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BUCKLEY-LEVERETT DISPLACEMENT THEORY FOR WATERFLOODING PERFORMANCE IN STRATIFIED RESERVOIR

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Abstract

A model is developed for performance prediction of waterflooding performance in stratified reservoirs using the Buckley-Leverett displacement theory. The paper discusses the flow process in heterogeneous reservoirs and the inherent assumptions underlining the model have been examined. The resulting formulation was used to forecast performance prediction and the inverse solution was used to determine the permeability distribution from displacement data. The results show that Dystra-Parsons advance theory may be applied to flow in heterogeneous porous media for processes that exhibit linear relation. The displacement performance of Dystra-Parsons two dimensional model approximates the one dimensional frontal advance theory with unit mobility ratio displacements. The performance relationship between the one dimensional model and the two dimensional is more qualitative at non-unity mobility ratios. However, true quantitative agreement is not attainable due to nonlinear relationship. The effect of mobility ratio and the Dykstra-Parsons permeability variation coefficient (V_{DP}) on the performance is investigated.

Keywords: oil and water displacement; recovery; water cut; pseudo relative permeability; mobility ratio; permeability variation.

1. Introduction

Most oil-bearing formations across the world are normally described by heterogeneous petrophysical properties. The most significant property that affects waterflooding performance is the absolute permeability and its variation normal to the direction of flow. This variation causes the displacing fluid to advance faster in zones of higher permeability and thus results in earlier breakthrough in such layers. In this case, the conventional frontal advance theory of Buckley-Leverett [1] and its graphical equivalent of Welge tangent construction method cannot be applied to the reservoir as a single layer. The reservoir is divided into a number of layers, each is assumed to be homogeneous with a constant permeability.

Different analytical models are available in the literature for waterflooding performance of stratified reservoirs [2-6,9-11,13]. Stiles [11] assumed the displacement velocity in a layer to be proportional to its absolute permeability neglecting the effect of mobility ratio. Dykstra-Parsons [2] developed a model for non-communicating layers without crossflow between layers while Hiatt [5] presented a model for communicating layers with complete crossflow. Warren and Cosgrove [13] applied Hiatt's model to stratified systems with a log normal permeability distribution. Hearn [6] developed expressions for the pseudo relative permeability functions for communicating stratified reservoirs. Reznik *et al.* [10] extended the Dykstra-Parsons method to continuous real-time basis. El-Khatib [3-4] investigated the effect of crossflow on the performance of stratified reservoirs and presented a closed form analytical solution for communicating stratified systems with log-normal permeability distributions.

All of the above mentioned models used to predict waterflooding performance in stratified reservoirs assume piston-like displacement in the different layers of the reservoir. Under this assumption, only oil flows ahead of the displacement front with a relative permeability k^0_{ro} at the irreducible water saturation. All recoverable oil is displaced by water leaving only the residual oil saturation behind the displacement front with water flowing with a relative permeability k^0_{rw} at the residual oil saturation. Only the end points of the rock relative permeability data k^0_{ro} and k^0_{rw} are used in these models. These two values with oil and water viscosities define the mobility ratio which is an important factor affecting the performance. When the displacement front in a given layer reaches the outlet face (producing well), no more oil flows from that layer and the production is completely water. All the recoverable oil ($1 - S_{wi} - S_{or}$) is produced from the layer at the time of water breakthrough. This leads to a highly optimistic performance prediction, i.e. higher fractional recovery and lower water cut. This is particularly aggravated at high mobility ratios where early breakthrough occurs with appreciable amounts of recoverable oil left behind the displacement front.

As opposed to piston-like displacement, the frontal advance theory shows that the saturation at the displacement front S_w* is less than ($1 – S_{or}$) and is determined by drawing a tangent to the fractional flow curve (f_w , S_w) from the point of initial conditions (S_{wi},0). The point of tangency determines the outlet water saturation and water cut, S_w* and f_w* at the time of water breakthrough. The intercept of the tangent with the horizontal line of $f_w=1$ determines the average saturation S_{avw} which is also below ($1 – S_{or}$). The oil recovery from the layer at time of breakthrough is ($S_w – S_{wi}$) and is equal to the reciprocal of the slope of the tangent line. After breakthrough, as water injection continues, more of the oil that is left behind the displacement front is recovered and the water fraction f_w increases steadily approaching the value of one as the oil recovery approaches the ultimate oil recovery of ($1 - S_{wi} - S_{or}$).

The outlet saturation and the water cut at any time after breakthrough are those at the point of tangency to the fractional flow curve of a line with slope equals to the reciprocal of the pore volumes of water injected into the layer (dimensionless time). Again, the intercept of the tangent with the horizontal line of $f_w=1$ locates the average saturation which determines the oil recovery at that time. Essentially all calculations of waterflood or other oil recovery process performance are based on the use of relative permeability functions which describe the local movement of phases based on their saturations. Several approaches have featured in the waterflooding where relative permeabilities are determined from the displacement data. For instance Buckley-Levertt [1]), Welge and Jonson [14], Johnson *et al.* [8], Jones and Roszelle [7] are routinely used to determine relative permeability functions from measurement of fractional flow of water leaving a core and the pressure drop across the core. This paper investigates the extent of the application of one dimensional frontal advance theory on flow in heterogeneous media. The advantages and limitations of the fractional flow in heterogeneous reservoirs was examined through the application of the fractional flow theory on displacement in stratified reservoir and the results juxtaposed with the results produced by the Dystra-Parsons theory on the same displacement. Moreover, the determination of pseudo relative permeabilities for the same system was considered.

Assumptions

The following assumptions are made:

1. The system is linear, horizontal and of constant thickness.
2. The flow is isothermal, incompressible and obeys Darcy's law.
3. Capillary and gravity forces are negligible
4. The system is divided into a number of homogeneous layers each with uniform thickness and constant permeability.
5. The system is a non-communicating with no crossflow allowed between the different layers.
6. The relative permeability characteristics are the same for all layers.
7. The initial fluid saturation is uniform at the irreducible water saturation.
8. The porosity is assumed constant in all layers. For convenience, the layers are arranged in decreasing order of permeability.

2. Materials and Methods

The summary of the methodology and the phases used in the study is described next. The section also provides in-depth explanation of the concepts and theories, which were employed during the process.

2.1. Theory of Fractional Flow

A flow of fluid representation displacement is based on the idea that a local change of a component is expressed only as a function of its neighborhood concentration.

Porosity volume of a hexadron: $\emptyset dxdydz$ (1)

Mass of fluid in a unit volume: $\rho\emptyset dxdydz$ (2)

Rate of fluid mass change during unit time: $\frac{\partial(\rho\emptyset)}{\partial t}dxdydz$ (3)

The total change of fluid mass during time dt: $\frac{\partial(\rho\emptyset)}{\partial t}dxdydzdt$ (4)

Obviously the total change of fluid mass should equal total mass differences between inflow and outflow during time dt, that is

$$-\left[\frac{\partial(\rho\vartheta_x)}{\partial x}+\frac{\partial(\rho\vartheta_y)}{\partial y}+\frac{\partial(\rho\vartheta_z)}{\partial z}\right]dxdydzdt=\frac{\partial(\rho\emptyset)}{\partial t}dxdydzdt \quad (5) \text{ or}$$

$$\left[\frac{\partial(\rho\vartheta_x)}{\partial x}+\frac{\partial(\rho\vartheta_y)}{\partial y}+\frac{\partial(\rho\vartheta_z)}{\partial z}\right]=\frac{\partial(\rho\emptyset)}{\partial t} \quad (6)$$

$$\frac{\partial(\rho\emptyset)}{\partial t}-\frac{\partial(\rho f_{\emptyset}\bar{\vartheta})}{\partial x}=0 \quad (7)$$

where $\emptyset$ is the concentration of the displacing fluid, $\bar{\vartheta}$ is the velocity, ρ is the density, and f is the volume fraction of the flowing fluid. If fluid is incompressible, (ρ) is constant and $\bar{\vartheta}$ is one space dimension time function,

$$\frac{\partial(\emptyset)}{\partial t}+\frac{\bar{\vartheta}\partial f_{\emptyset}}{\partial x}=0 \quad (8)$$

If $f_{\emptyset}$ is a function of $\emptyset$ only then

$$\frac{\partial f_{\emptyset}}{\partial x}=\frac{\partial f_{\emptyset}}{\partial\emptyset}\frac{\partial\emptyset}{\partial x} \quad (9)$$

Eq. 8 becomes

$$\frac{\partial(\emptyset)}{\partial t}+\vartheta\frac{\partial f_{\emptyset}}{\partial\emptyset}\frac{\partial\emptyset}{\partial x}=0 \quad (10)$$

Because $\emptyset$ is a function of space and time

$$\partial(\emptyset)=\left[\frac{\partial(\emptyset)}{\partial t}\right]dt+\left[\frac{\partial\emptyset}{\partial x}\right]dx \quad (11)$$

If $\emptyset$ is taken as constant, $\partial(\emptyset)=0$, then the velocity of the concentration ($\emptyset$) is

$$\left[\frac{dx}{\partial t}\right]_{\emptyset}=-\frac{\left[\frac{\partial(\emptyset)}{\partial t}\right]_{dt}}{\left[\frac{\partial(\emptyset)}{\partial x}\right]_{dx}} \quad (12)$$

Replacing Eq. (10) into Eq. (12), reduces it to the form

$$\left[\frac{dx}{\partial t}\right]_{\emptyset}=\vartheta\frac{\partial f_{\emptyset}}{\partial\emptyset} \quad (13)$$

Eq. (13) is similar in kind to the Backley-Leverett frontal displacement theory Eq. (14) if the local concentration is similar or the same as water saturation.

$$\left[\frac{dx}{\partial t}\right]_{Sw}=v\frac{\partial f_w}{\partial Sw} \quad (14)$$

From the Bacley-Leverett frontal displacement theory, the fractional flow is made as a functional part of water saturation and mobility ratio and is expressed as Eq. (15)

$$\mathrm{f_w}=\frac{1}{1+\frac{\mathrm{k_{ro}}\,\mu_\mathrm{w}}{\mathrm{k_{rw}}\,\mu_\mathrm{o}}} \quad (15)$$

Welge [14] proposed material balance approach for determination of the oil recovery determination from relative permeability data. Johnson *et al.* [8] and Jones and Roszelle [7] developed a model and for determination of relative permeabilities from dynamic displacement and subsequently outlined a simple regime for the determination of relative permeability functions from graphs of oil recovery and reduced pressure drop as a function of pore water injected. Performance prediction and interpretation of displacement performance have featured primarily in problems of waterflooding in a homogeneous medium.

However, these principles appear general and can be used in the displacement of other complicated systems contained in reservoirs composed of stratified layers.

2.2. Waterflooding performance in stratified reservoirs

Dystra and Parsons [2] discussed on displacement in stratified reservoirs. Their work assumed the reservoir to be composed of a number of individual layers that only relates at the wellbore. The individual layers are assumed to compose of homogeneous properties but different from each other. Dystra and Parsons considered the flow of oil to be described by piston-like displacement where the static properties of the individual layers such as initial saturations, porosities and relative permeabilities are considered the same for each layer. Based on the understanding, the position of the flood front at any time was derived analytically. An expression of coverage or oil recovery at breakthrough of successive layers was derived as below.

The term coverage, denoted by C, was defined as the fraction of the reservoir depleted by water. Eq. (16) is an expression for the coverage when the thickness of each layer is hi. The (n) layers are arranged in order of descending permeability with layer (j) representing the most recent layer flooded out.

$$\mathrm{C} = \sum_{i=1}^{j} \frac{\mathrm{h_i}}{\mathrm{h}} + \sum_{i=j+1}^{n} \frac{\alpha_\mathrm{i} \mathrm{h_i}}{\mathrm{h}} \tag{16}$$

The term αi is given by

$$\alpha_\mathrm{i} = \frac{\mathrm{M} - \sqrt{\mathrm{M}^2 + \frac{\mathrm{k_i}}{\mathrm{k_j}}(1 - \mathrm{M}^2)}}{\mathrm{M} - 1} \tag{17}$$

where: M is the mobility ratio based on end points of the relative permeability curves. Where the fractional flow expression may be defined in terms of the producing water-oil ratio (WOR) as

$$f_\emptyset = \frac{\mathrm{WOR}}{\mathrm{WOR+1}} \tag{18}$$

The WOR (water-oil ratio) corresponding to the flood out of layer j is given by

$$F_{WO} = \frac{\sum_{i=1}^{j} k_i}{\sum_{i=j+1}^{n} \left(\frac{k_i}{\sqrt{M^2 + \frac{k_i}{k_j}(1 - M^2)}} \right)} \tag{19}$$

The permeability of the individual layers may be determined from the cumulative probability distribution.

$$\mathrm{p(k)} = 0.5 + 0.5 erf \left[\frac{\ln\left(\frac{k}{k_m}\right)}{\sqrt{2}\sigma_k} \right] \tag{20}$$

where k_m is the mean and σ_k is the standard deviation of the distribution. In most cases however, the standard deviation is replaced by the Dystra-Parsons variation coefficient V_{DP} which are related in the following expressions,

$$\sigma_k = \ln \left[\frac{1}{(1 - V_{DP})} \right] \tag{21}$$

The Dystra-Parsons variation coefficient V_{DP} is estimated from the CDF plot by the following Eq.

$$V_{DP} = \frac{k_{50} - k_{84.1}}{k_{50}} \tag{22}$$

where $k_{50}$and $k_{84.1}$ are the values of permeability at 50% and 84.1% of cumulative probability. They are separated by one standard deviation of distribution σ_k. A value of V_{DP} of zero represents a homogeneous (constant K) while a value of 1 represents a totally heterogeneous reservoir. From Eq. (20), k can be obtained as

$$k = k_m exp\left[\sqrt{2\sigma_k}\, erf^{-1}\big(1 - 2\mathrm{p(k)}\big) \right] \tag{23}$$

This can be reduced as

$$ln\, k = ln\, k_m + \sqrt{2\sigma_k}\, erf^{-1}(2\mathrm{p(k)} - 1) \tag{24}$$

Eq. 24 can be written as linear Eq. in the form

$$y = mx + c \tag{25}$$

With $y = ln\, k$, $c = ln\, k_m$, $m = \sqrt{2\sigma_k}$, and $x = erf^{-1}(2\mathrm{p(k)} - 1)$

Values of x for different data points are computed by evaluating $erf^{-1}(2\mathrm{p}(\mathrm{k})-1)$. Eq. (25) is fitted to a straight line by the least square method. The slope of the straight line is $\sqrt{2\sigma_k}$ and the intercept is $ln\,k_m$. After these parameters, values of k are generated for $\mathrm{p}(\mathrm{k})$ ranging between 0 and 1.

2.3. Calculation of oil Recovery

Average coverage or recovery for a given outflow saturation was determined by the application of the Welge's material balance model. Fractional flow curves were determined from Eq.s 18 and 19 with permeability variation and mobility ration as variables. Average coverage as a function of injected water was obtained through the application of the Welge's material balance on the fractional flow curves. The gradient of the fractional flow curve was determined and the average water saturation calculated using Eq. (26)

$$\mathrm{S_{av}} = S_w^* + \left[\frac{1-\mathrm{f_{w^*}}}{\frac{df_w}{dS_w}}\right] \qquad (26)$$

The average outlet saturation $\mathrm{S_{av}}$ moves with velocity $\frac{df_w}{dS_w}$, hence the total pore volume injected is

$$\mathrm{t_D} = \left[\frac{1}{\frac{df_w}{dS_w}}\right] \qquad (27)$$

where $\mathrm{t_D}$ is the dimensionless time in pore volumes injected, $\mathrm{t_D} = \frac{\mathrm{qt}}{\emptyset AL}$. Therefore the average saturation at time $\mathrm{t_D}$ is given as

$$\mathrm{S_{av}} = S_w^* + (1-\mathrm{f_{w^*}}) \times \mathrm{t_D} \qquad (28)$$

The time scale of the displacement process is triggered by the water injection rate. Since the solution depends on time through the dimensionless value of pore volumes injected, the recovery curves are valid for constant-pressure injection, constant-rate injection or variable-rate injection. Hence the fractional flow and saturation derived above on one dimensional flow, can be used directly to calculate displacement performance. Based on the recovery performance of one dimensional flow in the Dystra-Parsons model, a similar calculation was made with the two dimensional form of Dystra-Parsons model. The waterflood performance forecast was put in the form of oil recovery and dimensionless pressure drop as a function of volume injected. Similarly, performance curves were generated for different permeability ratios using analytical results of Resinik *et al.* [10]. The unique thing about their model is it extends the Dystra-Parsons model beyond production performance only at layer breakthrough times to production performance on a continuous basis.

2.4. Pseudo Relative Permeability and Function Flow Curves

It is important to extend the discussion beyond the level of calculating the recovery given the description of the reservoir to the determination of the reservoir properties from displacement data. The property described here as pseudo relative permeability can be determined from the fractional flow model of the displacement given the availability of pressure data. The procedure is extended to two dimensional representation where layer permeabilities are determined from water production rate and overall pressure drop data. Displacement data may be expressed as graphs of water production and pressure drops all as a function of time. From the Dystra-Parsons model, the water production rate and pressure drop can be used to determine the individual layer permeabilities. The permeability thickness product of layer i from Darcy is given by Eq. (29)

$$k_i h_i = \frac{q_i}{\mathrm{w}}\frac{\mu_\mathrm{w}}{\mathrm{k_{rw}}}\frac{\mathrm{L}}{\Delta\mathrm{p}} \qquad (29)$$

Similarly the permeability thickness product of layer j=i+1,

$$k_j h_j = \frac{q_j - q_i}{\mathrm{w}}\frac{\mu_\mathrm{w}}{\mathrm{k_{rw}}}\frac{\mathrm{L}}{\Delta\mathrm{p}} \qquad (30)$$

where $\Delta\mathrm{p}$, is the pressure drop at the time of breakthrough of the layer.

The same procedure may be applied to subsequent layers to determine the permeability thickness product of each layer. For equal thickness layer, the permeability of each layer can be determined. Jones and Roszelle [7] presented a material balance technique that simplifies the procedure of pseudo relative permeability from displacement data. Firstly the fractional flow as a function of the outflow saturation is determined from the oil production curve. The slope of the oil production curve is calculated to determine $\mathrm{t_D}$ from Eq. 27and then the outflow saturation is determined from Eq. 28 where $(1-\mathrm{f_{w^*}})$ is obtained as the slope of the oil production curve. Then from the values of S_w^* and $\mathrm{f_{w^*}}$, the ratio of relative permeabilities can be obtained as a function of saturation using Eq. 15. The individual relative permeabilities can be obtained from the dimensionless pressure drop as follows

$$\mathrm{k_{ro}} = \frac{1-\mathrm{f_{w^*}}}{-\mathrm{t_D}\frac{d\left(\frac{1}{I_r}\right)}{dt_D}+\frac{1}{I_r}} \tag{31}$$

$$\frac{1}{I_r} = \frac{\left(\frac{\Delta \mathrm{p}}{\mathrm{q}}\right)}{\left(\frac{\Delta \mathrm{p_i}}{\mathrm{q_i}}\right)} \tag{32}$$

where $\frac{1}{I_r}$ refers to the dimensionless pressure drop and i initial values.

3. Results and Discussion

Effects of permeability variation and mobility ratio

Fractional flow curves as a function of outflow saturation, determined from Eq.s (18) to (19) with permeability variation and mobility ratio are shown in Figures 1 to 3. To investigate the effect of the reservoir heterogeneity, permeability distributions were generated using a log-normal distribution with values of V_{DP} of 0.2, 0.4, 0.6 and 0.8. The fractional flow increases with increasing mobility ratio as well as increasing permeability ratio. The more heterogeneous reservoirs show earlier water breakthrough there is higher water cut and increasing water saturation.

3.1. Oil Recovery Performance

The application of Welge material balance on the fractional flow curves above, gives average coverage as a function of volume injected as shown in Figures 4 to 6. As expected, the oil recovery decreases as mobility ratio increases. The oil recovery is higher for increasing heterogeneity of the reservoir. The effect is more evident in high mobility ratios. The reason is that for the case of high mobility ratios, the displacement front is very diffuse thus delay the water breakthrough and decreases oil recovery. For all mobility ratios recovery increases as pore volume injected increases. Figures 7 and 8 show the performance in terms of water cut as a function of fractional oil recovery. Earlier water breakthrough and higher water cut at the same fractional recovery is observed for higher mobility ratios.

3.2. Comparison of recovery performance in frontal advance model and Dystra-Parsons model

The curves for oil recovery as a function of volume injected are shown in Figures 9 to 11. The oil recovery in pore volume is equal to average coverage. A comparison was made for both one and two dimensional frontal advance formulation. If the computed results for both cases were completely consistent, Figures 4 to 6 would have been identical to Figures 9 to 11. However, comparison of the corresponding graphs show that calculated performance is identical only for unit mobility ratio. For instance, Figure 12 shows the difference in performance from one and two dimensional approaches, for mobility ratio of 0.5 and 5.0, and permeability variation of 0.8. It is observed that the one dimensional model underestimates the oil recovery for mobility ratio of 0.5, and over estimate the recovery for mobility ratio of 5. The difference in performance increases as mobility ratio increases departs from unity and as permeability variation increases. The difference occurs due to the fact that instability in saturation is a function of local speed which changes from layer to layer as injected fluid replaces original fluid with different viscosity. This is best explained by Figure 13. It is observed that the characteristic curve is only linear only at mobility ratio of 1. Hence, the two dimensional flow in Dystra-Parsons model approximate the one dimensional frontal advance approach only at unit mobility ratio.

3.3. Computation of Pseudo Relative Permeability

Figures 14 and 15 show the graphs of dimensionless pressure drop as a function of water injected in pore volume for the Dystra-Parsons model at various values of permeability variation and mobility ratio. It is observed that the dimensionless pressure decreases with increasing mobility ratio with the water injected in pore volume and will approach unity when mobility ratio approaches unity. Figure 16 shows the pseudo relative permeabilities determined from the displacement performance using Jones and Roszelle material balance technique Eq. (31). Pseudo relative permeabilities were determined from the Dystra-Parsons model for permeability variation of 0.6 and mobility ratio range of 0.5 to 5. It is observed that the pseudo relative permeability functions depend on both permeability variation and mobility ratio. Evaluation of the displacement data shown in Figures 17 and 18 show the production rate and pressure drop as a function of time for displacement in a ten layer system with a mobility ratio of 5.0 with reservoir property shown in Table 1. Evaluation of the displacement through Eq. (29) yields the individual layer permeabilities as shown in Figure 19. The values of the permeabilities were determined based on the reservoir properties in Table 1. Thus, if pressure drop and oil recovery data are available, Dystra-Parson model can be used to determine the reservoir properties.

Table 1 Reservoir properties

Length (ft)	650	Residual Oil Saturation (%)	20
Width (ft)	320	Relative Permeability to Oil at S_{wc}	1.0
Thickness (ft)	19.5	Relative Permeability to water at (1-S_{or})	1.0
Porosity (%)	19	Oil Viscosity (cp)	5.0
Initial Water Saturation (%)	19	Water Viscosity	1.0

4. Conclusion

The paper presents a model for application of Bakley-Leverett frontal advance theory to the displacement of water and oil in stratified reservoir systems. Jones and Roszelle analysis was used to develop pseudo relative permeabilities for heterogeneous reservoirs. The two dimensional Dystra-Parsons model for stratified systems approximates the one dimensional frontal advance theory with the effect of the heterogeneity in the fractional flow theory. There is an agreement between the two theories where mobility ratio approaches unity with a characteristic linear relationship. For nonunit mobility ratio, the relationship is more qualitative. The resulting pseudo relative permeability functions depend on the permeability variation, V, and the mobility ratio, M.

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References

[1] Buckley, S.E. and Leverett, M.C.: Mechanism of Fluid Displacement in Sands, 1942, Trans., AIME 146, 107.

[2] Dykstra, H. and parsons, R.L.: The Prediction of Oil recovery by Waterflooding, (1950), Secondary Recovery of Oil in the United States, 2nd ed., API 160–174.

[3] El–Khatib, N.A.: The Effect of Crossflow on Waterflooding of Stratified Reservsoirs, April, 1985, Soc. Pet. Eng. J., 291–302.

[4] El–Khatib, N.A.: Waterflooding Performance of Communicating Stratified Reservoirs with Log-Normal Permeability Distribution, Dec. 1999, SPEREE 2,542-49.

[5] Hiatt, N.W.: Injected–fluid Coverage of Multi–well Reservoirs with Permeability Stratification, 1958, Drill and Prod. Prac., API 165, 165–194.

[6] Hearn, C.L.: Simulation of Stratified Waterflooding by Pseudo Relative Permeability Curves, July 1971, J. Pet. Tech., 805–813.

[7] Jones, S. C. and Roszelle, W. O.: Graphical Techniques for determining Relative Permeability from Displacement Experiments, May 1978, J. Pet. Tech., 807-817.

[8] Jonhson, E. F., Bossler, D. P. and Naumann, V. O.: Calculations of Relative Permeability from Displacement Experiments, 1959, Trans., AIME 216, 370-372.

[9] Pande, K.K., Ramey, H. J. Jr., Brigham, W.E. and Orr, F.M. Jr.: Frontal Advance Theory for Flow in Heterogeneous Porous Media, April 8–10, 1987, paper SPE 16344 presented at SPE California Regional Meeting, Ventura, California.

[10] Reznik, A.A., Enick, R.M. and Panvelker, S.B.: "An Analytical Extension of the Dykstra–Parsons Vertical Stratification Discrete Solution to a Continuous, Real–Time Basis, 1984, Soc. Pet. Eng. J. 24, 643–656.

[11] Stiles, W.E.: "Use of Permeability Distribution in Water-flood Calculations, 1949, Trans., AIME 186, 9-13.

[12] Snyder, R.W. and Ramey, H.J. Jr.: Applications of Buckley-Leverett Displacement Theory to Noncommunicating Layered Systems, Nov. 1967, J. Pet. Tech., 1500-06.

[13] Warren, J.E. and Cosgrove, J.J.: "Prediction of Waterflood Behavior in a Stratified System, June 1964, Soc. Pet. Eng. J. 149–157.

[14] Welge, H.J.: "A Simplified Method for Computing Oil Recovery by Gas or Water Drive, 1952, Trans., AIME 195, 91–98.

[15] Willhite, P.G. (1986): Waterflooding, SPE Textbook Series, Vol 3, Richardson, TX. 139-45.

Supplement: Figures 1-19

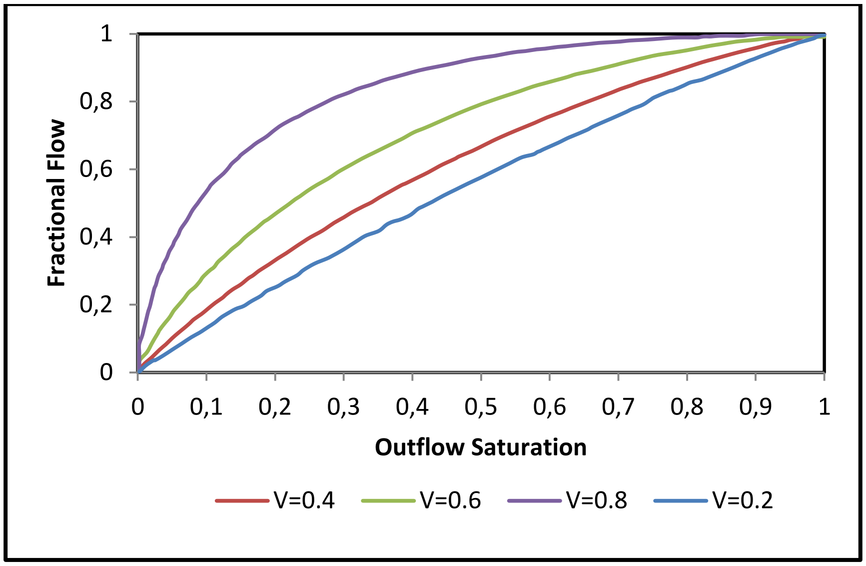

Figure 1 Fractional flow curves for the Dystra-Parsons model with mobility ratio of 0.5

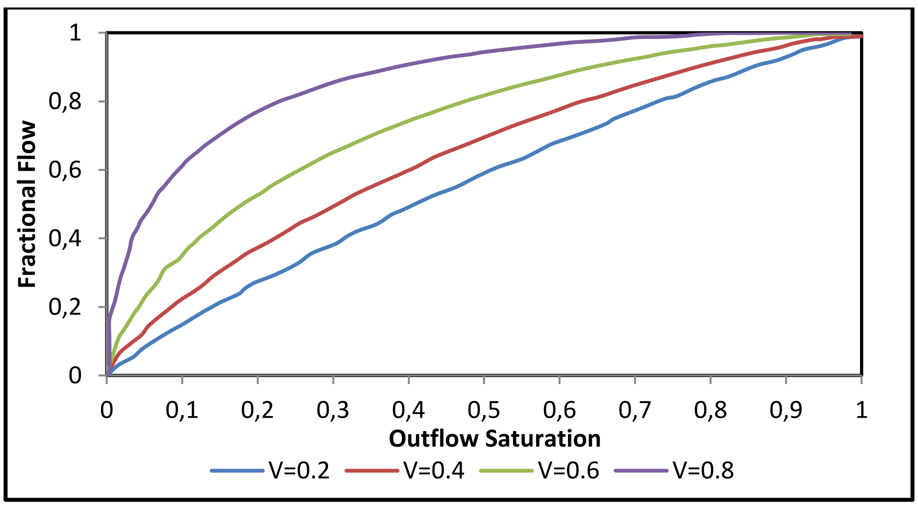

Figure 2 Fractional flow curves for the Dystra-Parsons model with mobility ratio of 1.0

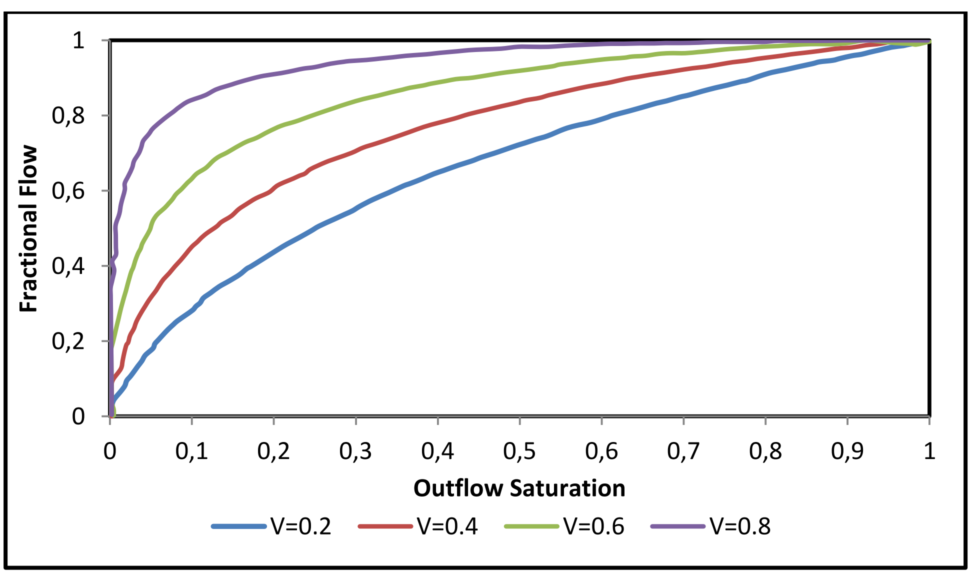

Figure 3 Fractional flow curves for the Dystra-Parsons model with mobility ratio of 5.0

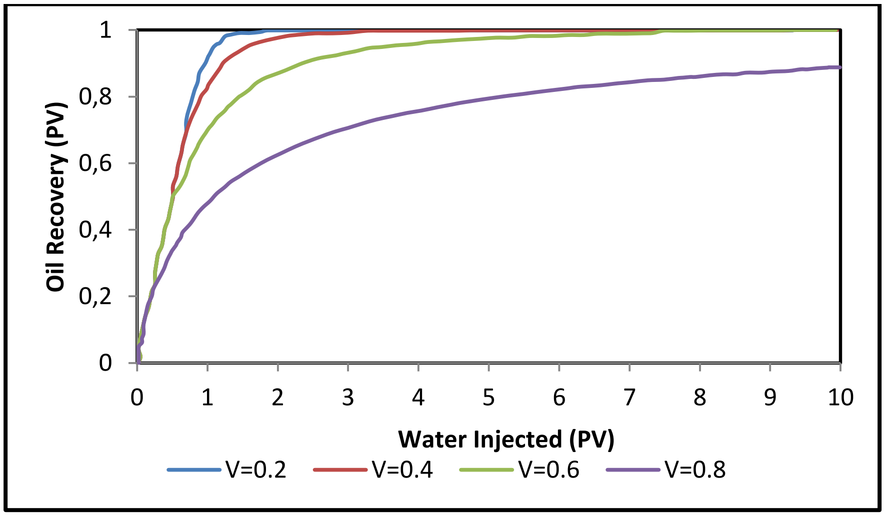

Figure 4 Oil recovery calculated using the one-dimensional frontal advance model for mobility ratio of 0.5

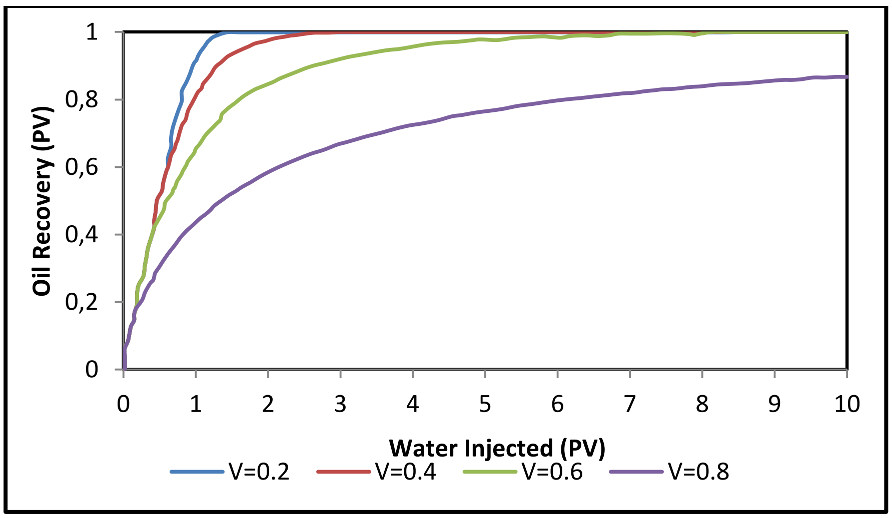

Figure 5 Oil recovery calculated using the one-dimensional frontal advance model for mobility ratio of 1.0

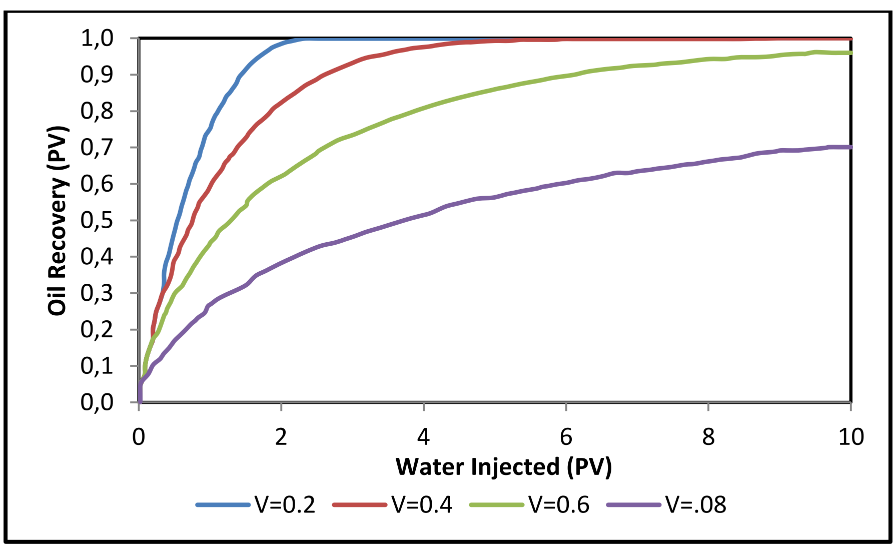

Figure 6 Oil recovery calculated using the one-dimensional frontal advance model for mobility ratio of 5.0

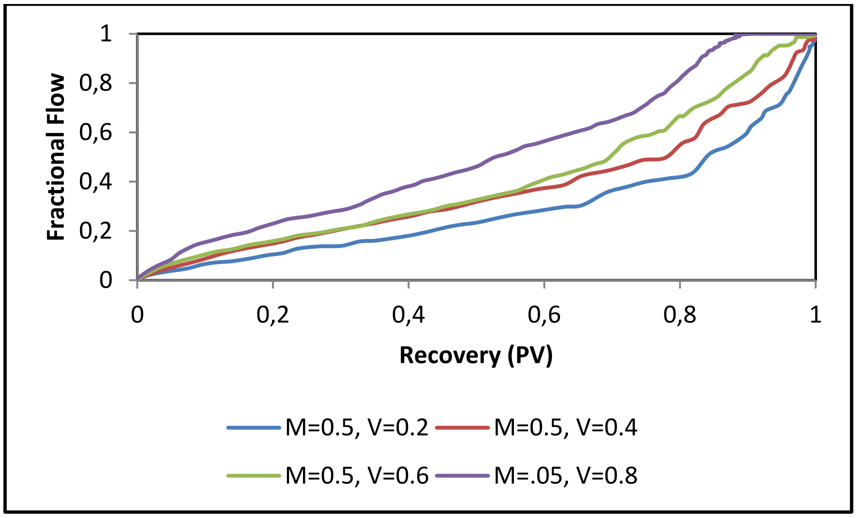

Figure 7 Effect of mobility ratio on oil recovery for mobility ratio of 0.5

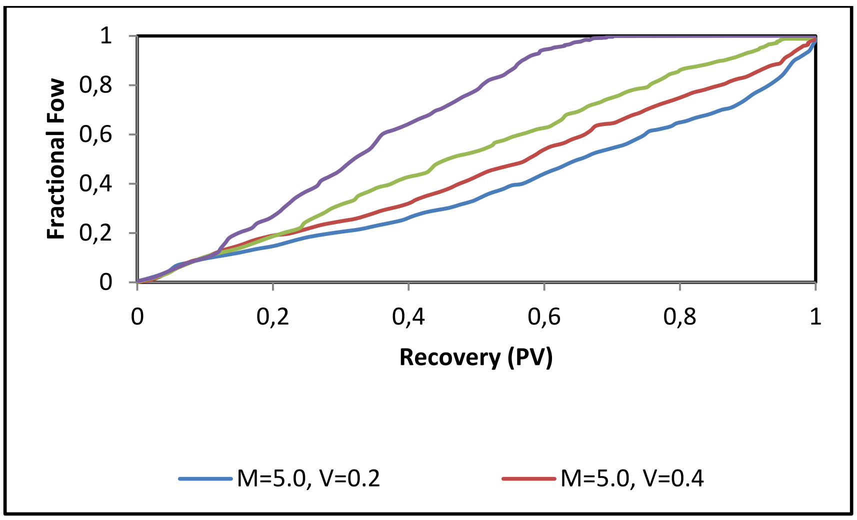

Figure 8 Effect of mobility ratio on oil recovery for mobility ratio of 5.0

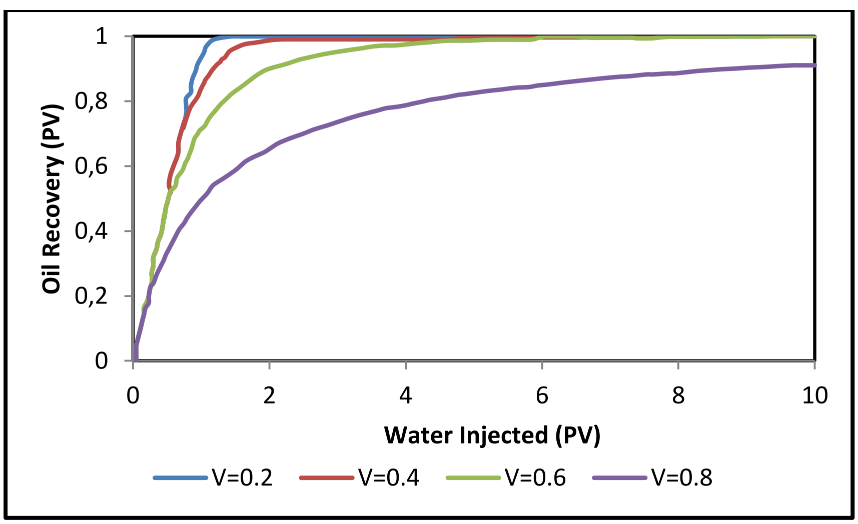

Figure 9 Oil recovery calculated using the two-dimensional Dystra-Parsons model for mobility ratio of 1.0

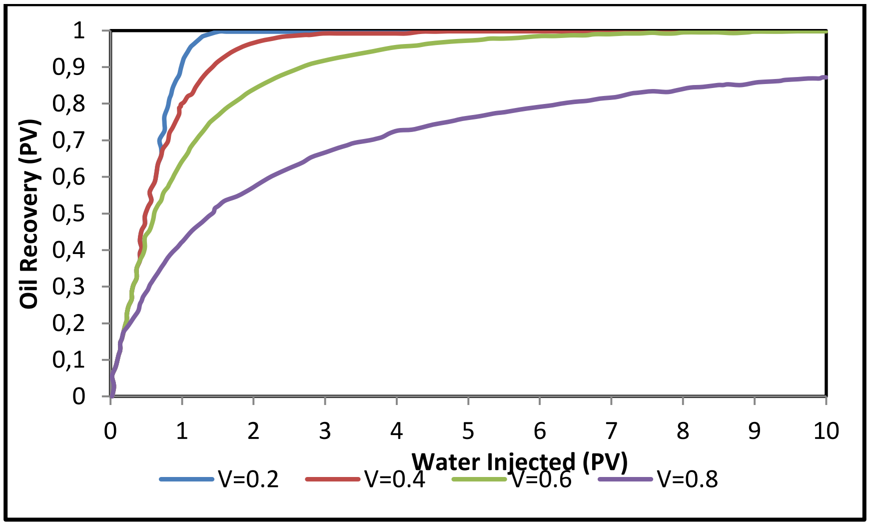

Figure 10 Oil recovery calculated using the two-dimensional Dystra-Parsons model for mobility ratio of 1.0

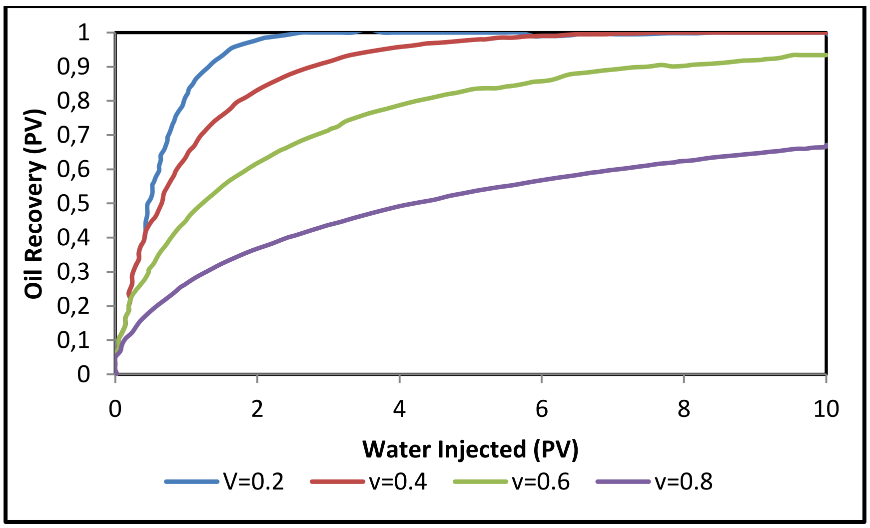

Figure 11 Oil recovery calculated using the two-dimensional Dystra-Parsons model for mobility ratio of 5.0

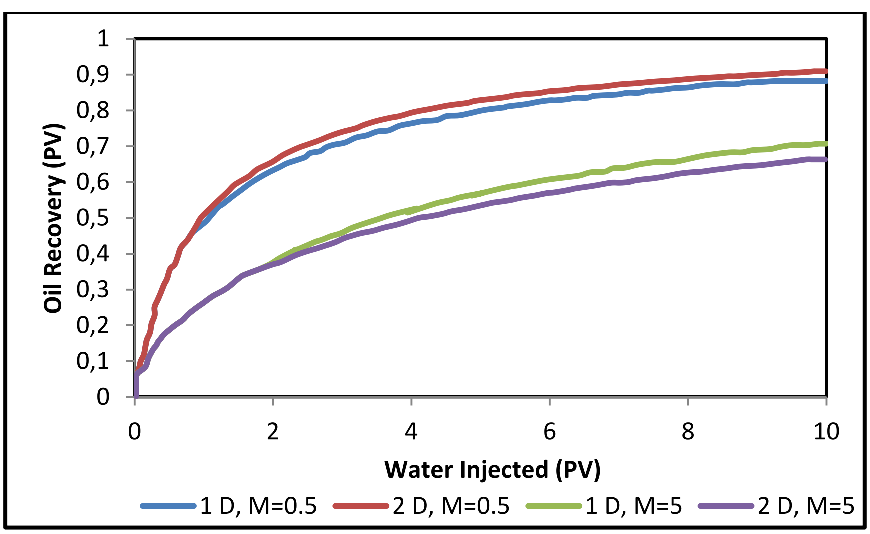

Figure 12 Comparison of oil recovery curves from one and two dimensional analysis for mobility ratios of 0.5 and 5 with permeability variation of 0.8

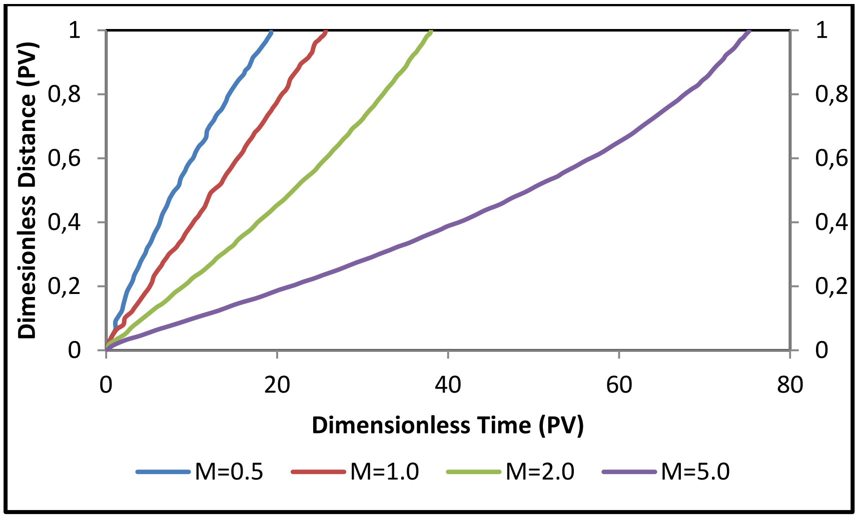

Figure 13 Characteristic curve for water saturation of according to two dimensional Dystra-Parsons model for permeability variation of 0.6

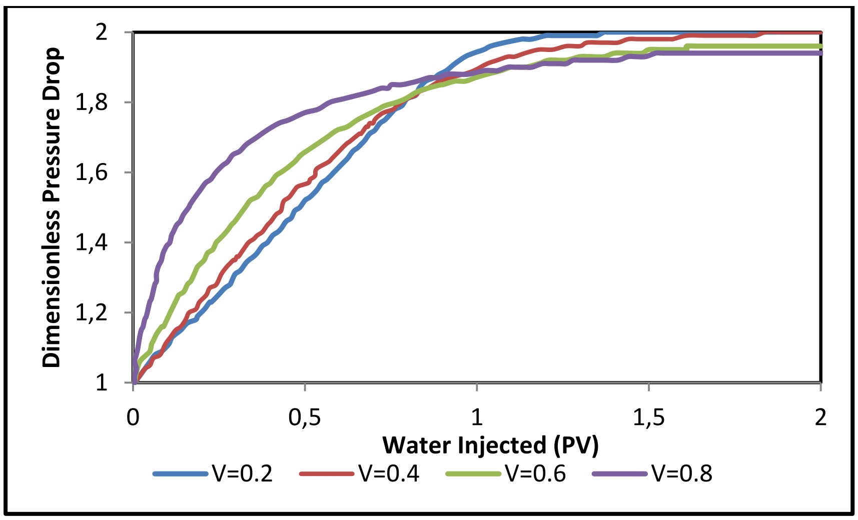

Figure 14 Dimensionless pressure drop calculated from the Dystra-Parsons model for mobility ratio of 0.5

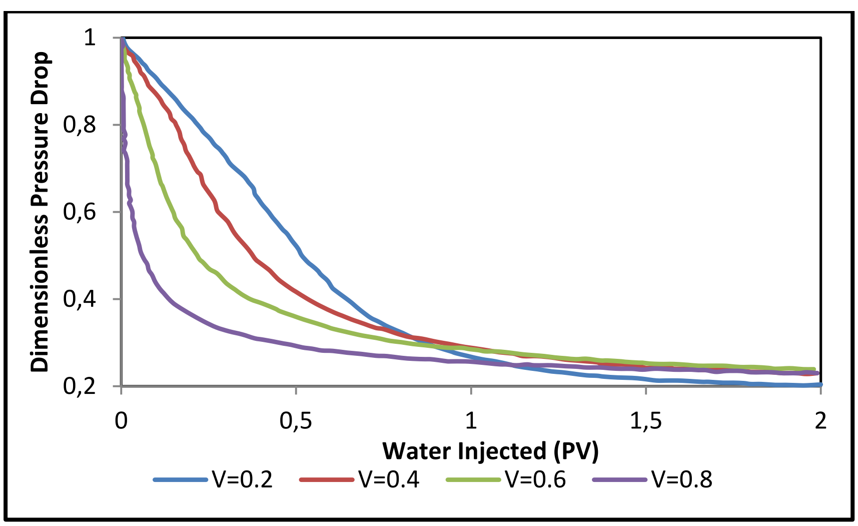

Figure 15 Dimensionless pressure drop calculated from the Dystra-Parsons model for mobility ratio of 5.0

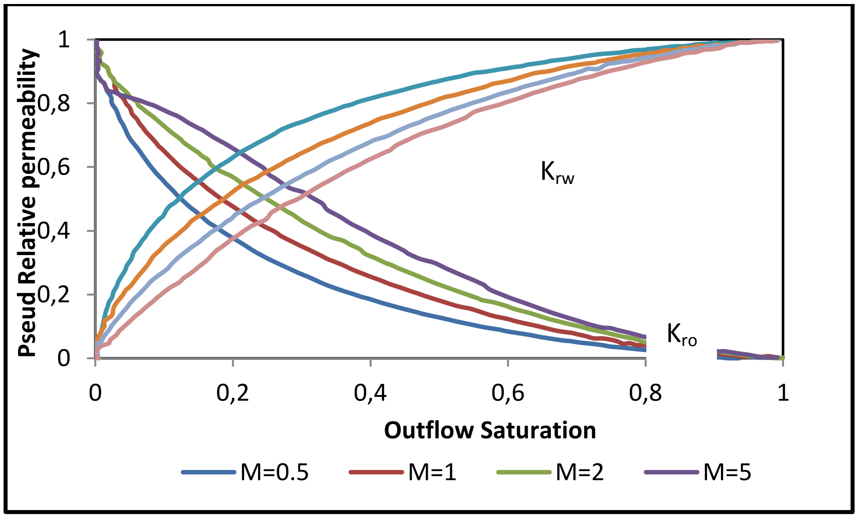

Figure 16 Pseudo Relative permeability from factional flow of displacement in a system of stratified reservoirs for permeability variation of 0.8

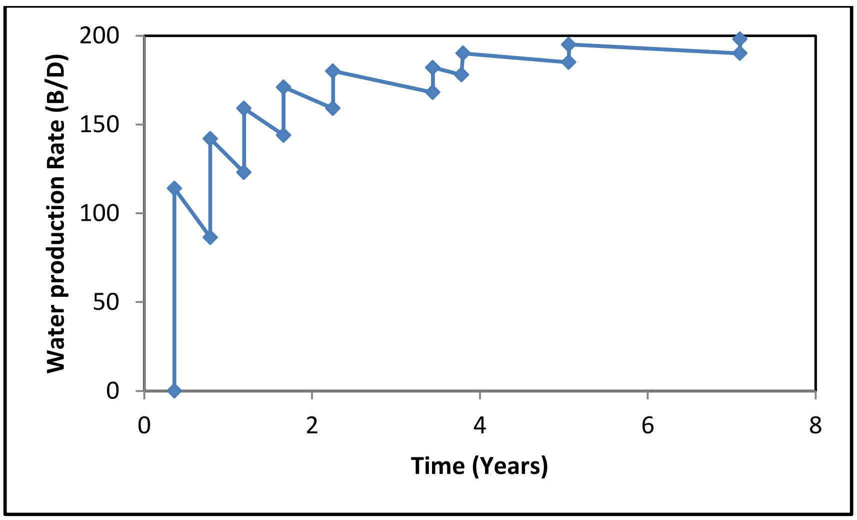

Figure 17 Water production rate for displacement in a stratified system

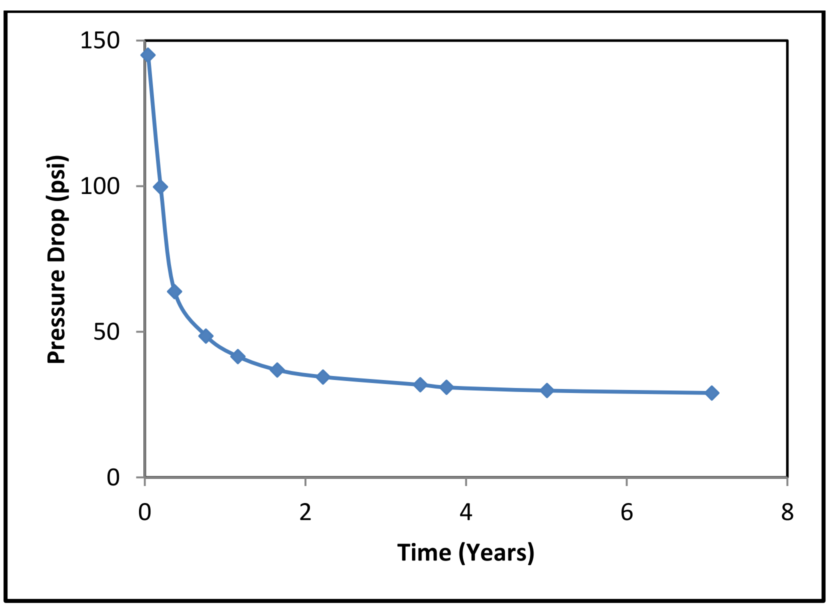

Figure 18 Pressure drop for displacement in a stratified system

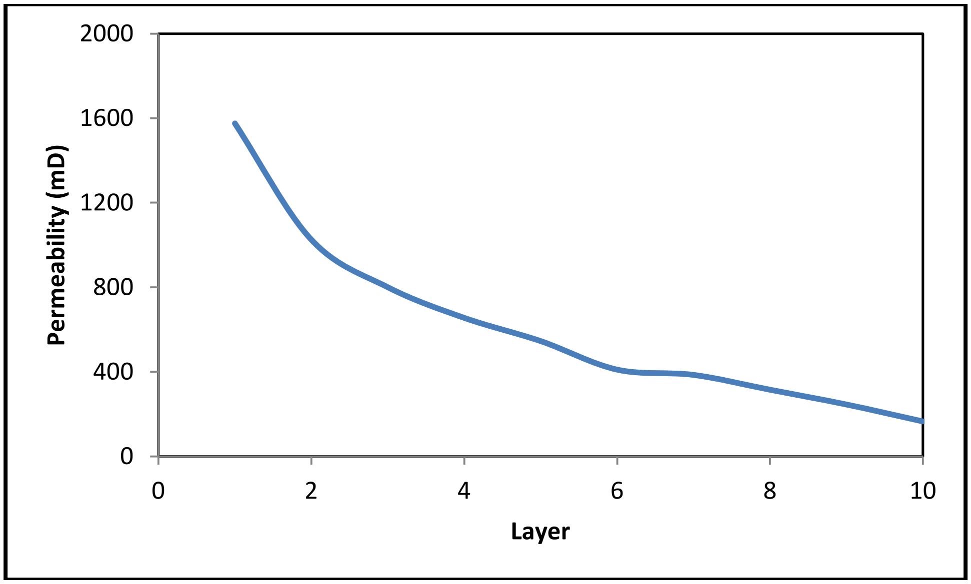

Figure 19 Layer permeability determined from displacement data