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Employing Ensemble Regression with Bourgoyne and Young Equations to Predict Penetration Rate for Oil Well Drilling in a Southern Iraqi Field

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Abstract

In oil well drilling, the rate of penetration (ROP) is a crucial factor that affects drilling safety, entire cost, and time management. Many models and approaches have been developed to predict ROP, with the Bourgoyne and Young model (BYM) currently being the most widely used, accurate, and comprehensive method. However, some studies indicate BYM limitations, such as its inability to capture complex parameter interactions and outlier point effects. This study proposed a new approach. It integrates ensemble regression techniques with BYM equations to analyse and predict ROP. This methodology enhances ROP prediction by integrating physics-based equations with ensemble regression. Real-time data collected from drilling logs of three oil wells in the southern part of Iraq are examined. Each data point is for a 0.25-meter increment and includes drilling parameters necessary for employing BYM. The results demonstrate that this approach provides an accurate ROP prediction, with an excellent statistic R^2 value of almost 1, zero P-value, and low residuals. This indicates an overall improvement in this approach and its ability to be an accurate tool in future drilling plans.

Keywords: *: Ensemble regression; Bourgoyne and Young model; Rate of penetration; Iraqi field.*

1. Introduction

Numerous optimization studies focused on the rate of penetration (ROP), trying to manage drilling costs and time, and decreasing risks, which are critical aspects of the oil industry. To achieve this goal utilizing various approaches, including the prior studies about the selection of optimal drilling parameters [1-3] and by statistical regression optimization that Bourgoyne and Young developed, it is a numerical model specifically for the oil industry to analyse and predict the ROP by taking into account eight factors that are impacting drilling efficiency including formation strength, normal compaction of the formation, under-compaction of the formation, downhole differential pressure, bit diameter and weight on bit, drill pipe rotary speed, bit tooth wear, and bit hydraulics [4].

BYM is widely used in drilling optimization studies at various oil well locations around the world, including the Arabian Gulf Area, Canadian Offshore well [5], Khangiran Iranian Gas Field, Iraqi Oilfields [6-11,32], Shadegan Oilfield, Kinabalu East Field, Persian Gulf Area [7], Volcanic geothermal field in Indonesia [8], Presalt layer [9] and MW-17 Well in Menengai, Kenya, and Gachsaran formation of Ahwaz oil field [10].

Although the BYM has demonstrated its efficacy in predicting and simulating drilling behavior, certain studies [12-13,30] indicate that it may not always offer accurate outcomes. This could be due to various factors, including the number of data points utilized for regression and multicollinearity problems. Prior researchers have endeavored to improve the BYM's precision by improving the regression methodology or adjusting the model.

The oil industry has witnessed the emergence of new computerized technologies and applied them in drilling optimization. These advancements have paved the way for various methods to enable real-time ROP optimization. Among these techniques are mechanical specific energy [14], mathematical optimization [15-16], remote systems [17-18], automated systems [19-20], artificial neural networks (ANN) [21-24], machine learning methods, pattern recognition [25], and support vector machines (SVM) [26-31].

Various studies have explored diverse ensemble models and their efficacy [27-29]. Although previous research did not study the potential advantage of using the BYM equation in ensemble regression, the BYM equation with its eight sub-equations depends on comprehensive hypotheses based on extended prior studies; each equation establishes a clear connection between drilling variables and ROP, and using it as a training dataset feeds to ensemble regressions has high benefits.

The ongoing study suggested integrating ensemble regression methods with BYM equations. MATLAB programming scripts estimate and predict ROP in oil wells from the southern Iraq field using multiple offset data points gathered from accurate drilling log records. Different case studies are employed with different conditions depending on various data points and depths. These techniques enhance ensemble accuracy and the advantage of BYM equations to generate more precise ROP predictions, highlighting the benefits of employing ensemble models for BYM equations with real-world examples.

2. BYM and Ensemble

Burgoyne and Young proposed and developed the most comprehensive and accurate mathematical model for predicting ROP in 1974. The model considers the impact of various drilling variables on the penetration rate and assumes that the effect of each parameter, such as weight on bit, drilling pipes rotation, bit tooth wear, and other factors, is independent of one another.

$$ROP = exp\left(a1 + \sum_{2}^{8} a_j x_j\right) \tag{1}$$

a1 to a8 represent BYM constants derived by employing MLR based on offset data for the well at the same local drilling conditions.

The eight functions in Eq. 1 are based on Bourgoyne's assumptions, which are built upon previous studies in the field, regarding each drilling variable's influence on the drilling rate (ROP):

$$x1 = 1 \tag{2}$$

$$x2 = e^{a2\,(10000-D)} \tag{3}$$

$$x3 = D^{0.69}(g-9) \tag{4}$$

$$x4 = D\,(gp - \rho c) \tag{5}$$

$$x5 = \ln\left[\frac{\frac{W}{d_b} - [\frac{W}{d_b}]_t}{4 - [\frac{W}{d_b}]_t}\right] \tag{6}$$

$$x6 = \ln\left(\frac{60}{N}\right) \tag{7}$$

$$x7 = -h \tag{8}$$

$$x8 = \ln\left(\frac{F_j}{1000}\right) \tag{9}$$

Ensemble regression is a technique that combines the knowledge of multiple and various regression models and gets the benefits and advantages from all the models concerned to improve the accuracy and precision of the overall predictive. This approach is based on the "wisdom of crowds" principle, which works better than individual models. Ensembles have three methods:

- The first method, bagging, builds models by training data models from the original data set. Each model works independently from other models and produces its predictions, which are combined to make a final prediction. This approach helps to reduce variability by improving consistency and reducing the impact of outliers and noisy data. Boosting, Eq.10 is often considered homogeneous weak learners. It learns them independently and in parallel and combines them by following a deterministic averaging process.

$$F(x) = \left(\frac{1}{M}\right) * \Sigma f_{m(x)} \quad (10)$$

- The second method, boosting, is a method in which all the weak models are successively trained, and each model corrects the deficiencies and limitations of the previous models. The final forecast is a composite of weights from all the models' images. This method helps to overcome limitations and increase accuracy through the model adjustments iteration; boosting Eq.11 is often considered homogeneous weak learners, learns them sequentially in a very adaptive way (a base model depends on the previous ones), and combines them following a deterministic strategy).

$$F(x) = \Sigma \alpha_m * f_{m(x)} \quad (11)$$

- The stacking method's third approach improves forecasts by combining models. It involves training a meta-model on the results of individual samples for each sub-model. The meta-learner then uses the predictions of the initial models to make a final prediction. The idea is to harness the power of models to understand complex relationships; stacking, Eq.12 is often considered heterogeneous weak learners, learn them in parallel and combine them by training a meta-model to output a prediction based on the different weak model's predictions.

$$F(x) = g\left(h_{1(x)}, h_{2(x)}, \ldots, h_{M(x)}\right) \quad (12)$$

Ensemble offers several advantages over traditional regression models. It can provide:

- A framework for improving prediction accuracy.
- Increasing model robustness.
- Capturing complex relationships.
- Selecting and combining different models.

The ensemble methods use models trained on different data subsets or algorithms for oil well movement, including the number that can be predictive. Various factors, such as drilling parameters, geological conditions, and operational variables, produce accurate and reliable forecasts.

3. Data and methodology

The study analyses data from three wells within recorded data at each 0.25m depth interval; the considered recorded parameters include measured depth (m) with true vertical depth (m), weight on Bit (Kpsi) with pipe rotation (RPM), standpipe pressure (psi) with pump flow (bbl), initial mud weight (lb/gal) with equivalent mud weight (lb/gal), bit size (inches) with nozzle size (1/32-inch), formation pressure (lb/gal), and bit jet impact force at each data.

The depth and data points for the three wells are as follows;

Oil Well #1 depth range is from 31 to 2641 meters, with 10,440 data points.

Oil Well #2, depth range: 32 to 2946 meters, data points: over 11,650.

Oil Well #3, depth range: 33 to 3025 meters; data points: over 11,960.

Integrating the Bourgoyne and Young model equations with ensemble regression involves collecting and pre-processing relevant data, such as many drilling parameters, formation properties, and penetration rates, using advanced monitoring techniques instead of solely relying on daily or final drilling reports. The 'ROP log' was the primary data source during drilling operations. Computer systems and digital encoders linked to the drilling rig were employed to monitor parameters such as ROP, WOB, rotary speed, and depth, among others.

An ensemble regression was applied to the BYM equation. Each model was trained within the ensemble using training data extracted from the three wells in the southern Iraqi field. The penetration rate is predicted from BYM using ensemble boosting, aggregation, and stacking methods. The ensemble's performance was evaluated based on testing data using appropriate metrics, and the ensemble was iteratively improved until a satisfactory level of prediction accuracy was achieved. Subsequently, multiple linear regression (MLR) was conducted for the BYM equation on the same dataset as an individual model. The three methods of ensemble regression employed in the BYM equation are compared with linear multiple regression on a diverse set of data points.

The performance of the three ensemble model methods for predicting ROP by BYM is evaluated using different statistical metrics, such as mean squared error (MSE), root mean squared error (RMSE), mean absolute error (MAE), and coefficient of determination (R^2).

4. Results and discussion

Plots and statistics for four study cases are presented to comprehensively analyse the study objectives across different wells, depths, and intervals. The first case pertains to the first well, the second case to the second well, and the third case to the third well. The fourth case involves a composite data analysis from all three wells at the same depth and depth interval. In the fourth case, data points are derived by sequentially selecting data points from the respective wells, starting with the first point from well 1, the second point from well 2, the third point from well 3, and so on, repeating this pattern for all data points within the depth interval.

The number of data points increases when the depth intervals are expanded since each 0.25-meter data point is included. Additionally, since there are three ensemble methods besides MLR, the plots may appear cluttered, but they follow the same pattern as those for the four short-interval study cases.

Table 1. Case 1 details and statistic.

Case	Well Number	From Depth, m	To Depth, m	P-value
1	J38P	1831.25	1931.25	0
Models || Statistic	MSE	RMSE	MAE	R^2
MLR	10.515	3.2426	2.3765	0.7167
Bagging	3.9372	1.9842	1.3739	0.8939
Stacking	1.4125	1.1885	0.8464	0.9619
Boosting	0.0070	0.0842	0.0551	0.9998

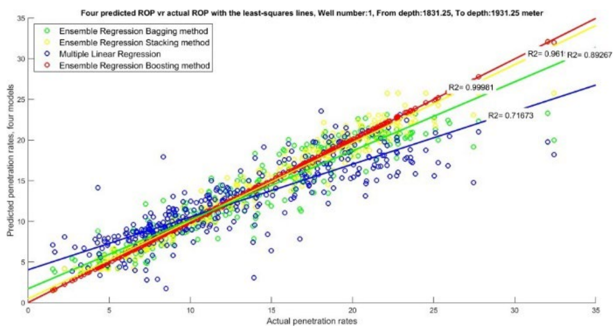

Figure 1-a. Predicted vs. predicted ROP of the ensemble methods and MLR, case 1.

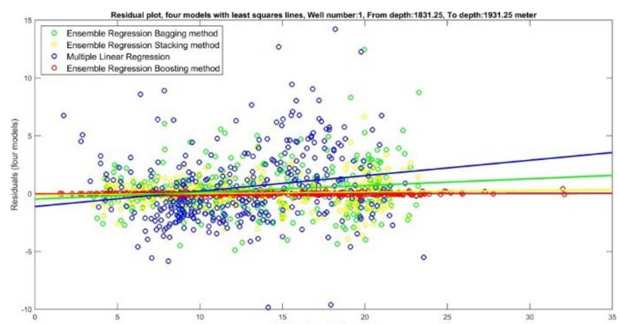

Figure 1-b. Residuals vs. predicted ROP of the ensemble methods and MLR, case 1.

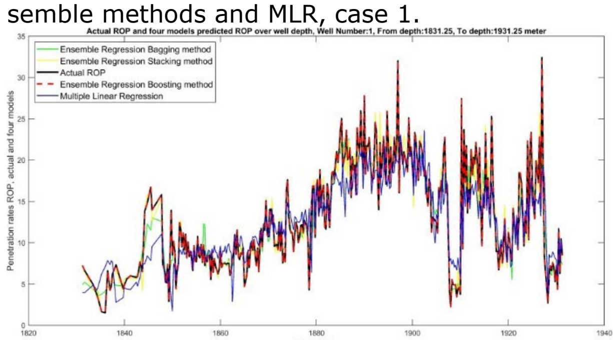

Figure 1-c. Actual and predicted ROP vs. well depth of ensemble methods and MLR, case 1.

These plots, above and beyond, for the different case studies try to put the predicted ROP results for all three ensemble methods besides the BYM in the same plots for all the cases to compare with the actual ROP and with each other's predicted ROP.

Figures 1-a, 2-a, 3-a, and 4-a clearly compare these methods; the ensemble boosting method's predicted ROP almost exactly matches the actual ROP, while the two other ensemble methods, bagging, and stacking, have excellent matches with the actual ROP.

Table 2: Case 2 details and statistic

Case	Well Number	From Depth, m	To Depth, m	P-value
2	J52P	2232.25	2282.25	0
Models || Statistic	MSE	RMSE	MAE	R^2
MLR	8.1487	2.8546	1.9877	0.5353
Bagging	3.7848	1.9454	1.2955	0.7434
Stacking	1.7250	1.3134	0.9413	0.8804
Boosting	0.00009	0.0099	0.0070	0.9999

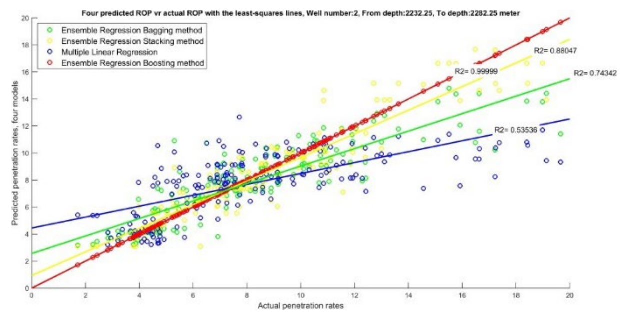

Figure 2-a. Actual ROP vs Predicted ROP of the ensemble methods + MLR, case 2.

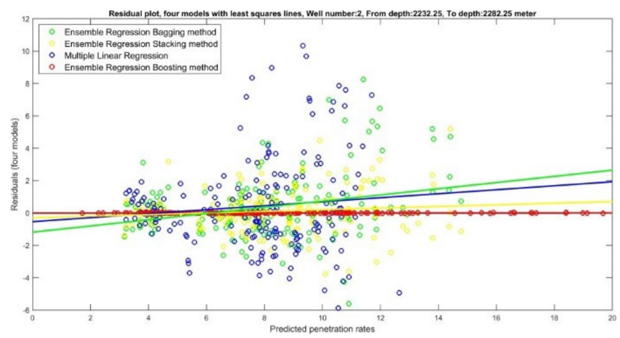

Figure 2-b. Residuals vs. Predicted ROP of the ensemble methods + MLR, case 2.

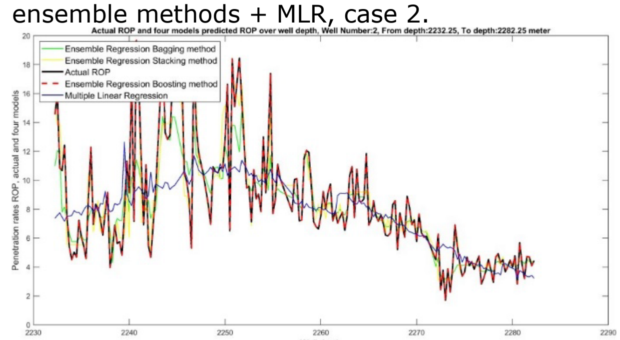

Figure 2-c. Well depth vs. actual + Predicted ROP of ensemble methods + MLR, case 2.

Analysing the residuals figures 1-b, 2-b, 3-b, and 4-b, which represent the difference between actual and predicted ROP for the three ensemble boosting, bagging, and stacking methods besides the MLR, it is evident that the ensemble boosting method signs are very close to the zero line which indicate an almost exact match with actual ROP also the boosting method is distributed along with x-axes that represent the predicted ROP giving another evident that it almost has exact match with actual ROP; also the boosting least square line is horizontal with the zero line meaning the identical of all the positive and negative residual point.

The bagging and stacking ensemble methods show more residual value than the boosting method, meaning their matching is less than that of the boosting method, and their distribution with the x-exes is less than that of the boosting method, representing excellent matches.

The MLR predicted ROP with actual ROP has more residual than others. However, it is still in a good range, representing a good and optimum match to the whole situation of the actual ROP and the related variables for all the points; also, the MLR signs are not distributed along with the x-axes but only in the good optimum range.

Table 3. Case 3, details and statistic.

Case	Well Number	From Depth, m	To Depth, m	P-value
3	J88P	1833	1883	0
Models || Statistic	MSE	RMSE	MAE	R^2
MLR	8.1093	2.8476	2.1917	0.6175
Bagging	6.0934	2.4684	1.7215	0.6149
Stacking	2.6994	1.6430	1.0998	0.8394
Boosting	2.03E-07	0.0004	0.0003	1

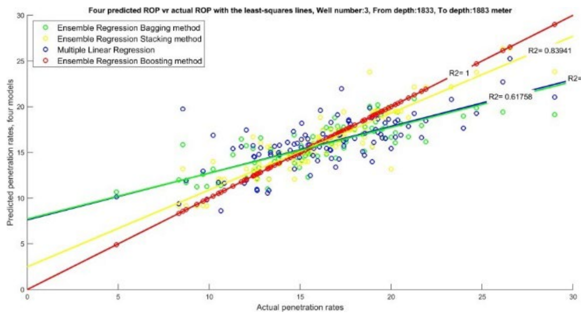

Figure 3-a. Actual ROP vs. Predicted ROP of the ensemble methods + MLR, case 3.

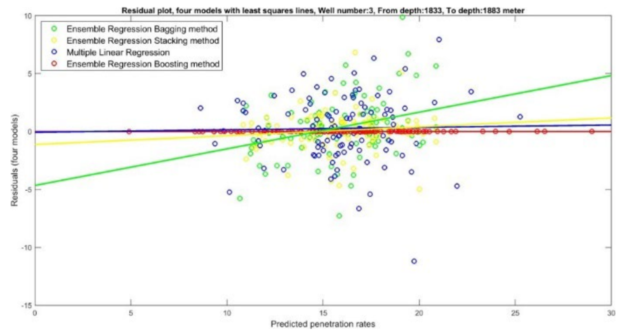

Figure 3-b. Residuals vs Predicted ROP of the ensemble methods + MLR, case 3.

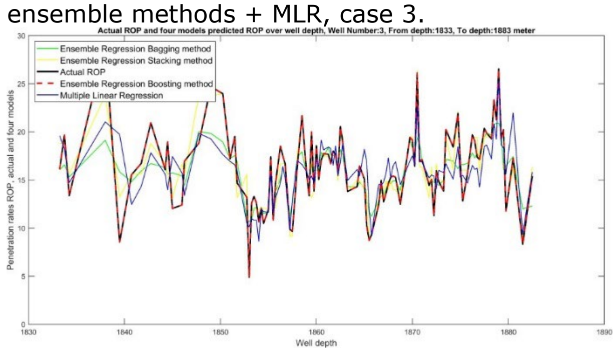

Figure 3-c. Well Depth vs Actual + Predicted ROP of ensemble methods + MLR, case 3.

Analysing Figures 1-c, 2-c, 3-c, and 4-c show the ROP curves with oil well depth. The plots show that the actual ROP curve is merged with the ensemble boosting method ROP curve, which improves the almost exact match between them.

Other curves representing the ensemble bagging and stacking method do not coincide with the actual ROP curve, and inside the excellent ROP limit, these lines show less match than the boosting method.

The BYM predicted ROP curve shows an average behavior almost in the middle of the upper and lower limit of the actual ROP curve, which shows a good optimum match with the actual ROP.

Table 4. Case 4, details and statistic.

Case	Well Number	From Depth, m	To Depth, m	P-value
4	J(38+52+88)P	2033	2083	0
Models || Statistic	MSE	RMSE	MAE	R^2
MLR	11.665	3.4154	2.2599	0.5796
Bagging	4.3529	2.0863	1.3264	0.8058
Stacking	2.9514	1.7179	1.1608	0.8683
Boosting	0.00018	0.0135	0.0085	0.9999

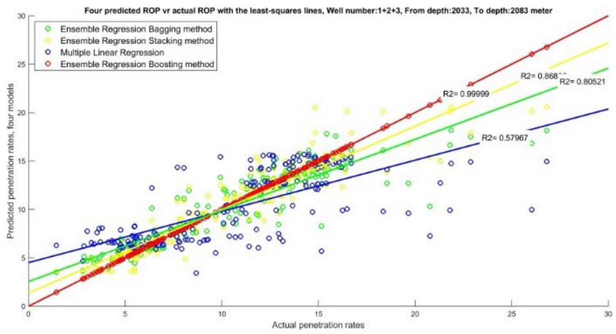

Figure 4-a. Actual ROP vs Predicted ROP of the ensemble methods + MLR, case 4.

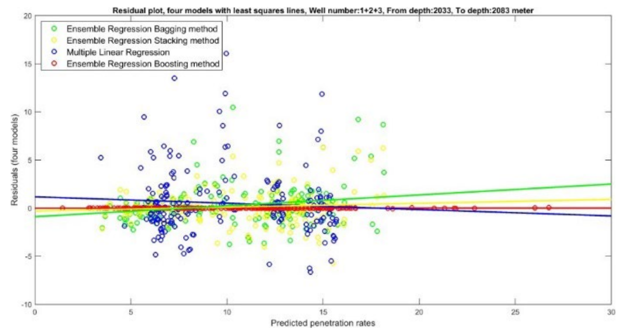

Figure 4-b. Residuals vs Predicted ROP of the ensemble methods + MLR, case 4.

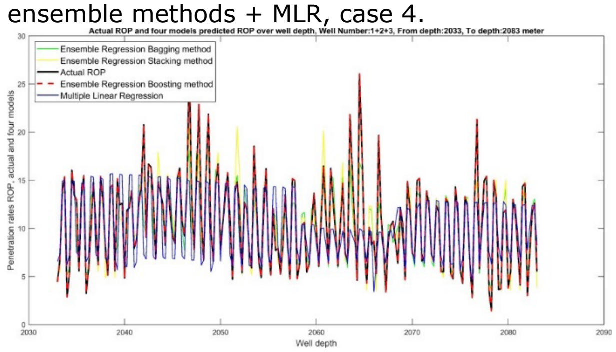

Figure 4-c. Well Depth vs Actual + Predicted ROP of ensemble methods + MLR, case 4.

The various figure styles utilized in different case studies complement each other, as the ensemble regression method proves to be highly advantageous when combined with the BYM equation, resulting in an accurate prediction of ROP through the boosting method; the bagging and stacking methods also yield excellent matches between actual and predicted ROP, providing significant benefits for future decision-making in oil well plans when engineers require precise predictions.

5. Conclusions

The ensemble regression method was successfully employed with the BYM equation to capture the ROP in a southern Iraqi oil field at different wells and depths. The model was improved by using other methods, such as boosting, bagging, and stacking, in addition to MLR. Analysis of the data and results statistics show that the ensemble boosting method accurately predicts the ROP. Bagging and stacking methods also effectively predicted ROP but resulted in slightly higher residual values. The finding provides valuable insights, helps improve ROP predicting techniques, and opens avenues for more studies.

Nomenclatures

$[\frac{W}{d_b}]_t$	*The drilling begins at Threshold WOB 1000lbf/inch.*
D	*Oil well, true vertical depth (ft).*
F(x)	*the model prediction for input x.*
Fj	*Hydraulic jet impact force beneath the drilling bit, force lbf.*
$f_{m(x)}$	*the prediction of the* m_{th} *base model.*
gp	*Formation pore pressure gradient (lbm/ft).*
h	*Drilling bit fractional tooth dullness.*
M	*the number of base models.*
N	*drilling pipe rotation(RPM)*
x1	*Represents the effect of variables not considered in the model on ROP.*
x2	*The effect of increased rock strength is due to normal compaction with depth on ROP.*
x3	*The impact of under-compaction experienced in abnormally pressured.*
x4	*related to the effect of the hydrostatic and formation differential pressure on ROP.*
x5	*the impact of bit weight on ROP.*

x6 *related to the drilling pipe rotary speed on ROP.*
x7 *Models the impact of tooth wear on ROP.*
x8 *Models the effect of bit hydraulics on ROP.*
a_m *represents the weight assigned to the* m_{th} *base model.*
ρc *Drilling fluids, Equivalent circulating density.*

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