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Boundary Layer Analysis of Micropolar Nanofluid with GO Nanoparticles in Water, Methanol and Kerosene over a Horizontal Circular Cylinder

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Abstract

The considered problem is devoted to the investigating of the mixed convection micropolar nanofluid flow over a horizontal circular cylinder. The constant wall temperature boundary condition is also possessed into account. To be more particular, the physical case of micropolar nanofluid in the presence of graphene oxide suspended in three base fluids, such as water, methanol, and kerosene are mathematically modeled in terms of PDEs. These equations are first converted into dimensionless form and then solved with the help of Keller-Box method, numerically. The model used for the micropolar nanofluid depends on the impacts of nanoparticles volume fraction χ, material parameter K and mixed convection parameters λ. Numerical solutions for local skin friction C_f and local Nusselt number Nu, as well as temperature, velocity, and angular velocity profiles are discussed through figures and tabular form. It is noticed that C_f, Nu and temperature are increased with increase χ. Also, GO–water based micropolar nanofluid has a higher temperature, velocity and angular velocity compared with GO–kerosene or GO–methanol). The accuracy of the present results is notarized by equating them with the previous literature..

Keywords: *Heat transfer; Mixed convection; Micropolar nanofluid; Keller–Box method; Circular cylinder.*

1. Introduction

The nanofluid expression was first conducted by Choi and Eastman [1] to explain the reduction with size less than 100 nm particles in the base fluid [2], such as water, ethylene glycol and kerosene [3]. The distinct discussions in fluid dynamics from different researchers formed it so clear that the presence of nanoparticles in the fluids produces to increasing the thermal conductivity of the fluid itself and therefore enhances the heat transfer characteristics [4]. Recently, the nanofluids have established wide usages in medical engineering and industrial which are frequently developing [5-6]. Nanofluids are engineered colloids involving a base fluid (e.g., water, kerosene). Nanofluids typically utilize mineral or mineral oxide nanoparticles, such as silver, iron, titanium oxide, and graphene oxide, and generally, the base fluid carries a conductivity property, such as kerosene, water, or ethylene glycol [7-8]. A considerable number of experimental and theoretical researches have been taken out by many studies on the thermal conductivity of nanofluids, it can be found in [9–13]. Important investigation on the boundary layer flow (BLF) of a nanofluid over a horizontal circular cylinder has been considered by Tham *et al.* [14] using Tiwari and Das [2] model. Najib *et al.* [15] studied the BLF and mass transfer on stagnation point over a shrinking cylinder in a nanofluid. Swalmeh *et al.* [16] considered the micropolar nanofluid BLF on the solid sphere under free convection effects. More studies on various sides of nanofluids over circular cylinder can be found in [17–21].

The non–Newtonian fluids have received great notice in the recent few years. Boundary layer theory has been utilized swimmingly to different non–Newtonian fluid models. To obtain examples of non-Newtonian fluids, we can mention liquid soaps, cosmetic products, dairy products such as cheese and butter and biological fluids like blood and saliva. Depending on the special properties of Non–Newtonian fluids, these fluids can divide into several kinds, such as Jefferly, Casson, Eyring–Powell, Crreau, Walter's–B and micropolar fluids. The micropolar theory was first conducted by Eringen [22-23]. In this theory, the micropolar fluid explains the micro–rotational impacts and micro–inertia. Later on, important studies have been done on the micropolar fluid to search for the significant results associated with various flow problems. The numerical solutions of convection BLF in micropolar over a circular cylinder by using Keller–Box method were considered by Nazar [24]. Heat transfer in micropolar fluid on a vertical cylinder has been analyzed by Rehman and Nadeem [25]. Furthermore, Swalmeh *et al.* [10,26], applied the Keller–Box method to examine BLF in the presence of micropolar nanofluid with constant wall temperature and constant heat flux. Lately, other notable researches on micropolar fluids comprise those by [27-28].

The heat transfer characteristics in the existence of mixed convection BLF about a circular cylinder have a wide extent in applied engineering, such as solar power, microelectronics to nuclear reactors [29–31]. Recently, the many papers in mixed convection BLF for a circular cylinder with various types of fluids were presented by several researchers as [32–35]. Depending on the previous researches, mixed convection BLF of a micropolar nanofluids on a horizontal circular cylinder with constant wall temperature is not studied. Furthermore, the current article inspects the mixed convection flow of a micropolar nanofluid over a horizontal circular cylinder. The pertinent PDE's are transformed to non–dimensional forms by using appropriate conversion and then solved by applying the Keller–Box scheme. The relevant notes of this investigation are recorded in the conclusions.

2. Basic equations

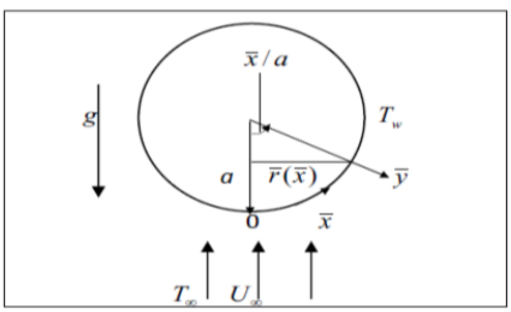

Figure 1. Systematic diagram of the present problem

A steady BLF of a micropolar nanofluid from a horizontal circular cylinder with radius a, with constant heat flux, which is immersed in a viscous and incompressible fluid, is taken. Furthermore, the free stream velocity is directed vertically upward with $q\_w > 0$ for assisting flow and $q\_w < 0$ for opposing flow as shown in Figure 1.

Under Tiwari and Das [2], the governing equations of micropolar nanofluid for the contemporary problem are:

$$\frac{\partial \hat{u}}{\partial \hat{x}}u + \frac{\partial \hat{v}}{\partial \hat{y}} = 0, \tag{1}$$

$$\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} = \hat{u}_e\frac{d\hat{u}_e}{d\hat{x}} + \left(\frac{\mu_{nf}+\kappa}{\rho_{nf}}\right)\frac{\partial^2 \hat{u}}{\partial \hat{y}^2} + \frac{\kappa}{\rho_{nf}}\frac{\partial \hat{H}}{\partial \hat{y}},$$
$$+\frac{(\chi\rho_s\beta_s + (1-\chi)\rho_f\beta_f)}{\rho_{nf}} g(T - T_\infty)\sin\left(\frac{\hat{x}}{a}\right), \tag{2}$$

$$\rho_{nf} j\left(\hat{u}\frac{\partial \hat{H}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{H}}{\partial \hat{y}}\right) = -\kappa\left(2\hat{H} + \frac{\partial \hat{u}}{\partial \hat{y}}\right) + \phi_{nf}\frac{\partial^2 \hat{H}}{\partial \hat{y}^2}, \tag{3}$$

$$\hat{u}\frac{\partial T}{\partial \hat{x}} + \hat{v}\frac{\partial T}{\partial \hat{y}} = \alpha_{nf}\frac{\partial^2 T}{\partial \hat{y}^2}. \tag{4}$$

The nanofluid properties are defined by Swalmeh *et al.* [36], as follows

$\rho_{nf} = (1-\chi)\rho_f + \chi\rho_f, \mu_{nf} = \frac{\mu_f}{(1-\chi)^{2.5}}, \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}},$

$$(\rho C_p)_{nf} = (1-\chi)(\rho C_p)_f + \chi(\rho C_p)_s, \phi_{nf} = \left(\mu_{nf} + \frac{\kappa}{2}\right) j, \tag{5}$$
$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\chi(k_f - k_s)}{(k_s + 2k_f) + \chi(k_f - k_s)},$$

In a particular case, when $(\chi = 0, K = 0)$, symbolizes viscous Newtonian fluid. Here, α_{nf} μ_{nf}, φ_{nf}, $(\rho C_p)_{nf}$ are the nanofluid thermal diffusivity, viscosity, spin gradient viscosity, heat capacity. Besides, other quantities are listed in nomenclature. The used boundary conditions [37] are

$$\hat{u} = \hat{v} = 0, T = T_w\,,\ \hat{H} = -\frac{1}{2}\frac{\partial \hat{u}}{\partial \hat{y}} \text{ as } \hat{y} = 0,$$
$$\hat{u} \to \hat{u}_e(\hat{x}), T \to T_\infty\ H \to 0 \text{ as } \hat{y} \to \infty, \tag{6}$$

where $\hat{u}_e(\hat{x}) = U_\infty \sin\left(\frac{\hat{x}}{a}\right)$ is the local free–stream velocity. The non–dimensional variables are recognized as follows

$$x = \frac{\hat{x}}{a}, y = Re^{2/5}\left(\frac{\hat{y}}{a}\right), r(x) = \frac{\hat{r}(\hat{x})}{a}, \theta = Re^{\frac{2}{5}}\left(\frac{T - T_\infty}{\frac{aq_w}{k}}\right),$$

$$u = \frac{\hat{u}}{U_\infty}, v = Re^{2/5}\left(\frac{\hat{v}}{U_\infty}\right), v = Re^{2/5}\left(\frac{\hat{v}}{U_\infty}\right), H = \left(\frac{a}{U_\infty}\right) Re^{-2/5}\,\hat{H}, \tag{7}$$

such that $Re = U_\infty \frac{a}{v_f}$ and v_f is called the kinematic viscosity of the base fluid.

Substituting the above variables (5) and (7) into equations (1)–(4), we get the next dimensionless equations for this problem

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\partial u_e}{\partial x} + \frac{\rho_f}{\rho_{nf}}(D(\chi) + K)\frac{\partial^2 u}{\partial y^2}$$
$$+ \frac{1}{\rho_{nf}}\left(\chi\rho_s\left(\frac{\beta_s}{\beta_f}\right) + (1-\chi)\rho_f\right)\lambda\theta \sin x + \frac{\rho_f}{\rho_{nf}} K\frac{\partial H}{\partial y}, \tag{9}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr\left[\dfrac{\frac{k_{nf}}{k_f}}{(1-\chi) + \chi\frac{(\rho c_p)_s}{(\rho c_p)_f}}\right]\frac{\partial^2\theta}{\partial y^2}} \tag{10}$$

$$u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y} = -\frac{\rho_f}{\rho_{nf}} K\left(2H + \frac{\partial u}{\partial y}\right) + \frac{\rho_f}{\rho_{nf}}\left(D(\chi) + \frac{K}{2}\right)\frac{\partial^2 H}{\partial y^2}. \tag{11}$$

In the the above system $D(\chi) = \frac{1}{(1-\chi)^{2.5}}$, and Prandtl number $Pr = \frac{v_f}{\alpha_f}$, as well as micropolar parameter $K = \frac{\kappa}{\mu_f}$, and the mixed convection parameter $\lambda = \frac{Gr}{Re^2}$. In constant wall temperature boundary condition case, the Grashof number is computed as $Gr = g\beta_f\left(\frac{aq_w}{k}\right)\left(\frac{a^3}{v_f^2}\right)$. The boundary condition (5) becomes

$$u = v = 0, \theta = 1, H = -\frac{1}{2}\frac{\partial u}{\partial y} \text{ as } y = 0,$$
$$u \to u_e(x) = \left(\frac{3}{2}\right)\sin x,\ u \to 0, \theta \to 0, H \to 0 \text{ as } y \to \infty. \tag{12}$$

We suppose the following variables to solve equations (8)–(11) and corresponding conditions (12) as

$$\psi = xf(x,y), \theta = \theta(x,y), H = xh(x,y). \tag{13}$$

Substituting above equation into (9–11), we acquire the following PDEs

$$\frac{\rho_f}{\rho_{nf}}(D(\chi) + K)\frac{\partial^3 f}{\partial y^3} + f\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \frac{1}{\rho_{nf}}\left(\chi\rho_s\left(\frac{\beta_s}{\beta_f}\right) + (1-\chi)\rho_f\right)\lambda\frac{\sin x}{x}\theta$$
$$+ \frac{\sin x \cos x}{x} + \frac{\rho_f}{\rho_{nf}} K\frac{\partial h}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right), \tag{14}$$

$$\frac{1}{Pr\left[\frac{k_{nf}/k_f}{(1-\chi)+\chi\frac{(\rho c p)_s}{(\rho c p)_f}}\right]}\frac{\partial^2\theta\partial\theta}{\partial y^2\partial y}\left(\frac{\partial f\partial\theta}{\partial y\partial x}-\frac{\partial f\partial\theta}{\partial x\partial y}\right) \tag{15}$$

$$\frac{\rho_f}{\rho_{nf}}\left(D(\chi)+\frac{K}{2}\right)\frac{\partial^2 h}{\partial y^2}+f\frac{\partial h}{\partial y}-\frac{\partial f}{\partial y}h-\frac{\rho_f}{\rho_{nf}}K\left(2h+\frac{\partial^2 f}{\partial y^2}\right)=x\left(\frac{\partial f}{\partial y}\frac{\partial h}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial h}{\partial y}\right). \tag{16}$$

The boundary condition (12) becomes

$$f=\frac{\partial f}{\partial y}=0, \theta=1, h=-\frac{1}{2}\frac{\partial^2 f}{\partial y^2} \text{ as } y=0,$$

$$\frac{\partial f}{\partial y}\rightarrow\left(\frac{3}{2}\right)\sin x\ \theta\rightarrow 0, h\rightarrow 0 \text{ as } y\rightarrow\infty. \tag{17}$$

The engineering interesting physical quantities, in dimensional form, are local skin friction C_f and Nusselt number Nu can be written as

$$C_f=\left(\frac{a}{\mu_f U_\infty}\right)Re^{-1/2}\left((\mu_{nf}+\kappa)\frac{\partial\hat{u}}{\partial\hat{y}}+\kappa\hat{H}\right)_{\hat{y}=0}, Nu=Re^{-1/2}\left(\frac{k_{nf}a}{k_f(T_\infty-T_f)}\frac{\partial T}{\partial\hat{y}}\right)_{\hat{y}=0}.$$

Using the equation (6) and boundary conditions (11), local skin friction C_f and Nusselt number Nu

$$C_f=Re^{-1/2}\left(D(\chi)+\frac{K}{2}\right)x\frac{\partial^2 f}{\partial y^2}(x,0), Nu=Re^{-1/2}\left(\frac{k_{nf}a}{k_f(T_\infty-T_f)}\frac{\partial T}{\partial\hat{y}}\right)_{\hat{y}=0}. \tag{18}$$

3. Results and discussion

Keller–Box scheme, along with finite differences reduction technique as conducted by Cebeci and Bradshaw [38] is implemented for computations of nonlinear equations (14) to (17). and sketched for several values of micropolar nanofluid parameters for the physical quantities, such as C_f, Nu, $\theta(x,0)$, $f^{'}(x,0)$ and $h(x,0)$, at various positions x in case of assisting $(\lambda>0)$ and opposing $(\lambda<0)$ flow. The data regarding thermo–physical characteristics of the used fluid and nanoparticles are listed in Table 1. Before we find the impacts of micropolar nanofluid, it needs to compare the values of C_f with Nazar *et al.* [39] at $K=0$ and $\chi=0$ as displayed in Table 2. We have found the values of C_f are in perfect agreement.

Table 1. Thermo–physical characteristics of GO–water, GO–kerosene GO–methanol

Material	ρ (kg/m^3)	C_p (J/kg − K)	K (W/m − K)	$\beta\times10^{-5}(K^{-1})$	Pr
Water	997.1	4179	0.613	21	6.2
Kerosene	783	2090	0.145	99	21
Methanol	792	2545	0.2035	149	7.38
GO	1800	717	5000	28.4	

Table 2. Values of C_f for $K=0$ and $\chi=0$ (Newtonian fluid), for different values of λ.

x	λ							
	-4	-3	-2	-1	-0.5	0.0	0.74	0.75
0°	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$\pi/18$	0.07987	0.1806	0.2665	0.3443	0.3810	0.4166	0.46761	0.4682
	(0.0801)	(0.1806)	(0.2662)	(0.3438)	(0.3804)	(0.4160)	(0.4669)	(0.4675)
$\pi/9$	0.1152	0.3269	0.5016	0.6583	0.7320	0.8034	0.9053	0.9067
	(0.1149)	(0.3261)	(0.5000)	(0.6564)	(0.7301)	(0.8014)	(0.9031)	(0.9045)
$\pi/6$		0.4043	0.6754	0.9138	1.0253	1.2110	1.2858	1.2878
		(0.4024)	(0.6718)	(0.9098)	(1.0211)	(1.1284)	(1.2813)	(1.2833)
$2\pi/9$		0.3737	0.7602	1.0860	1.2363	1.3806	1.5851	1.5878
		(0.3704)	(0.7535)	(1.0790)	(1.2292)	(1.3733)	(1.5775)	(1.5802)
$5\pi/18$			0.7199	1.1540	1.3457	1.5281	1.7848	1.7882
			(0.7181)	(1.1434)	(1.3350)	(1.5172)	(1.7737)	(1.7771)
$\pi/3$			0.5466	1.1014	1.3392	1.5623	1.8729	1.8770
			(0.5295)	(1.0866)	(1.3246)	(1.4577)	(1.8580)	(1.8621)
$7\pi/18$				0.9127	1.2078	1.4727	1.8550	1.8493
				(0.8929)	(1.1889)	(1.4583)	(1.8260)	(1.8307)
$4\pi/9$				0.5545	0.9326	1.2705	1.7023	1.7078
				(0.5280)	(0.9190)	(1.2480)	(1.6800)	(1.6855)

x	λ							
	-4	-3	-2	-1	-0.5	0.0	0.74	0.75
$\pi/2$					0.5243	0.9305	1.4573	1.4638
					(0.4813)	(0.9154)	(1.4289)	(1.4352)
$5\pi/9$						0.4612	1.1245	1.1321
						(0.4308)	(1.0847)	(1.0922)
$11\,\pi/18$							0.7003	0.7001
							(0.6543)	(0.6637)
$2\pi/3$								0.0427
								(0.0380)

The behaviours of C_f and Nu for the graphene oxide in differently based fluids such as water, methanol and kerosene under different parameters, such as fraction of nanoparticle volume fraction χ and micropolar parameter K, with various values of x are shown in the Figures 2 to 5. It is obvious that from these figures, when χ increases that lead to increasing Nu and C_f. It is also noticed that GO has higher Nu for kerosene based micropolar nanofluid than water and methanol based micropolar nanofluids, for various value nanoparticle volume fraction χ, but GO kerosene has higher C_f compared with GO for water and methanol for various value of χ. Figures 4 and 5 explain the impact of the mixed convection parameter λ on Nu and C_f, respectively. It is seen from these figures that an increase of the mixed convection parameter λ leads to decreases on Nu and C_f. Furthermore, Nu of GO kerosene is higher than GO (water/methanol) for every value of the mixed convection parameter λ. Also, C_f of GO kerosene is lower than GO (water/methanol) when $(\lambda > 0)$ (heated cylinder). On the other hand, the opposite happens behaviour is seen in case of $(\lambda < 0)$ (cooled cylinder), i.e. C_f for GO kerosene is higher than GO (water/methanol). This is because of the density of water has a highest than methanol and kerosene.

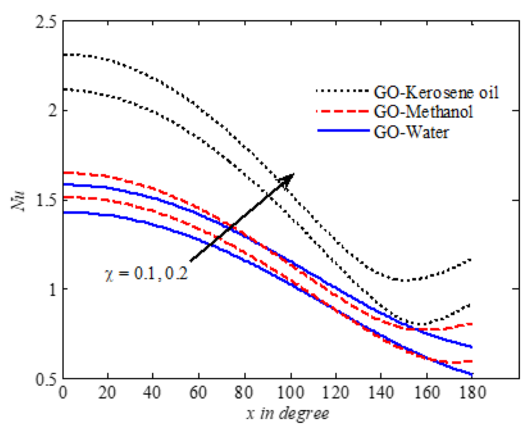

Figure 2. Nuunder different variation of x and χ

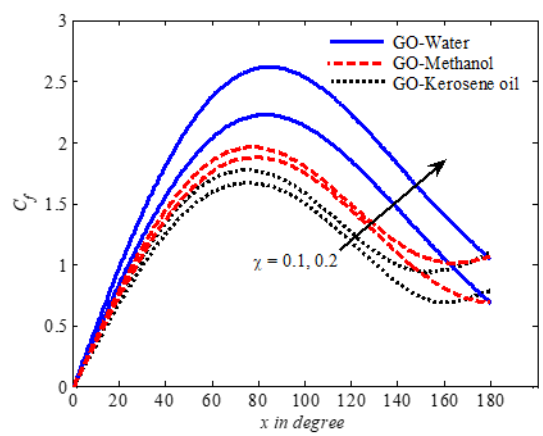

Figure 3. C_f under different variation of x and χ

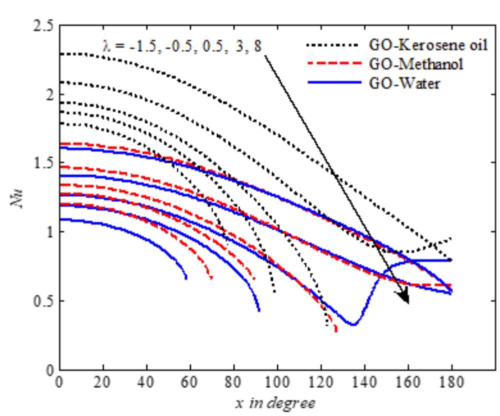

Figure 4. Nuunder different variation of $x$and λ

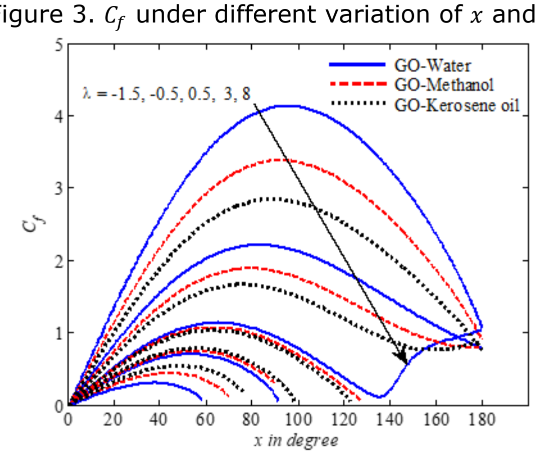

Figure 5. C_f under different variation of $x$and λ

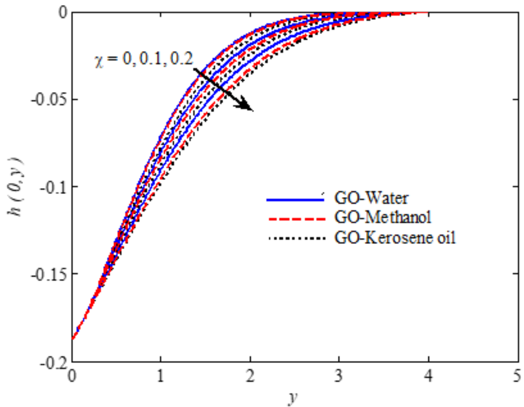

Figure 6. $h(0,y)$ at $x \approx 0$ for various values of χ

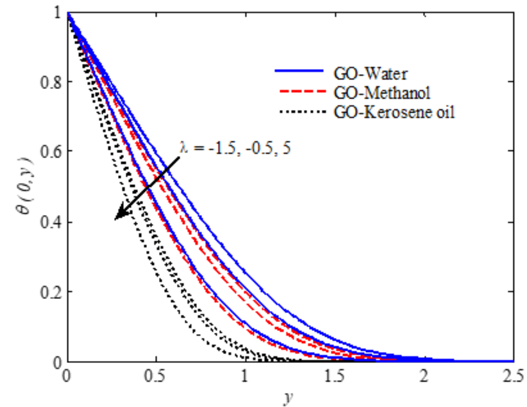

Figure 7. $\theta(0,y)$ at $x \approx 0$ for various values of λ

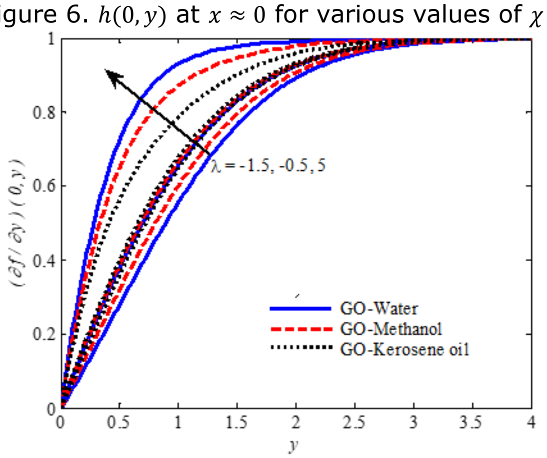

Figure 9. $(\frac{\partial f}{\partial y})(0,y)$ at $x \approx 0$ for various values of λ

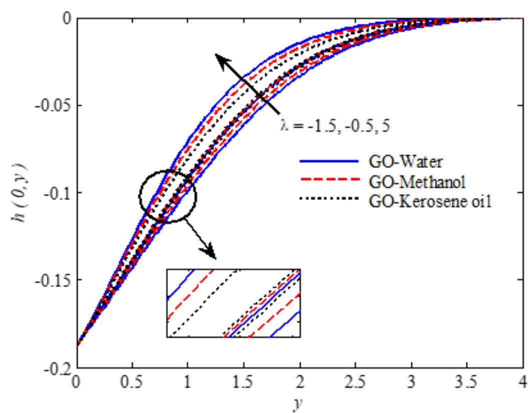

Figure 10. $h(0,y)$ at $x \approx 0$ for various values of λ

Figures 6 to 10 display impacts of the nanoparticle volume fraction χ, and the mixed convection parameter λ on the physical quantities, such as temperature, velocity, and angular velocity profiles, at the lower stagnation point of the circular cylinder $x \approx 0$. The GO suspended in three based fluids namely water, methanol, and kerosene are considered in this study. It can be observed that from Figures 6 to 10 the temperature field increase as increase the values of parameter χ, but the velocity and angular velocity decrease. Moreover, GO water has a higher temperature, velocity and angular velocity compared with GO (kerosene/methanol) for every values nanoparticle volume fraction χ.

5. Conclusions

The mixed convection BLF over a circular cylinder with micropolar nanofluid was investigated. Different parameters effects such as mixed convection and micropolar parameter parameters, nanoparticle volume fraction on GO–water (methanol/kerosene) were discussed.

We could the following conclusions:

a) The C_f and Nu with different values of x for GO (water/methanol/kerosene) based micropolar nanofluid decrease, when the value of χ increases.
b) An increase in the mixed convection parameter λ leads to a decrease of both C_f and Nu.
c) The GO kerosene has higher in Nu compared with GO (water/methanol) for various values of χ and λ.
d) An increase in the value of χ leads to an increase in the temperature, however temperature decrease when the mixed convection parameter λ increases.

Nomenclature

T_∞	*Ambient temperature,*	α_{nf}	*Thermal diffusivity of nanofluid*
T_f	*Temperature of fluid*	λ	*Mixed convection parameter*
U_∞	*Free stream velocity.*	μ_{nf}	*Viscosity of the nanofluid*
k_f	*Thermal conductivity of base fluid*	κ	*Vortex viscosity*
k_{nf}	*Thermal conductivity of nanofluid*	ν_f	*Kinematic viscosity of the fluid*
k_s	*Thermal conductivity of GO*	χ	*Nanoparticle volume fraction*
q_w	*Surface heat flux*	ρ_f	*Density of base fluid*
$u_e(x)$	*boundary layer*	ρ_s	*Density of GO*
a	*Radius of sphere*	ρ_{nf}	*Density of the nanofluid*
f	*Dimensionless stream function*	$(\rho C_p)_{nf}$	*Heat capacity of the nanofluid*
g	*Acceleration due to gravity*	ϕ	*Spin gradient viscosity*
K	*Micropolar parameter*	θ	*Dimensionless temperature*
Pr	*Prandtl number*	ψ	*Stream function*
Re	*Reynolds number*	μ_{nf}	
Gr	*Grashof number*		

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