and positive coefficients greater than one.

Here the significance of the relationship between each injector-producer well pair is determined by a statistical significance hypothesis test. The types of statistical hypotheses are the null hypothesis H_o and the alternative hypothesis H_1. The null hypothesis is the hypothesis which requires no action to be taken. That is, no changes need be made. When trying to determine the significance between each injector-producer well pair, the null hypotheses for the statistical model is that there is no significant relationship between each injector- producer well pairs. The alternative hypothesis H_1 is a statement that does not agree with the null hypothesis.

The alternative hypothesis is accepted if the null hypothesis is to be rejected. In the SLM model, the alternative hypothesis states that the injector-producer well pair relationship is significant.

The choices of decisions to be made for the statistical significance hypothesis test will be to reject the null hypothesis H_o (and conclude H_1) or to not reject the null hypothesis.

The types of errors of a hypothesis test are the type I and type II errors. The α (P-value) term accounts for the probability of making a type I error which occurs when H_o is incorrectly rejected. Historically, the most frequently used α value has been 0.05. The P-value can be used to test the significance level of the relationship between each injector-producer well pair. A P-value less than 0.05 would then indicate that the relationship between the injector-producer well pair is significant and a P-value greater than 0.05 indicates that the relationship between the injector producer well pair is non-significant. The significant *b* values provide a key for further information about the effectiveness of the displacement which should help in the provision of recommendations for operational changes that might improve the displacement.

From the results of the hypothesis test, well pairs with significant relationships are represented with *b* values greater than 0 and the non-significant injector-producer relationships are represented with *b* values equal to 0. There are 46 well pairs with insignificant relationships. Only one producer is strongly correlated with injector I1; four producer wells (P3, P4, P10, P12 and P14) are strongly correlated with injector I3; three producer wells (P3, P15 and P18) are correlated with injector I10; two producer wells (P2 and P7) are also correlated with injector I11 and injector I12 has correlation with 6 producers (P3, P12, P13, P15, P17, and P18). Injector wells I2, I4, I5, I6, I8 and I9 have no correlation with the producer wells. Similar to the cases with and without diffusivity filters, there are well pairs with weighting coefficient greater than 1 and this relationship is statistically significant.

Figure 20 shows the magnitude of the relationship between each injector-producer well pair represented with the significant *b* values. As shown in Figure 20, injector I12 appears to have a stronger connectivity with producers P3, P15, P17 and P18 followed by Injector I3 with strong connectivity with producers (P3, P4, P12 and P14) which is consistent with the previous cases.

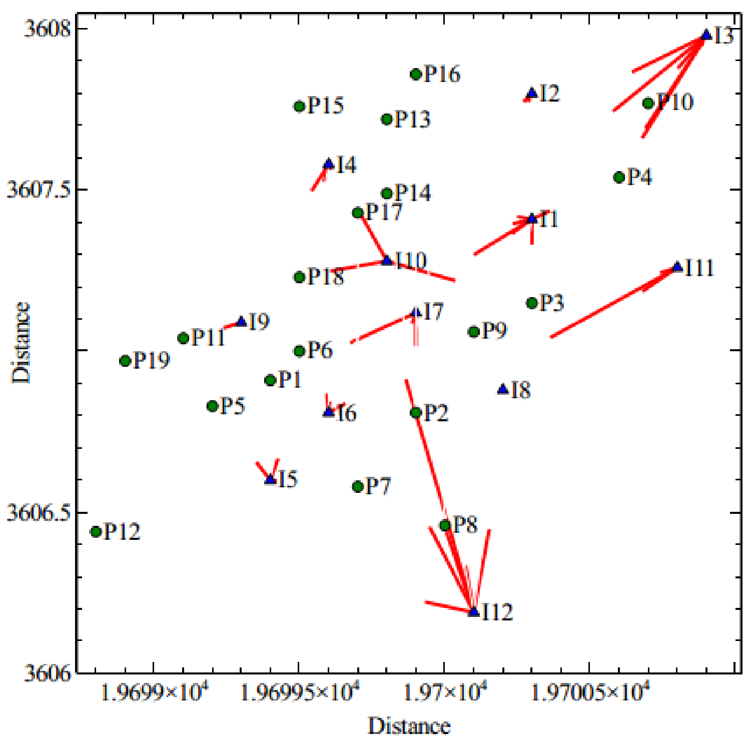

Figure 20 Representation of the significant *b* values for the SL M

Unlike the previous cases, injector I8 has no connectivity with producers in the system. This implies that the area around I8 has less effective sweep than would be suggested by MLR models. Producer P3 is being influenced by injectors I3, I10 and I12. Similarly, producer P12 is being influenced by injectors I3 and I12. Again, producers P15 and P18 are only being influenced by injectors I10 and I12. Producer P7 is not being influenced by injector I12; P13 and P16 are also not being influenced by injector I3 and P6, P14 and P17 are also not being influenced by injector I10.

The magnitude of connectivity between these producers and the surrounding injectors especially P6 and P16 is very low. The result of the SLM suggests that there is something preventing flow from injectors I12, I3 and I10 to these set of producers. One possibility is a barrier or fault in the portion of the field where producers P7, P6, P13, P14, P16 and P17 are located relative to injectors I3, I10 and I12. The suggested barrier or fault seems to be preventing the injection fluid from the adjoining injectors. The lack of connectivity between injectors and producers P6, P8, P9, P11, P16 and P19 confirms Table 1 as they are among the low rate performing producers. Injectors I3 and I12 seem to be having the highest influence on producers P12 and P17 respectively which are much further away from it in comparison to nearby producers.

3.5.1. Relationships between significant *b* Values and Reservoir Characteristics

Various plots showing the relationship between the *b* values for each injector-producer well pair and their cumulative recovery, average production, and other reservoir characteristics such as permeability and porosity for each producer were evaluated.

As shown in Figure 21, the distribution of the *b* values is approximately Log-Normal. So plots of the natural log of the *b* values for each producer and their cumulative recovery, average production, and other reservoir characteristics should give better desired relationships. While the correlations were better for the natural log of the *b* values than using the values themselves, the correlation coefficients were very small.

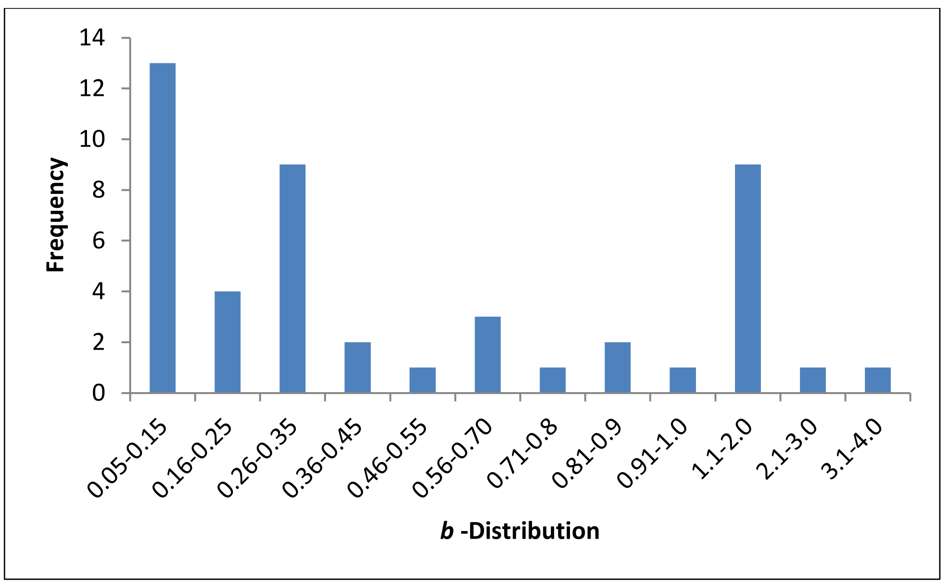

Figure 21 Log Normal Distribution of *b* values

Figure 22 shows an example plot showing the relationship between *ln(b)* values for each producer and the square of the distance between each injector-producer well pair. The relationship is negative which is expected because, at least in a heterogeneous environment, as the distance between each injector-producer well pair increases, there should be a decrease in the effect an injector has on a producer. The R^2 value for this relationship is very low which indicates a weak correlation.

The relationship between the summation of the *b* values for each producer and the oil production for each producer is shown in Figure 23 and is positive as expected. A greater $\sum b$ value for a producer *j* would imply that there is a larger effect of the injectors acting on that producer which would result in an increase in the average production (oil) from that well. It is important to note that there is a positive trend between the $\sum b$ and oil production yet the R^2 value is very low which shows very low correlation.

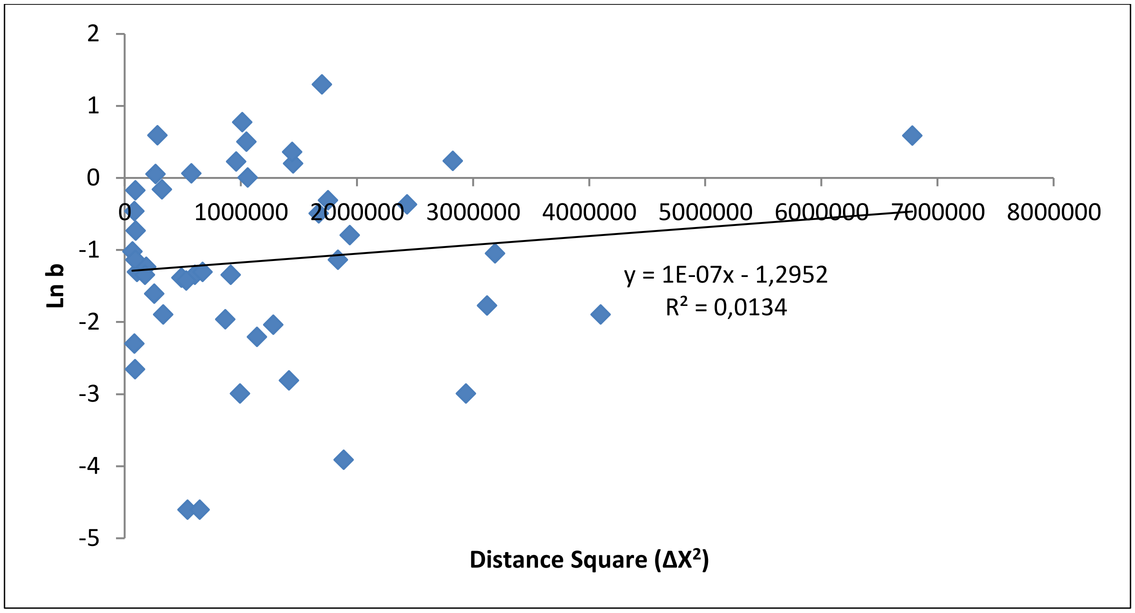

Figure 22 A weak relationship between the Log Normal *b* values and distance.

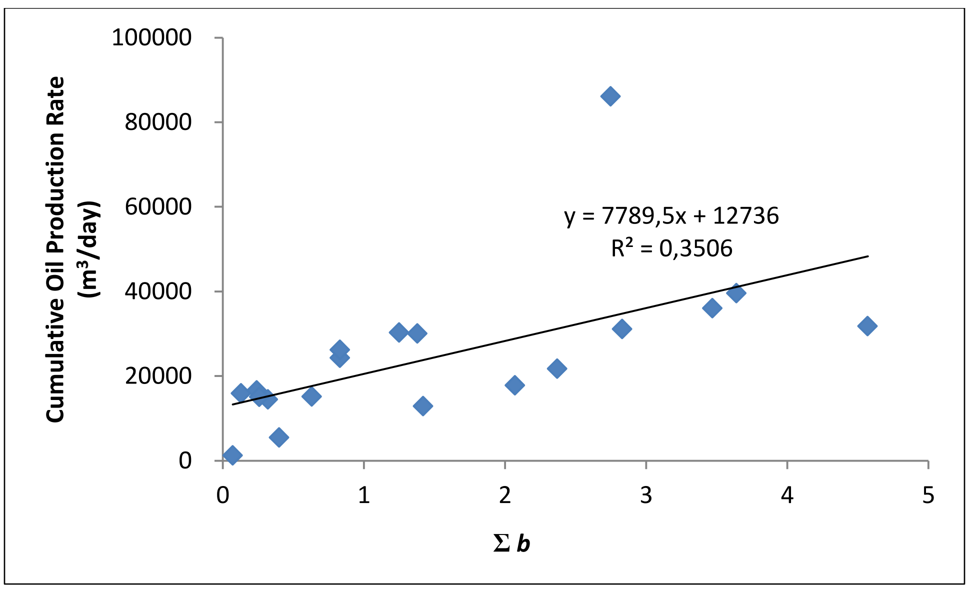

Figure 23 Relationship between the average oil production for each producer and summed *b* values for each producer

The most common measures of heterogeneity in the industry are the Dykstra-Parsons coefficient, V_{DP} and the Lorenz coefficient, L_C. Both measures range from zero to one where higher values scaled between about one-half and one correspond to higher heterogeneity (Lake and Jensen [9]). Therefore, a value of zero is for a completely homogeneous reservoir while a value of one is for an infinitely heterogeneous reservoir.

The Dykstra-Parsons coefficient is based on permeability distribution and is calculated as follows

$$V_{DP} = \frac{k_{50} - k_{84.1}}{k_{50}} \tag{15}$$

where the k_{50} term in equation "is the median permeability and the $k_{84.1}$ term is the permeability one standard deviation above k_{50} on a log-normal plot" (Dykstra and Parsons [6]; Jensen and Lake [10]).

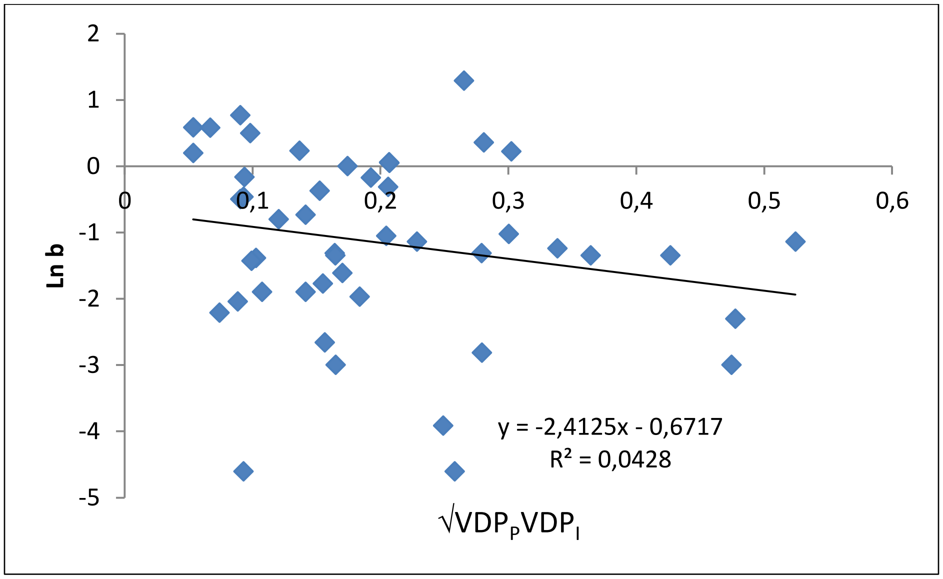

Figure 24 Relationship between the log normal b values for each producer and the Dykstra Parsons coefficient for each injector-producer well pair.

A high Dykstra-Parsons coefficient would imply heterogeneity in the reservoir which may cause an increase or decrease in the effect an injector *i* has on a producer *j*. Figure 24 shows a negative relationship between the natural log of *b* values for each producer and the Dykstra Parsons coefficient for each injector-producer well pair. This suggests that there is a decrease in the magnitude of connectivity between injector-producer well pairs in the more heterogeneous parts of the reservoir. Since the Dykstra-Parsons coefficient is based on permeability distribution, the decrease in magnitude of connectivity would suggest that due to the variability in permeability in the heterogeneous parts of the field, movement of fluid would be directed towards areas with higher permeabilities; thereby causing a decrease in connectivity between producer-injector well pairs in areas with lower permeability.

Figure 25 shows the ultimate relationship between the cumulative production rate and cumulative injection rate. There is a positive relationship between the cumulative injection and cumulative production rate. This shows that increasing the injection rate translate into increase in production rate. The coefficient of determination R^2 shows a better relationship. The performance of any arbitrary injection rate can be simply inferred.

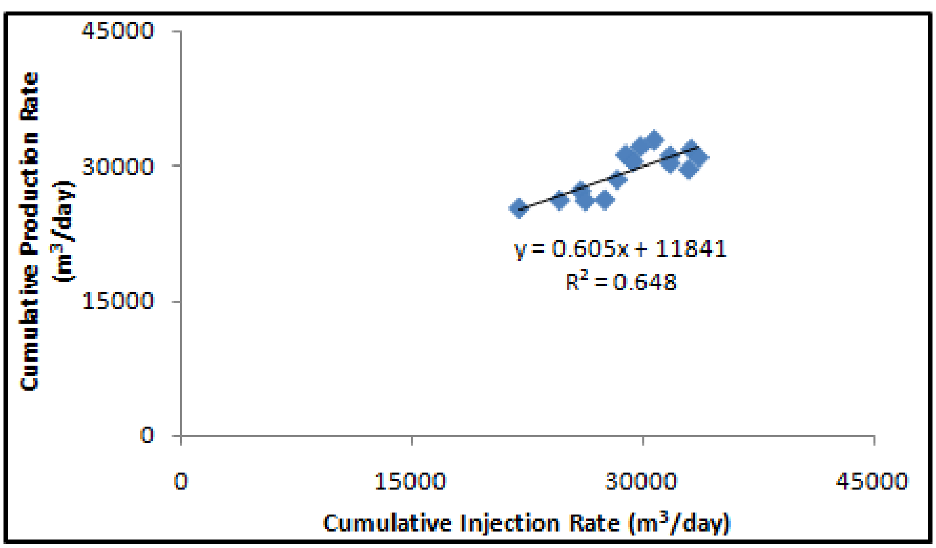

Figure 25 relationship cumulative rate and cumulative injection rate

4. Conclusions

The determination of the connectivity between injector-producer well pairs to evaluate the effectiveness of the displacement process is a difficult task if reservoir information is limited to injection and production rate data. Various statistical methods have been used to quantify connectivity to guide operational changes to improve the displacement process.

Compared to the Multivariate Linear Regression Model with SEN and SEP, the Simple Linear Model is less complicated and requires fewer ad hoc assumptions. SLM can be used to evaluate the relationship between each injector-producer well pair for a selected time period.

The significance of the *b* values which represents the relationship between each injector-producer well pair was obtained from the Simple Linear Model. Plots showing the relationships between the significant *b* values for each injector-producer well pair and their cumulative recovery, average production, and other reservoir characteristics were inconclusive based on the low correlation coefficients obtained. The Simple Linear Model and hypothesis test suggest that injector wells I1, I3, I7, I10, I11 and I12 have the most significant degree of connectivity.

Producer P17 shows a very strong connectivity with injector I12 and Injectors I2, I4, I5, I6, I8, and I9 shows little connectivity with its surrounding producers. To improve sweep in the non performing producers, the surface pressure for wells P6, P8, P9, P11, P16 and P19 should be reduced if possible. The increase in pressure drawdown should increase the productivity for these producer wells. Cutting back on the production from well P17 which shows a strong connectivity with injector I12 may help increase reservoir pressure in the area and allow for fluid to flow from Injectors I1, I3 and I10 to producers to the of their well location. By increasing the surface pressure for well P17, the operating flow rate for this well should decrease and more fluid will be directed away from this area. SLM results suggest that producer P6, P8, P9, P11 P16 and P19 have very little connectivity with injectors in the system. Producer P17 emerges as the best producing well from the SLM this is confirmed by Table 1.

Injector I6 has the highest total injection rate for the time period being analyzed and this raises a concern as to where that injection fluid is going. When injector I6 is shut-in or significantly cut back it will affect all the nearby producers and hence improve in production rate. If there is no rate change observed then injector I6 should be shut-in permanently. Injector I8 and I9 should be shut-in permanently as they have no effect on any of the surrounding producers in the system. The average daily injection rate for I3 is 10611.21m^3/day. Injector I3 performs very well yet it has the lowest injection rate. The rate for this injector should be enhanced since it will affect the nearby producers in the system. The injection rate of I10 should also be enhanced to improve on its effect on the surrounding producers. The SLM model gives real time connectivity between injector-producer well pairs and the hypothesis test gives meaning to the significance of the relationships between each injector-producer which provides information about the effectiveness of the displacement process.

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