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Determination of Well Performance for Gas Wells: A Novel Prediction Method Using Taylor Series

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Abstract

Computational methods of determining the operating pressure (P_{wf}) and flowrate (q_{sc}) of gas wells are particularly important in the petroleum industry because it saves time and serves as a platform for petroleum engineers to perform complex sensitivity analysis on a production system. Sadly, methods to computationally determine the point of intersection between the IPR (Inflow Performance Relationship) and VLP (Vertical Lift Performance) curves are scarce in literature due to their complexity. This study seeks to determine dry gas wells' performance by predicting their bottomhole flowing pressure and corresponding gas flowrate, using the concept of Taylor series expansion. A computer program (TAYNOD) was developed to implement the proposed mathematical model. In addition, a graphical method of computing P_{wf} and q_{sc} was employed to validate the developed prediction model. The operating pressure and flow rate results were painstakingly read from the charts to obtain the exact solution. The exact values were then contrasted with the results from the developed prediction model. Nevertheless, it was established that the predicted values of P_{wf} and q_{sc} matched adequately with an R^2 of 0.937.

Keywords*: Computer program; Taylor series; Nodal analysis; Gas well performance; Computational method.*

1. Introduction

The Inflow Performance Relationship (IPR) of a well is a non-linear mathematical expression of the oil or gas volumetric production rate (qo or qg) and the bottom-hole flowing pressure (pwf), which is the reservoir pressure at the well-reservoir interface [1]. The performance of a given gas well can be determined by performing a matching of IPR (Inflow Performance Relationship) and VLP (Vertical Lift Performance) curves. The concept of matching these two curves (IPR and VLP) is known as nodal analysis [2]. Over the years, nodal analysis has proven to be the most popular means through which the performance of oil and gas wells are determined [3-4]. The IPR curve describes the flow of reservoir fluids from the reservoir to the wellbore, while the VLP curve demonstrates fluid flow from the wellbore to the surface [4-6]. Also, the point of intersection of the two curves represents the operating point of the gas or oil well as shown in Figure 1.

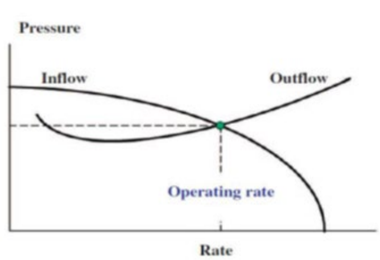

Figure 1. A typical nodal analysis chart for oil and gas wells [11]

However, graphical method has been the commonly used method to determine well performance in the oil and gas industry. Although the graphical method of performing nodal analysis has recorded huge success in literature, there is a need to compute the operating pressure and flowrate of a gas well without necessarily having to plot a chart first. Guo *et al.* [7] proposed that an iterative method, such as Newton Raphson, can be used to determine the point of intersection between the two curves (IPR and VLP).

Nonetheless, the method was deemed computationally exhausting and tasking to implement [7]. Consequently, methods to computationally determine the point of intersection between the IPR and VLP curves are scarcely available in the literature. As a result, most research works available in the literature are only centred on determining the performance of oil and gas wells using the graphical method [8-10]. Moreover, there is a serious need to be capable of promptly determining the operating flowrate and bottomhole flowing pressure of gas wells without using graphs on spreadsheets. This computational ability will assist engineers in performing sensitivity analysis speedily on a typical gas production system by varying relevant parameters such as wellhead pressure, tubing size, etc. Notwithstanding, this paper presents a novel computational method of performing nodal analysis for dry gas wells using Taylor series expansion.

2. Methodology

2.1 Mathematical modelling

This section briefly describes the procedures involved in using Taylor series expansion to determine the point of intersection between two functions. It should be noted that the two curves (IPR and VLP) to be analysed are polynomial functions whose roots represent the point of intersection of the curves. Therefore, given two multi-variable functions: u(x,y) and v(x,y), by applying the Taylor series expansion method, the following relationships are obtained [12]:

$$u_{i+1} = u_i(x_i, y_i) + (x_{i+1} - x_i)\frac{\partial u}{\partial x} + (y_{i+1} - y_i)\frac{\partial u}{\partial y} \quad (1)$$

$$v_{i+1} = v(x_i, y_i) + (x_{i+1} - x_i)\frac{\partial v}{\partial x} + (y_{i+1} - y_i)\frac{\partial v}{\partial y} \quad (2)$$

The roots of the two curves u(x,y) and v(x,y) corresponds to the values of x and y where u_{i+1} and v_{i+1} equals to zero. Thus, eq. 1 and 2 can be reduced to the following equations.

$$0 = u_i(x_i, y_i) + (x_{i+1} - x_i)\frac{\partial u}{\partial x} + (y_{i+1} - y_i)\frac{\partial u}{\partial y} \quad (3)$$

$$0 = v(x_i, y_i) + (x_{i+1} - x_i)\frac{\partial v}{\partial x} + (y_{i+1} - y_i)\frac{\partial v}{\partial y} \quad (4)$$

Collecting like terms,

$$x_i\frac{\partial u}{\partial x} + y_i\frac{\partial u}{\partial y} - u_i(x_i, y_i) = x_{i+1}\frac{\partial u}{\partial x} + y_{i+1}\frac{\partial u}{\partial y} \quad (5)$$

$$x_i\frac{\partial v}{\partial x} + y_i\frac{\partial v}{\partial y} - v_i(x_i, y_i) = x_{i+1}\frac{\partial v}{\partial x} + y_{i+1}\frac{\partial v}{\partial y} \quad (6)$$

Let $\mathrm{A} = x_i\frac{\partial u}{\partial x} + y_i\frac{\partial u}{\partial y} - u_i(x_i, y_i)$

and $\mathrm{B} = x_i\frac{\partial v}{\partial x} + y_i\frac{\partial v}{\partial y} - v_i(x_i, y_i)$ Thus,

$$A = x_{i+1}\frac{\partial u}{\partial x} + y_{i+1}\frac{\partial u}{\partial y} \quad (7)$$

$$B = x_{i+1}\frac{\partial v}{\partial x} + y_{i+1}\frac{\partial v}{\partial y} \quad (8)$$

Applying Crammer's rule to solve eq. 7 and 8 simultaneously,

$$x_{i+1} = \frac{1}{\frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial x}} \times \begin{vmatrix} A & \frac{\partial u}{\partial y} \\ B & \frac{\partial v}{\partial y} \end{vmatrix} \quad (9)$$

$$x_{i+1} = \frac{A.\frac{\partial v}{\partial y} - B.\frac{\partial u}{\partial y}}{\frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial x}} \quad (10)$$

Similarly,

$$y_{i+1} = \frac{B.\frac{\partial v}{\partial x} - A.\frac{\partial u}{\partial y}}{\frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial x}} \quad (11)$$

Applying eq. 10 and eq. 11 to the IPR and VLP of dry gas wells, the ordinate is taken as P_{wf} and the abscissa becomes q_{sc}. Thus, recall the back pressure equation for dry gas wells [6],

$$q_{sc} = C\left(P_r^2 - P_{wf}^2\right)^n \quad (12)$$

where q_{sc}= gas flow rate, Mscf/day; P_r = average reservoir pressure, psi; n = exponent; C = performance coefficient, Mscf/day/psi^2

Therefore, eq. 12 is the equation needed to specify the Inflow Performance Relationship of the dry gas wells. Meanwhile, in specifying the vertical lift performance (VLP) curve, the average temperature and pressure method was employed in this study. The method is summarised as follows [7];

$$P_{wf}^2 = \mathrm{Exp}(s)P_{hf}^2 + \frac{6.67\times10^{-4}\,[\mathrm{Exp}(s)-1]f_m Q_{sc}^2\bar{z}^2\bar{T}^2}{d_i^5 cos\theta} \tag{13}$$

where $\bar{z}$ = average compressibility of gas; $\bar{T}$ = average temperature; θ = inclination angle of tubing; P_{wf} = bottomhole flowing pressure; Q_{sc} = gas flowrate; d_i = internal tubing diameter; P_{hf} = tubing head pressure; L = tubing length.

$$s = \frac{0.0375\gamma_g L cos\theta}{\bar{z}\bar{T}} \tag{14}$$

Zeroing equation 12 and 13 to obtain two basic functions, the equation is given by,

$$u(q_{sc}, p_{wf}) = q_{sc} - C(P_r^2 - P_{wf}^2)^n = 0 \tag{15}$$

$$v(q_{sc}, p_{wf}) = P_{wf}^2 - \left[\mathrm{Exp}(s)P_{hf}^2 + \frac{6.67\times10^{-4}\,[\mathrm{Exp}(s)-1]f_m Q_{sc}^2\bar{z}^2\bar{T}^2}{d_i^5 cos\theta}\right] = 0 \tag{16}$$

Differentiating eq. 15 and 16, the following equations are obtained:

$$\frac{\partial u}{\partial P_{wf}} = 2nCP_{wf}(\mathrm{P_r^2} - P_{wf}^2)^{n-1} \tag{17}$$

$$\frac{\partial v}{\partial P_{wf}} = 2P_{wf} \tag{18}$$

$$\frac{\partial u}{\partial q_{sc}} = 1 \tag{19}$$

$$\frac{\partial v}{\partial q_{sc}} = \frac{1.334\times10^{-3}[Exp(s)-1]F_M q_{sc}\bar{z}^2\bar{T}^2}{d_i^5 cos\theta} \tag{20}$$

Therefore, the novel prediction method is given as;

$$q_{sc_{i+1}} = \frac{A.\frac{\partial v}{\partial p_{wf}} - B.\frac{\partial u}{\partial p_{wf}}}{\frac{\partial u}{\partial q_{sc}}.\frac{\partial v}{\partial p_{wf}} - \frac{\partial u}{\partial p_{wf}}.\frac{\partial v}{\partial q_{sc}}} \tag{21}$$

$$p_{wf_{i+1}} = \frac{B.\frac{\partial u}{\partial q_{sc}} - A.\frac{\partial v}{\partial q_{sc}}}{\frac{\partial u}{\partial q_{sc}}.\frac{\partial v}{\partial p_{wf}} - \frac{\partial u}{\partial p_{wf}}.\frac{\partial v}{\partial q_{sc}}} \tag{22}$$

where,

$$\mathrm{A} = q_{sc_i}\frac{\partial u}{\partial q_{sc}} + y_i\frac{\partial u}{\partial p_{wf}} - u\left(q_{sc_i}, p_{wf_i}\right) \tag{23}$$

and

$$\mathrm{B} = q_{sc_i}\frac{\partial v}{\partial q_{sc}} + y_i\frac{\partial v}{\partial p_{wf}} - v\left(q_{sc_i}, p_{wf_i}\right) \tag{24}$$

Also, q_{sc_i} and p_{wf_i} represents the initial guess for gas flowrate and bottomhole flowing pressure, respectively. In short, Eq. 21 and Eq. 22 will be solved iteratively until the difference between the future and previous values is less than 0.5, i.e., 5%. The models could be incorporated into a spreadsheet program or any programming language.

2.1.1. Model assumptions

Following are the assumptions of the model developed in this study:

1. The well involved is a dry gas well
2. Average compressibility factor is known
3. A turbulent flow regime was assumed at the wellbore of the gas well.

2.2. Computer modelling

This section details the computer method employed to incorporate the developed mathematical predictive model. However, this study uses C# programming language to implement the developed mathematical model. The flowchart for the developed computer program (TAYNOD) is indicated in Figure 2.

Table 1. Input data [5,7]

Parameters	Well-A	Parameters	Well-A
Tubing length	10,000ft	Wellhead pressure	800psi
Tubing roughness	0.0006	Gas specific gravity	0.71
Tubing internal diameter	2.259in	Tubing head temperature	150oF
Inclination angle	0	Reservoir pressure	2000psi
C- back pressure constant	0.01Mscf/d-psi^{2n}	Bottomhole temperature	200^{o}F
n-exponent	0.8		

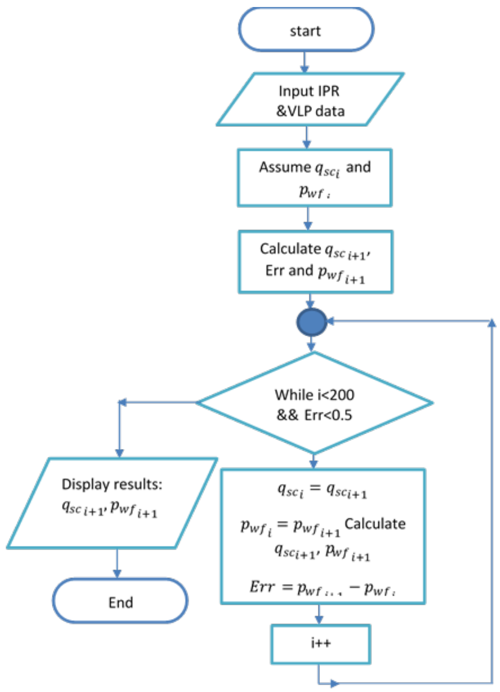

Figure 2. Overall flowchart for the developed computer program, TAYNOD

Invariably, published data were used to validate the developed computer model (TAYNOD). These data are represented in Table 1. To compare the predicted values of the operating flowrate and bottomhole flowing pressure with the graphical method, modified data was used. The modified data points are shown in Table 2. Finally, some software runs were performed on the developed computer program (TAYNOD). The results are presented in the next section of this paper.

Table 2. Modified data points [5]

Parameters	Mod1 (di=2in)	Mod 2 (di=3.5in)	Mod 3 (di=4.5in)	Mod 4 (di=5.5in)
Tubing length, ft	10,000	10,000	10,000	10,000
Tubing roughness	0.0006	0.0006	0.0006	0.0006
Tubing internal diameter, in	2	3.5	4.5	5.5
Inclination angle	0	0	0	0
Wellhead pressure, psi	700	800	850	900
Gas specific gravity	0.71	0.71	0.71	0.71
C- back pressure constant, Mscf/d-psi^{2n}	0.01	0.01	0.01	0.01
n-exponent	0.8	0.8	0.8	0.8
Tubing head temperature, ^{o}F	150	150	150	150
Bottomhole temperature, ^{o}F	200	200	200	200
Reservoir pressure, psi	2000	2000	2000	2000

3. Results and discussion

The results obtained from using the developed mathematical and computer model in this study are shown in Figure 4 and 5.

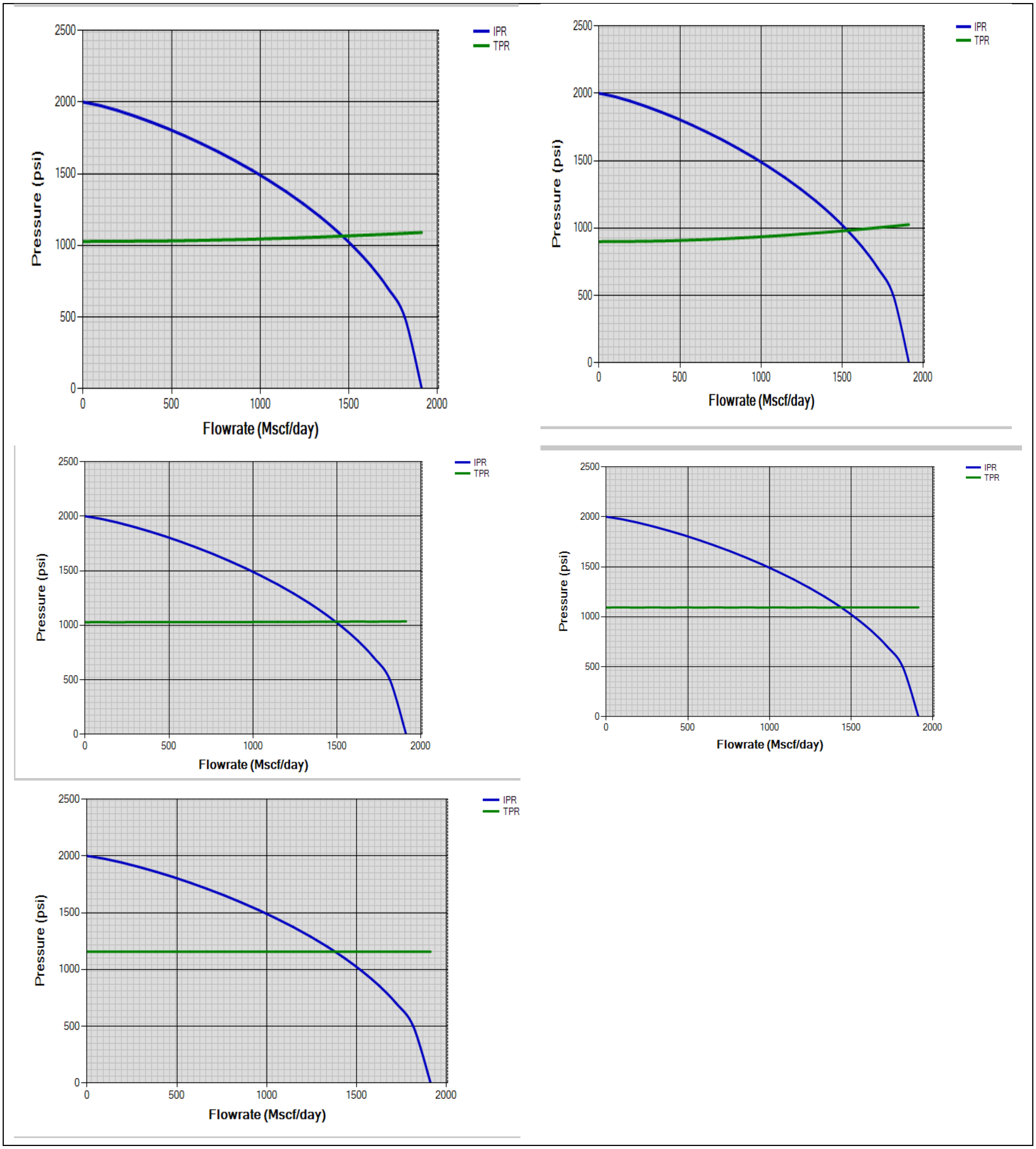

Figure 3. Graphical illustration of gas well performance curve using TAYNOD

As shown in Figure 4, it took the model barely three iterations to converge. In addition, the value of bottomhole flowing pressure was computed as 985.6psi and the operating flowrate was determined as 1552Mscf/day. Figure 4 represents the result for Well A.

However, Figure 5 shows the result obtained from a modified data point used for testing the developed prediction model. The result shows an operating flowrate of 1442Mscf/day and operating pressure of 1090psi (See Figure 5).

More so, to validate the developed prediction model, graphical plots of the gas wells considered in this study were used as the exact model. Some software runs were made using TAYNOD, and the plots, as shown in Figure 3, were presented. The results obtained from the charts were used as the exact values. The exact values were contrasted with the predicted ones using a statistical model like R^2. The result obtained from the evaluation revealed that the prediction has a substantial level of precision with an R^2 value of 0.937 (See Figure 6).

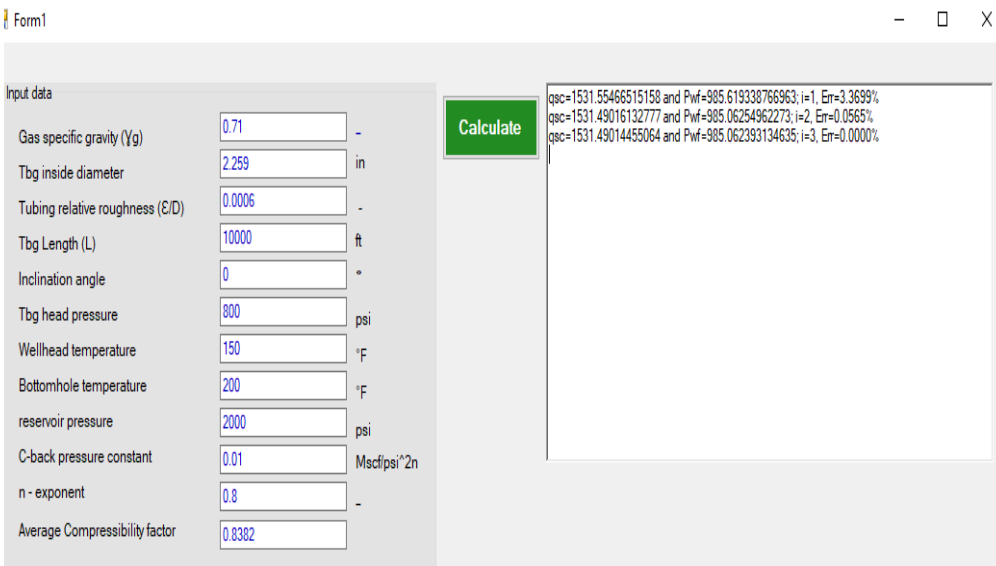

Figure 4. Prediction of gas well performance for Well A using TAYNOD

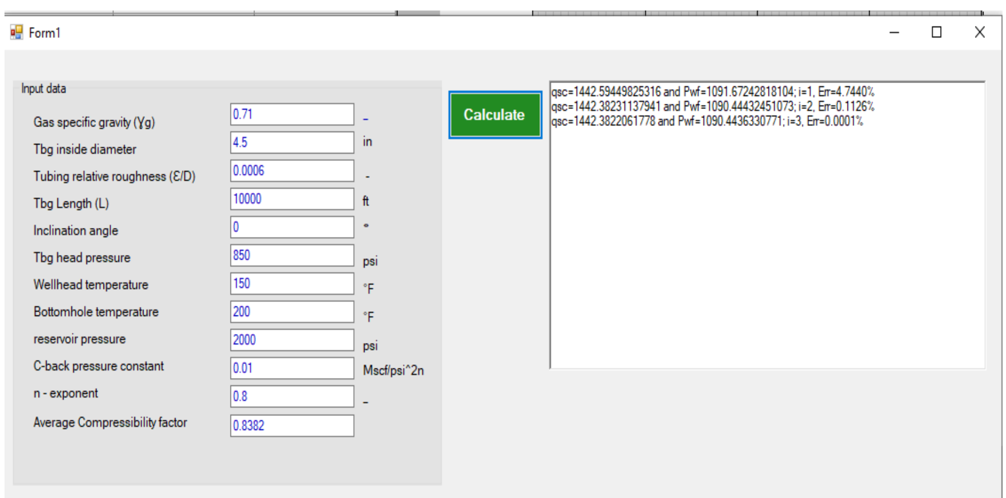

Figure 5. Prediction of gas well performance for one of the modified datapoints using TAYNOD

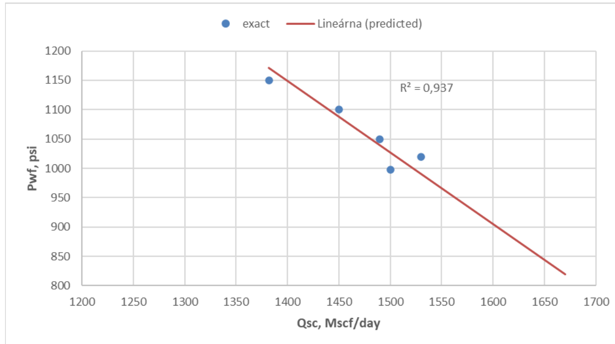

Figure 6. Validation of the prediction model

4. Conclusion

This study has successfully demonstrated that the performance of a dry gas well can be determined computationally by the use of Taylor series expansion. The developed mathematical model was put into a computer program, TAYNOD, where the operating pressure and flowrate of a gas well was determined speedily. The prediction of bottomhole flowing pressure and gas flowrate were performed in a timely manner with less computational stress – as it barely takes three or fewer iterations for convergence to be reached. To validate the prediction model, graphical nodal analysis was performed using the same data points as the prediction model. The operating pressure and flow rate results were painstakingly read from the charts to obtain the exact solution. The exact values were then contrasted with the results from the developed prediction model. Nevertheless, it was established that the predicted values of Pwf and qsc matched adequately with an R^2 of 0.937.

References

[1] Azim RA, Ramirez M, Mohammad R. Determination of inflow performance relationship for a vertical well in naturally fractured oil reservoirs: numerical simulation study. Petroleum & Coal, 2018; 60(2): 294-304.
[2] Jansen, DJ. Nodal Analysis of Oil and Gas. United State of America, Society of Petroleum Engineers, 2017.
[3] Azin R, Sedaghati H, Fatehi R, Osfouri S, Sakhaei Z. Production assessment of low production rate of well in a supergiant gas condensate reservoir: application of an integrated strategy. Journal of Petroleum Exploration and Production Technology, 2019; 9: 543-560.
[4] Nwanwe CC, Duru UI, Nwanwe OI, Chikwe AO, Ojiabo KT, Umeojiakor CT. Optimum tubing size prediction model for vertical multiphase flow during flow production period of oil wells. Journal of Petroleum Exploration and Production Technology, 2020; 10: 2989-3005.
[5] Okologume CW, Omonusi R. Tubing system performance profiling of dry gas wells using newton Raphson iteration method. Journal of Engineering Studies and Research, 2021; 27(2): 17-28.
[6] Ahmed T. Reservoir engineering handbook. Gulf professional publishing, 2018.
[7] Liu X, Guo B, Tan X. Petroleum Production Engineering. Gulf Professional Publishing, 2017: 30-160.
[8] Al-Qasim A, Almudairis F, AlAbdulatif Z, Alsubhi M. Optimizing Production using Nodal Analysis Applications. In Society of Petroleum Engineers Kuwait Oil & Gas Show and Conference, OnePetro, 2019.
[9] Fan Y, Sarica C. A novel approach for system instability prediction using nodal analysis. In Society of Petroleum Engineers Annual Technical Conference and Exhibition, OnePetro, 2019.
[10] Salaudeen I, Bopbekov D, Abdulkarim A. Optimization of petroleum production system using Nodal Analysis Program. Nigerian Journal of Technological Development, 2022; 19(1):1-8.
[11] Lea JF, Rowlan L. Gas well deliquification. Gulf Professional Publishing, 2019.
[12] Chapra SC, Canale RP. Numerical methods for engineers. 6th edition, New York, McGraw-Hill, Inc., 2010: 37-148.

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