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Different Fracture Fluids and Models in Hydraulic Fracture Technique. A Review

Amr Abu Zeina[1], Said Kamel[2], and Adel M. Salem[2,3]*

[1] *Khalda Petroleum Company, Cairo, Egypt*
[2] *Petroleum Eng. Dept., Faculty of Petroleum and Mining Engineering, Suez University, Egypt*
[3] *Future University in Egypt (FUE), Egypt*

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Abstract
Hydraulic fracturing encompasses rock failure, the formation of complex fractures, the transport of proppants, and the eventual closure of fractures. Complex fluids are used at nearly every stage of fracturing, particularly for creating and sustaining fractures with transporting and distributing proppant. The slick water fluid is widely used for tight unconventional reservoir due to those benefits of cost, proppant and polymer used and fracture height reduction, however, a viscoelastic surfactant base fluid that develop a shrinkage of viscosity reduction, also, an energized fluid enhances a reduction of water invasion to formation. A PKN and KGD models have a limitations of fracture height with an elastic response of PKN model, so, the P3D model is an extension development and relatively enhancement by a long side PL3D model that utilizes mesh systems. Finally, it is essential to accurately understand the assumptions and limitations of each model, as well as how these factors influence the modeling results and the overall treatment design.

Keywords: *Hydraulic Fracturing; Unconventional Reservoir; Surfactant; PKN model; KGD model.*

1. Introduction

Hydraulic fracturing (HF), commonly known as fracking, has revolutionized global energy production since its inception in the mid-20th century [1-2]. Originally developed to enhance recovery from conventional oil and gas reservoirs by creating fractures that increase permeability and facilitate the flow of oil and gas to the wellbore (Figure 1.) [3].

A significant breakthrough in hydraulic fracturing technology occurred with the development of horizontal drilling techniques which allowed wells to extend horizontally through targeted reservoir formations, significantly increasing contact area with hydrocarbon-rich zones (Figure 2.). This innovation was crucial for unlocking resources in unconventional reservoirs, such as shale formations, where natural permeability is low [4].

The combination of horizontal drilling with multi-stage fracturing techniques revolutionized shale gas and tight oil extraction. Multi-stage fracturing involves sequentially fracturing multiple sections along the horizontal wellbore, using packers to isolate each stage. This technique enables operators to maximize contact with reservoir rock, enhancing hydrocarbon recovery rates [5-6].

Hydraulic fracturing techniques have expanded beyond traditional oil and gas extraction to other energy sectors. Enhanced Geothermal Systems (EGS) which involves injecting fluid at high pressure into the rock to create fractures and enhance heat transfer, thereby increasing the efficiency and viability of geothermal energy production [7-8].

Moreover, innovative approaches, such as using alternative fracturing fluids like liquefied natural gas (LNG) or CO_2-based fluids, aiming to mitigate greenhouse gas emissions by injecting captured carbon dioxide (CO_2) into deep geological formations for long-term storage. Carbon Capture and Storage (CCS) in hydraulic fracturing has been explored to reduce water consumption and environmental footprint [9-10].

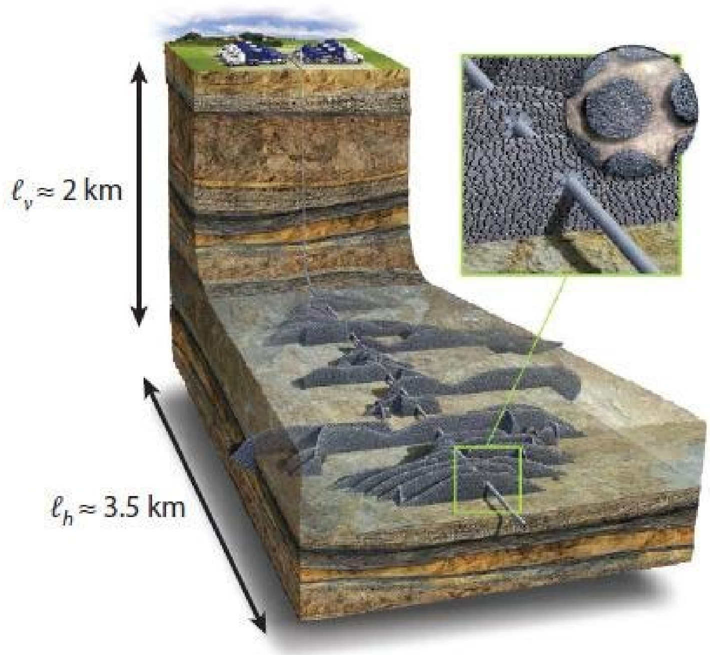

Figure 1. Schematic diagram of a contemporary fracturing job [3].

Despite its economic benefits, hydraulic fracturing remains a subject of significant environmental and regulatory scrutiny. Concerns include potential groundwater contamination, induced seismicity, surface water pollution, and methane emissions. The handling and disposal of fracturing fluids and wastewater are also critical environmental considerations. To address these concerns, stringent regulations and best practices have been implemented in many jurisdictions to ensure the safe and responsible use of hydraulic fracturing technologies [11].

Looking ahead, the future of hydraulic fracturing will likely involve further technological innovations aimed at improving efficiency, reducing environmental impact, and expanding applicability. Research and development efforts are focused on enhancing fracture modeling and simulation capabilities, optimizing fluid formulations, and developing sustainable practices for water management and chemical use.

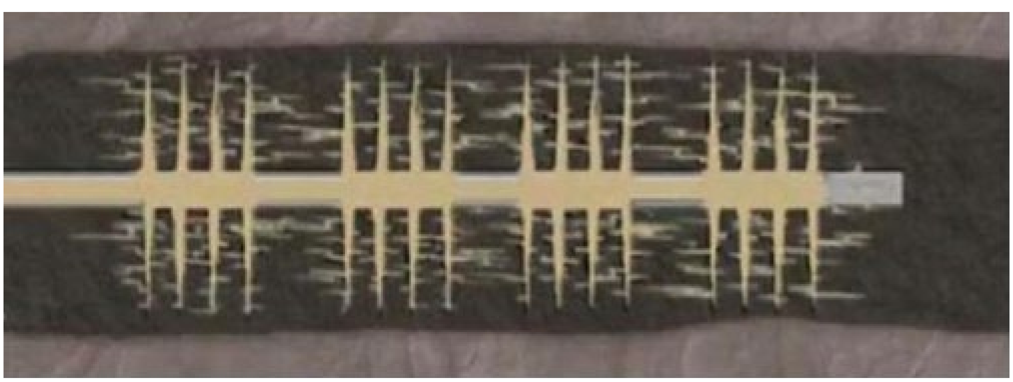

Figure 2. Schematic diagram of hydraulic fracture networks generated by multiple stages in a horizontal well [4].

1.1. History of hydraulic fracturing

Hydraulic fracturing, originated in the late 1800s with early acidizing techniques that dissolved carbonates and sandstones to enhance oil and gas extraction. The Van Dyke patent in the early 20th century introduced crucial innovations such as rubber packers for well isolation and targeted fluid injection, setting the stage for modern fracturing methods. The practical application of hydraulic fracturing began in 1947 when Stanolind Oil used gelled gasoline for the first time at the Hugoton Gas Field in Kansas, leading to a significant patent and commercialization by Halliburton. By 1949, this technology demonstrated considerable production improvements, rapidly adopted across the industry [12].

The technology saw substantial growth and refinement over the decades. In 1968, American Petroleum Corporation (now BP) performed a notable fracturing job in Oklahoma, using 0.5 million pounds of material. By 2008, hydraulic fracturing had become a global operation with over 50,000 frac stages annually, involving costs ranging from USD 10,000 to USD 6 million per job [13].

1.2. Reasons for hydraulic fracturing

More wells not produce at their optimum level in their natural state, but hydraulic fracturing (HFRAC) can address multiple challenges to efficient production. Radial flow from the reservoir into the wellbore is inefficient because the fluid must pass through successively smaller areas as it approaches the wellbore, causing "jamming" and reducing flow. By changing the flow pattern from radial to linear, HFRAC can significantly increase well productivity (Figure 3.) [14].

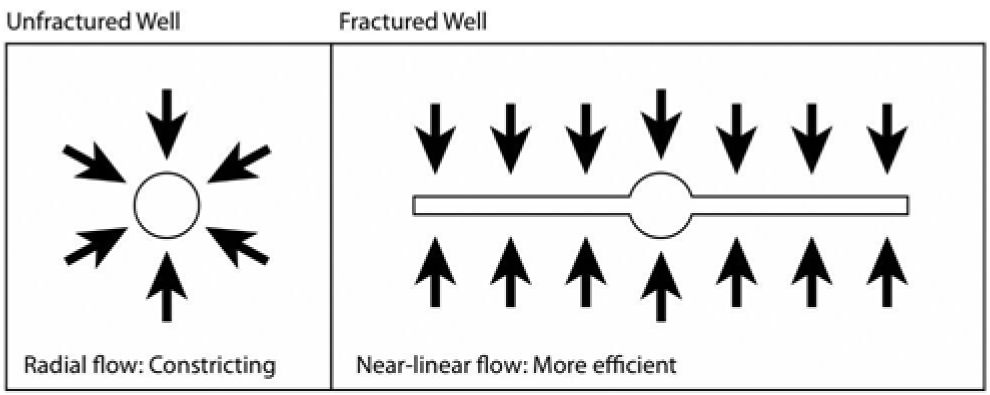

Figure 3. Mechanism of production by hydraulic fracturing.

Near wellbore permeability in most formations is reduced by drilling, cementing, and completion operations, which causes substantial reductions in production rates (Figure 4.) [15].

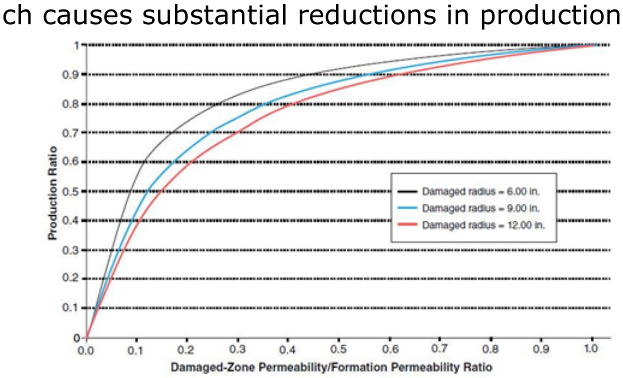

Figure 4. Effect of permeability damage on well productivity.

HFRAC can extend the reach of the wellbore far into the formation beyond the damaged area and reduce its negative effect on production. This reduces the risk of drilling into less or nonproductive zones. The net effect is that production is controlled by the properties of the reservoir reached by the fracture.

Production rates in most wells will eventually drop to a point where continued operation is no longer economically viable. Hydraulic fracturing (HFRAC) boosts the well's ultimate recovery factor, maintaining production at economically feasible levels. This makes HFRAC one of the most widely used completion techniques in oil and gas reservoirs. Although theoretically beneficial for all wells, this method is most commonly applied in medium to low permeability formations. In fact, in many low permeability reservoirs, wells are typically fractured before production begins to ensure economic viability.

2. Application of polymeric fluids in hydraulic fracturing

2.1. Guar-based fluids

Guar and its derivatives are widely used to viscosity water for fracturing applications, the long-chain, and high-molecular-weight polymer structure composed of mannose and galactose sugars. The polymannose backbone of guar is insoluble in water, but the galactose branches confer water solubility. The average molecular weights reported for guar derivatives are between 2 to 4 million Daltons [16].

Guar's insolubility in water is partly due to its polymannose backbone, where as few as six contiguous unbranched mannose units can form an insoluble helix. Consequently, guar can leave an insoluble residue of up to 6–10% by weight, which can damage the proppant pack used in hydraulic fracturing. Besides the initial insoluble residue, the use of enzyme breakers can generate additional residues. Over time, inappropriate breaking of the polymer's backbone by enzyme breakers can cause helices formation, further reducing the conductivity of the proppant pack. The development of these precipitates can take from a few hours to several days [17].

2.2. Crosslinking of guar

Borate, titanium (IV), zirconium (IV), and aluminum (III) ions are used as crosslinkers to enhance the rheological properties of water-soluble polymers, such as guar, for fracturing applications. These ions react with cis-OH pairs on the galactose side chains of guar. The selection of crosslinking agents depends on the pH, temperature, and type of polymer. Titanate and zirconate can function over a broad pH range (3–11), while borate ions are effective between pH 8 and 11, and aluminum works within a pH range of 3–5.

In terms of temperature, zirconate can be applied up to 400°F, borate and titanate up to 325°F, and aluminum below 150°F. Borate, effective at pH values of 7.5 and above, is less sensitive to shear rate and history than other metal ions, although early-time viscosity development may show some sensitivity. Borate ions are the most commonly used crosslinkers for guar, sourced from borax (sodium tetraborate decahydrate) and boric acid (plus caustic soda) with concentrations ranging from 0.024% to 0.09% by weight [18].

For high-temperature applications or when a delayed crosslink is needed to maintain low friction pressure before reaching the formation, low-solubility calcium or sodium-borate minerals such as colemanite and ulexite are used.

2.3. Slickwater fluids

The development of ultra-tight and tight unconventional reservoirs, such as tight gas and shale, has led to the increased use of slickwater, linear gel, and hybrid treatments for proppant transport. Slickwater treatments typically involve large volumes of water mixed with polyacrylamide or low concentrations of linear gel as friction reducers. This approach is characterized by higher injection rates (50–100 barrels per minute) and lower proppant concentrations (0.25–4 pounds per gallon) to counteract the natural limitations of slickwater, such as poor proppant transport and narrow fracture widths [19].

Slickwater treatments offer benefits like reduced costs due to lower proppant and polymer use, minimized gel damage within fractures, and reduced fracture height growth owing to their lower viscosity. For comparison, crosslinked fluids typically contain 20–40 pounds of polymer per thousand gallons, whereas slickwater jobs use only 5–10 pounds per thousand gallons.

Despite these advantages, slickwater treatments come with drawbacks, including the need for high fluid volumes and pumping rates, which can be cost-effective only near large water sources [20].

A significant case study revealed that slickwater treatments can create significantly wider and more complex fracture networks compared to crosslinked fluids for the same well. This complexity arises from deeper fluid penetration into micro- and nano-fractures, proppant settlement, and the formation of proppant monolayers between fracture faces. However, the lower viscosity of slickwater leads to narrower fracture widths and lower overall conductivity, presenting a trade-off between operational efficiency and fracture performance [21].

2.4. Viscoelastic surfactant (VES)-based fluids

The damage occurs to proppant packs caused by residues from incompletely broken fracturing fluids led to develop viscoelastic surfactant (VES)-based fluids. These fluids consist of hydrophilic and hydrophobic groups that self-associate to protect their nonpolar regions from the aqueous phase. When dissolved in water, these surfactants increase viscosity without the need for a crosslinker. The rod-shaped micelles swell and break into smaller spherical micelles, reducing fluid viscosity when exposed to organic and hydrophobic fluids like oil and gas, thus eliminating the need for additional breakers (Figure 5.) [22].

However, VES systems have some drawbacks, including high fluid leakoff volumes due to the absence of wall-building, high costs, and undesirable viscosity reduction at high temperatures. To address these issues, nanoparticle-modified VES systems were developed, demonstrating stability at high temperatures and better elastic behavior. These systems create pseudo-filtercakes during fluid loss [23].

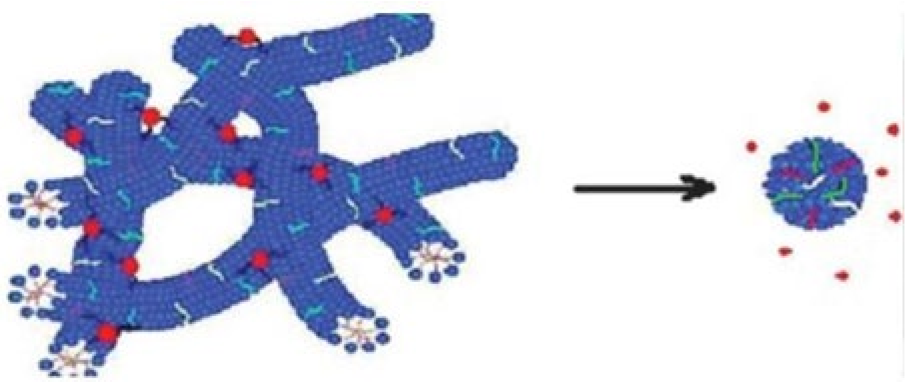

Figure 5. Association of internal breakers (light blue) and nanoparticles (red) with VES micelles.

In some cases, breakers have been introduced to viscoelastic surfactant (VES) fluids to degrade the molecules or act as compatible agents that break down the VES into micelles at reservoir temperatures, aiming to enhance fracture conductivity. These modifications are referred to as internal breakers, which are incorporated into the VES fluid and activate degradation when needed [24].

Additionally, the foaming of VES gels with nitrogen and CO_2, combined with the addition of cationic and anionic surfactants, has been reported to improve both viscoelastic moduli and leakoff properties. These foamed VES systems are effective at carrying proppants and leave minimal to no residue in the fracture, optimizing their performance in hydraulic fracturing applications [25].

2.5. Energized fluids

The growing interest in tight and ultra-tight unconventional formations with high clay content has led to the development of energized systems with significant gas fractions and minimal water content. These systems aim to mitigate damage from capillary pressure, relative permeability discontinuities, and the physical damage caused by invading fluids. They also significantly reduce fluid loss volumes, which can either decrease the total volume of water required for hydraulic fracturing or extend the fracture network for the same volume of injected water [26].

The main advantages of foam systems are limitation of water invasion into the formation, reducing liquid blocking caused by capillary pressure and permeability discontinuities near the fracture face, enhancement of hydraulic conductivity recovery by filling the near-fracture area with dissolved and free gas and also, minimization of water-sensitive clays and water contact, helping to preserve formation integrity.

These foam systems are reported to be effective in addressing challenges associated with hydraulic fracturing in complex reservoir conditions.

2.6. The proppant transport and distribution in hydraulic fractures

The migration and distribution of proppants in the fracture affect the fracture conductivity and the production of fractured wells. Kern et al. was firstly to carry out experimental research on proppant migration in a single fracture by an equipment of sand movement [27] (Figure 6.).

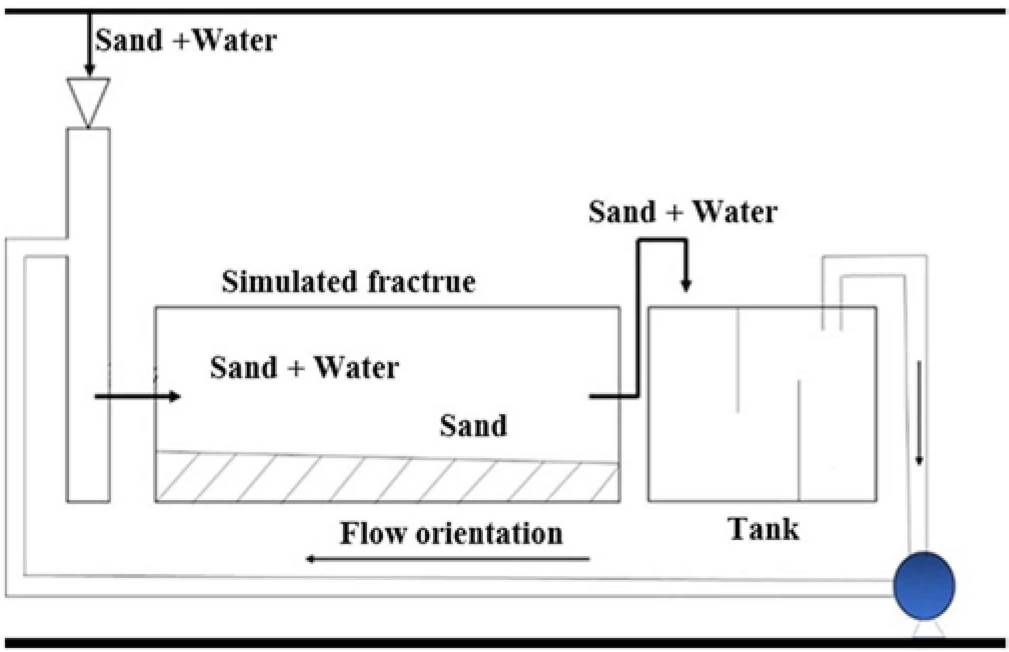

Figure 6. A sand movement equipment [27].

As hydraulic fracturing in tight reservoirs often results in more complex fracture networks, researchers such as Alotaibi and Miskimins, and Ashtiwi and Jennifer [28-29] have examined proppant migration in multi-branched fractures using modified experimental setups (Figure 7.). The proppant migration from a main fracture to secondary fractures depends on factors like fluid flow velocity, fracture width, and fluid viscosity. Proppant tends to accumulate at fracture intersections unless these conditions are met, as it requires a certain start-up speed to enter branch fractures.

The roughness of fracture walls plays a crucial role in proppant migration that can lead to particle agglomeration in narrow fractures, as demonstrated by Tomac and Gutierrez, and Zhang and Prodanovi [30-31]. found that larger proppants tend to accumulate at the fracture toe, while Liu and Sharma noted reduced sedimentation and horizontal movement as the particle size-to-width ratio nears unity [11]. Raimbay et al. highlighted that rough walls influence fluid flow paths and proppant stability [32], and Huang *et al.* observed that rough walls enhance vertical proppant distribution [33], though high viscosity and injection rates are necessary.

These fractures, with narrowing widths and multiple levels, require fracturing fluid flow rates to exceed a critical speed to ensure effective proppant migration.

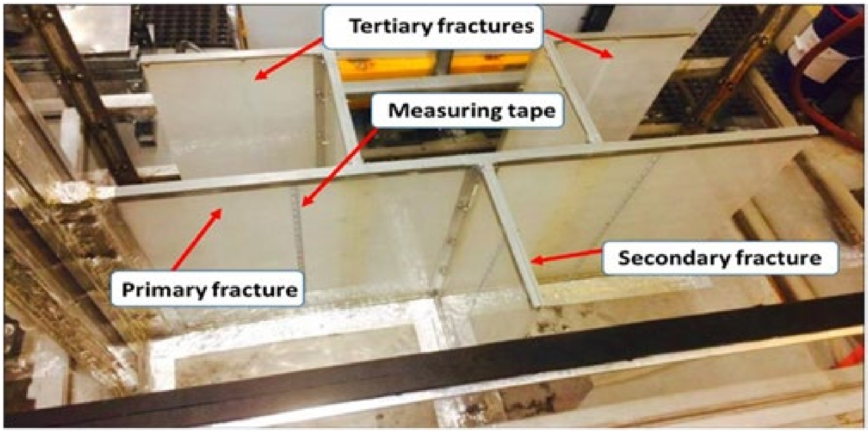

Figure 7. Proppant transports setup modification [29].

The complex fractures are formed after hydraulic fracturing in tight oil reservoirs. The complex fracture morphology has good self-similarity and can be described by the fractal binary tree model (Figure 8.) [34].

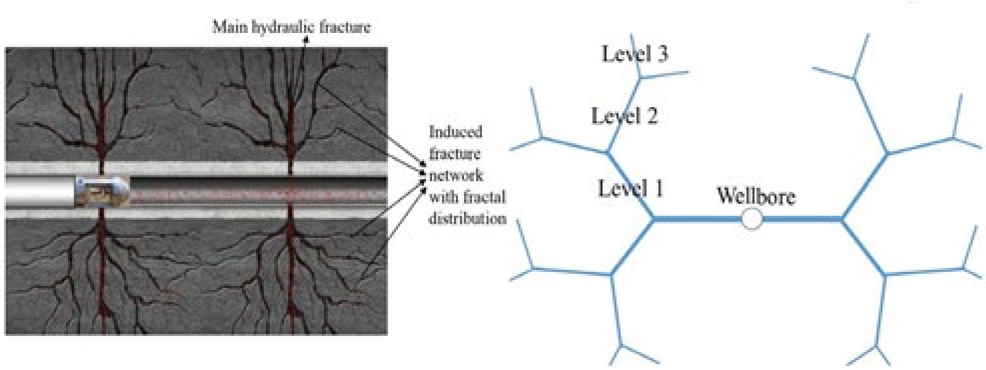

Figure 8. Schematic diagram of fracture morphology of actual and fractal tree [34].

3. Rheology and rheometry of fracturing fluids

Accurately modeling hydraulic fracturing requires understanding how fluid stress responds to shear rate and temperature, as these factors influence fracture geometry and pumping energy. Most numerical models simplify fluids as continuous media using shear-thinning viscosity models, although precise viscosity measurements are often deemed less critical by oilfield operators. However, viscosity significantly impacts leak-off rates and fracture dimensions, necessitating independent leak-off tests to evaluate fluid rheology's effect. Proppant transport optimization also hinges on understanding suspension flow behavior, with fracturing fluids often exhibiting viscoelastic properties such as normal stress differences and memory effects [35]. More accurate modeling of these fluids involves complex viscoelastic equations of state, like the Giesekus or Kaye-Bernstein-Kearsley-Zapas models, which, while more precise, are challenging to implement analytically and computationally [36].

3.1. Fracturing fluid rheometry

The bespoke design of fracturing fluids and the lack of universally predictive structure-function relationships necessitate direct measurement of complex fluid properties using rheometers. These instruments measure stress responses to various shearing motions, including steady-state, transient oscillations, and the initiation and cessation of stress or strain, capturing both linear and nonlinear viscoelastic properties. Pressure-driven flows through different geometries simulate downhole conditions, though extensional properties are challenging to measure due to difficulties in managing boundary conditions and flow dynamics [37]. Rheometric tests provide key material functions such as shear viscosity, normal stress differences, viscoelastic storage and loss moduli, compliance, relaxation modulus, extensional viscosity, and relaxation time. While linear viscoelasticity measures are well-defined, nonlinear measures better mimic real processes. Despite the availability of advanced commercial tools, accurate execution and interpretation of these experiments are crucial due to the inherent complexities of fracturing fluids [38].

- Slip and shear banding. Slip, adhesive failure, and shear banding on the measuring instrument will compromise results. Unavoidable in many cases, they can be quantified with direct measurement or inferred by repeated measurements varying the gap between parallel plates [39].
- Instabilities. Strongly viscoelastic fluids can exhibit purious shear thickening owing to elastic or inertial instabilities. Predictive theories for the onset of instabilities are available in most geometries as a function of material properties and experiment parameters [40].
- Particle migration. Gradients in shear rate and elastic stress and nonzero streamline curvature promote particle migration; most rheometric shear flows have circular streamlines. The measurement time should be short as compared with the timescale for particle migration.
- Boundary effects. In addition to slip, rheometer boundaries can induce ordering and disturb the orientation of non-spherical particles. The geometry should be large as compared with the particle size, typically greater than 10 particle diameters [41].
- Measurement at elevated temperature and pressure. Reproducing downhole temperature and pressure introduces problems in sample drying and containment and is generally available only for steady measurements above 100^{0}C (transient and dynamic measurements are challenging).
- Repeatability and mixing. Care must be taken to properly hydrate and mix additives in fluids, and also to ensure that the materials have not degraded or biologically decomposed when used over several days. In all cases the material should reflect the state of hydration/homogeneity used in the actual process.

In addition to advanced rheometry techniques, simpler index-based measures of rheological properties are employed, particularly outside academic settings. These indices, such as friction factor-Reynolds number correlations and the relative recovery of viscosity after extreme shear, provide straightforward diagnostic metrics related to pressure drop or torque in response to an ill-defined flow field. Although challenging to correlate with models and microstructural material theories, these indices are valuable for rough quality-control checks, ensuring the target fluid meets required specifications. However, they do not directly link the fluid's microstructure to its rheological properties [42].

3.2. Rheology of particle-laden fluids

Proppants play a critical role in maintaining fracture conductivity after hydraulic fracturing by preventing the closure of fractures once surface flow and pressure are reduced. These proppants are categorized by their strength, grain size, fines content, roundness, and density. Common materials include sand, resin-coated particles, ceramics, and glass spheres, with fibers occasionally added to modify fluid rheology and reduce settling rates. Proppant volume fractions vary significantly, ranging from 0-5% in slickwater treatments to over 20% in more viscous fluids. The mesh size, which indicates particle size, affects fluid viscosity; higher mesh numbers correspond to smaller particles [43].

In addition, shear thickening occurs in some fracturing fluids, where viscosity increases with shear rate, particularly above approximately 40% solid particle concentration [44]. This behavior contrasts with the viscosity increase due to added particles, where no-slip conditions enhance local viscous dissipation rates. Rheology of particle-laden non-Newtonian fluids involves complex behaviors and constitutive laws.

4. Hydraulic fracturing modelling

A variety of hydraulic fracturing models have been developed to enhance both the design of fracturing treatments and the understanding of the underlying physical mechanisms. Although hydraulic fracturing is inherently complex, substantial progress has been achieved over the past few decades by integrating theoretical concepts such as Linear Elastic Fracture Mechanics (LEFM) and fluid mechanics, along with extensive field and laboratory testing, as well as numerical modeling. This section primarily reviews the early classical models, which approach fracture geometry with varying levels of simplification. The latest hydraulic direction and the fracture length is much greater than the height. The aperture profile at any vertical section is restricted to be elliptical and is computed based on the plane strain assumption.

$$w = \frac{(1-\nu)}{G}\sqrt{(h^2-4z^2)(p-\sigma_0} \qquad (1)$$

where h is the fracture height, z is the coordinate in vertical direction, p is fluid pressure and σ_0 is the confining stress. In this scenario, fracture width depends solely on the local pressure, disregarding the non-local nature of the elastic response. The pressure gradient along the direction of fracture propagation is calculated using the traditional solution for laminar flow within an elliptical tube:

$$\frac{\partial p}{\partial x} = -\frac{64\mu q}{\pi h w_{max}^3} \qquad (2)$$

where w_{max} is the fracture width at centre.

The continuity equation for fluid flow is expressed as:

$$\frac{\partial A}{\partial t} + \frac{\partial q}{\partial x} + g_L = 0 \qquad (3)$$

where A is the cross-sectional area of the fracture.

As illustrated in the governing equations above, this model does not incorporate the rock's ability to resist fracturing, which is typically characterized by toughness or the strain energy release rate. According to Eq. (1), the displacement boundary condition at leading edge of fracture results in a zero pressure difference, and this is often used to detect fracture in this model. The above three equations were solved in a dimensionless manner by [45] Nordgren, along with the displacement boundary condition at fracture leading edge and the constant injection flow rate at injection point. Regardless of the strong assumptions made in the PKN model, it does present a clearly structured model ling framework for hydraulic fracturing, which includes the elasticity equation, the fluid flow equation and continuity equation are used in this model, enabling it to capture certain essential aspects of the dynamic propagation of hydraulic fractures. An enhanced version of the PKN model, which incorporates poroelastic effects, was developed in [46].

4.1. KGD model

Based on the work by [47] Geertsma and De Klerk developed a well-known hydraulic fracturing model, namely the KGD model. Unlike the PKN model, a plane strain condition is applied to the horizontal section, as illustrated in (Figure 9.).

The KGD model is further developed by [48] Carbonell, where the plane strain condition from the original KGD model is reserved, but some corrections are made to the governing equations with more rigorous solution methods. The improved KGD model is explained in detail in this section. The rock deformation is computed according to an elastic singular integral equation relating the net pressure to the fracture width [49].

$$w = \frac{1}{E'}\int_0^l G(\frac{x}{l},\frac{s}{l})p_n ds \qquad (4)$$

If an existing fluid lag is present with zero pressure: where E' is the plane strain modulus, l is half length of the fracture, l_f is half length of fluid channel and the integral kernel G is expressed as An inverse relation expressing the net pressure P_n by w is used in some other literatures [50] for the case without a fluid lag:

$$p_n = \frac{E'}{4\pi}\int_{-l}^{l}\frac{\partial w}{\partial s}\frac{ds}{s-x} \tag{5}$$

Alternatively, the propagation condition can also be implemented by computing the mode I stress intensity factor from fluid pressure distribution and fracture length. By using Bueckne-Rice function

$$K_l = 2\sqrt{\frac{l}{\pi}}\int_{0}^{l}\frac{p_n}{\sqrt{l^2-x^2}}dx \tag{6}$$

Pioneering work by Spence and Sharp, Detournay has highlighted the importance of scaling in deriving analytical solutions for hydraulic fracturing. Scaling transforms governing equations into dimensionless forms by normalizing coordinates, fracture length, net pressure, and width. This framework, which includes self-similar solutions for conditions like absent leakoff, helps derive accurate predictions for fracture width and net pressure, providing benchmarks for numerical simulations and addressing singularities near crack tips.

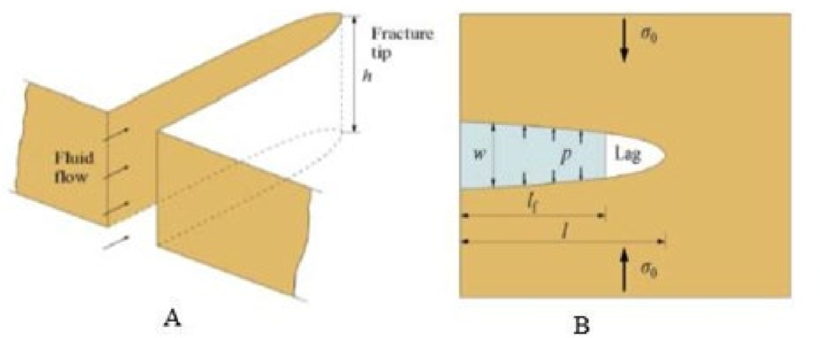

Figure 9. The KGD model: A) the model setup, and B) the plane strain assumption on the horizontal section.

4.2. Radial model

When the wellbore is aligned with the direction of the minimum principal stress, penny-shaped hydraulic fractures perpendicular to the wellbore are prone to formation. Geertsma and De Klerk's radial model uses axisymmetric assumptions instead of the plane strain assumption of the KGD model [47], applying similar governing equations to describe fracture propagation in an infinite linear elastic medium under confining stress. Subsequent enhancements to this model have included self-similar solutions for zero toughness (at the M vertex) and asymptotic solutions for large toughness (at the K vertex). Savitski and Detournay derived these solutions [51], while Bunger and Detournay further investigated toughness-dominated propagation with leakoff, finding good agreement between asymptotic and numerical solutions for mixed toughness cases [52]. For scenarios involving free surfaces, such as in environmental remediation and rock excavation, Zhang et al. developed a penny-shaped fracture model that incorporates the effects of nearby free surfaces [53]., modifying the radial model to address these additional complexities. The governing equations are listed below:

Elastic equation. The non-local elasticity relation between fracture width and fluid pressure is described using DDM and expressed as

$$D\left\{\frac{w}{R};\frac{R}{H}\right\} = \frac{p}{E'} \tag{7}$$

where D is the linear functional based on DDM, and R is the fracture radius.

Poiseuille's law

$$q = -\frac{w^3}{12\mu}\frac{\partial p}{\partial r} \tag{8}$$

Continuity equation without leakoff, the continuity equation is given as

$$\frac{\partial w}{\partial t}+\frac{1}{r}\frac{\partial (rq)}{\partial r}=0 \qquad (9)$$

Fracture propagation criterion

$$w\cong\frac{K'}{E'}(R-r)^{\frac{1}{2}}(R-r\ll R) \qquad (10)$$

Governing equations for hydraulic fracturing are normalized using either viscosity or toughness scaling systems, leading to solutions for zero-toughness and zero-viscosity scenarios. Zhang et al. provided a general numerical solution validated against self-similar solutions: an early-time solution for deep-buried cracks with zero confining stress and a large-time solution for the large toughness case [53]. The comparison between the PKN model and the KGD model were conducted in (Figure 10.).

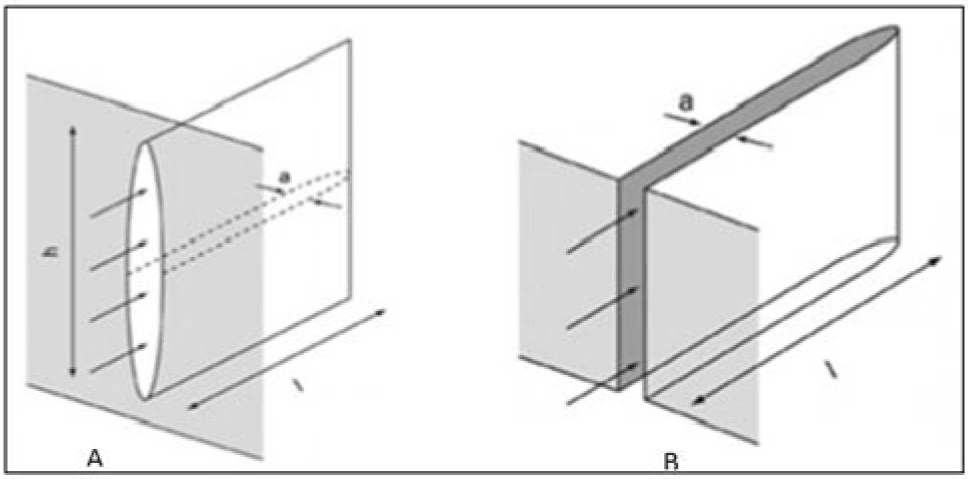

Figure 10. Fracture models. A) PKN model, and B) KGD model [45].

4.3. Pseudo 3D (P3D) model

The PKN model's limitations (constant fracture height and ignoring non-local elastic responses) led to the development of advanced models like the cell-based P3D model. This model, an extension of the PKN approach, computes fracture width in uncoupled vertical planes while allowing fracture propagation into adjacent formations. It incorporates a 1D flow simulation along the fracture direction and uses variations of the KGD model. The lumped model, developed by Cleary [54] and evolved into the **FracPro** software, also addresses these limitations. Key developments include a structured presentation of governing equations by Palmer and Carroll [55]., which account for varying in-situ stress and toughness, while the LEFM theory is applied to vertical planes with constant fluid pressure assumptions.

Carter's leakoff model is used here. Rahim and Holditch [56]. Developed a model (**TRI FRAC**) which is capable of modelling proppant transport, influence of multi-layers with asymmetric mechanical properties and in-situ stresses, the finite difference method (FDM) is employed to solve the governing equations, and field data, including well logs, are necessary for this simulator. This model has been utilized in numerous practical applications [57]. One notable implementation of the P3D model is the **MFrac** simulator (Meyer Fracturing simulators) is extensively used within the petroleum engineering sector.

Comparisons between 2D models (the PKN model and the KGD model) and the P3D model were conducted by Rahman [58]. The 2D PKN model provides propped fracture lengths that more closely match those predicted by the P3D model compared to the 2D KGD model, assuming the correct fracture height is used in all models. Nonetheless, it is important to note that all P3D models overlook the non-local elastic response. The assumption is applicable when the fracture length greatly exceeds the height (more than five times), but it could introduce significant errors in fracture geometry predictions under different conditions.

4.4. The planar 3D (PL3D) model

The planar 3D (PL3D) model was developed alongside the P3D model, much effort has been made to release the second assumption in the PKN model. This led to the development of the PL3D model, which can be based on either a moving or a fixed mesh system, as shown in (Figure 11.). Clifton and Abou-Sayed introduced the PL3D model, which utilizes a moving mesh system [59]., and subsequently enhanced it in. These developments have become the foundation for the widely utilized, commercial simulator **TerraFrac**.

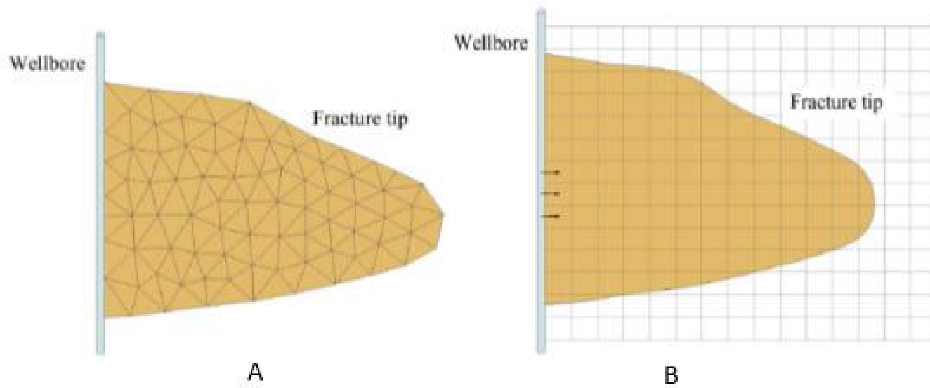

Figure 11. The planar 3D (PL3D) model: A) a moving mesh system, and B) a fixed mesh system.

The PL3D model for hydraulic fracturing employs Linear Elastic Fracture Mechanics (LEFM) to determine fracture propagation criteria and calculates the critical width using the mode I stress intensity factor, considering factors like leakoff, thermal effects, and proppant transport. Key implementations of the PL3D model include variations such as 3D HFRAC, which uses a surface integral method for rock deformation and FEM for fluid flow, and models that incorporate leakoff effects validated against PKN and KGD models. Advani et al. utilized FEM with a migrating mesh for dynamic fractures in multi-layered media [60], while Ouyang et al. integrated proppant transport and non-Newtonian fluid behavior into their model [61].

The fixed mesh PL3D model, foundational to commercial simulators like **GOHFER®**, employs tensile strength as a fracture criterion and simplifies leakoff simulation. Other approaches, such as HYFFIX and fixed mesh models by Siebrits and Peirce [62], use remeshing and stress intensity factors for fracture growth. Recent advancements, including combining near-tip asymptotic solutions for better accuracy and integrating structured meshes, demonstrate the PL3D model's comprehensive but computationally demanding framework, which, despite higher costs, offers more rigorous and accurate predictions of hydraulic fracture propagation compared to 2D and P3D models.

5. Comparison of common fracturing models

Table 1. summarizes the key features of the aforementioned classical hydraulic fracturing models. Warpinski conducted a comparative analysis of six hydraulic fracturing models [63], including 2D, P3D, and PL3D models, focusing on their predictions of fracture geometry and injection pressure based on actual field data. These classical models simplify rock deformation by solving elastic equations relating fracture width to fluid pressure, which reduces computational complexity and cost.

For example, the PKN, KGD, and radial models use 1D meshes to solve 2D or 3D fracture geometries, while the PL3D model uses a 2D mesh for 3D geometries and assumes 2D fluid flow. Although these models can be solved analytically or numerically, they are limited by their assumption of straight-line or planar fracture propagation and difficulty in incorporating natural fractures. To address these limitations and model more complex scenarios, advanced numerical methods have been developed, which are reviewed in subsequent sections.

Table 1. Overview of common hydraulic fracturing models.

Features	Model
Plane strain assumption for each horizontal cross section; 2D fracture geometry and 1D fluid flow	KGD
Constant height; Plane strain assumption for each vertical cross section; 3D fracture geometry and 1D fluid flow	PKN
Radial shape; 3D fracture geometry and 1D fluid flow	Radial model
Plane strain assumption for each vertical cross section; 3D fracture geometry and 1D fluid flow	Cell-based P3D
Half-ellipse fracture fronts	Lumped P3D model
Planar fracture; 2D fluid flow	PL3D

Selecting an appropriate fracture model. The choice of model affects treatment design and evaluation [64]. Selecting an appropriate model involves understanding key factors such as fracture geometry, fluid leak-off, and proppant transport. For instance, analytical 2D models are mainly for validation, and P3D models are suitable for simple formations. Table 2. Comparison between fracture models considering the pumping-rate dependency, determination of fracture opening, proppant transport and settling, stress shadow effect, and fractures contaminations.

Table 2. Comparison between fracture models.

Fractures containment at stress or slip barriers	Stress shadow effect	Proppant transport/ settling	Fracture geometry determination	Pumping-rate dependency effects	Model
Shear slip at top/bottom (assumption)	No	No	Analytical solution	Rheology-insensitive	KGD
Constant crack height (accumption)	No	No	Analytical solution	Too rheology-insensitive	PKN
Yes (barriers should be defined)	NO/Yes (Vertical across models)	Yes	Semi analytical solution	Port discharge Friction Rheology	P3D
Yes (barriers should be defined)	Yes	Yes	Numerical solution (FDM, FEM)	Port discharge Friction Rheology	3D Planar (Shear Decoupled)

6. Conclusion

Hydraulic fracturing (HF) is a method employed to extract petroleum from impermeable rock formations. This technique is widely used across various geological settings for energy extraction, storage, and in-situ stress assessments.

Complex fluids used in fracturing operations encompass essentially all aspects of rheology and non-Newtonian fluid dynamics—linear and nonlinear viscoelasticity, physicochemical gelation, transport and orientation of spherical and fibrous particles, control of slip/shear banding, and migratory and many-body particle interactions are present in abundance and intimately affect the ultimate hydrocarbon recovery that can be achieved.

The fracture conductivity damage by viscous fluids in ultra-tight formations found in unconventional reservoirs prompted the industry to develop an alternative fracturing fluid called "slickwater". It primarily consists of water with a minimal concentration of linear polymer. This low concentration polymer mainly helps to minimize friction loss along the flow paths. Due to limitations in local water availability and the risk of damaging formations, the industry has developed alternative fracturing fluids, including viscoelastic surfactants and energized fluids.

A fully coupled model for pressure-induced cohesive fracture in a saturated porous medium and its solution by the finite element method is of the discrete crack type and requires continuous updating the mesh as the crack tip progresses.

Choosing between a continuum or discontinuum approach depends on the complexity of the rock mass and fracture system, as well as the scale of the problem. Continuum models are advantageous for addressing large-scale problems, while discontinuum models are more effective for explicitly modeling natural fracture networks, multiple fractures, and fragmentation. Careful consideration should be given to the geological and geomechanical context, stress environment, data availability, and the scope of the investigation when selecting the appropriate fracture model.

3D planar-fracture models use simplified assumptions. The hydraulic fracturing is influenced by viscous fluid flow, fracture propagation, and fluid leak-off. Even in an idealized scenario with a homogeneous, isotropic, continuous formation, and under assumptions of plane strain, penny-shaped, and semi-infinite rectangular fractures, solving the fracture propagation equations involves dealing with several nonlinear, coupled hydro-mechanical processes on differing length and time scales.

Hydraulic fracturing involves the deformation of fracture surfaces, fluid flow within the fractures, and fracture propagation. Traditional models like the Perkins-Kernan-Nordgren (PKN) and Kirsch, Gross, and Dunne (KGD) provide foundational frameworks for simulating these processes. The PKN model is suited for fractures where length exceeds height, while the KGD model applies to fractures where height exceeds length. These 2D models are often extended to include terms for leak-off effects. To address more complex scenarios, such as non-uniform fracture shapes, pseudo-3D (P3D) models like cell-based and lumped models have been developed, though they face limitations in handling arbitrary fracture shapes and regional stress inversions. Advanced models such as the PL3D (Planar 3D) model use triangular or rectangular meshes to represent fractures of arbitrary shapes, overcoming some limitations of P3D models.

Nomenclature

KGD	*Khristianovich-Geertsma-De Klerk*
FDM	*Finite difference method*
PKN	*Perkins-Kern-Nordgren*
DDM	*Displacement discontinuity model*
P3D	*Pseudo 3D model*
PL3D	*planar 3D model*
FEM	*Finite element method*

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To whom correspondence should be addressed: Amr Abu Zeina, Khalda Petroleum Company, Cairo, Egypt, E-mail: amr.abuzeina@gmail.com