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EXISTENCE AND UNIQUENESS OF SOLUTION FOR LAYER DIFFUSION OF AN INCOMPRESSIBLE FLUID IN A HYDROCARBON RESERVOIR

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Abstract

In the study of an incompressible fluid in a hydrocarbon reservoir, developing a reliable model to predict the fluid behavior in the reservoir, based on diffusion mechanism and some mathematical parameters is necessary. In the present study, we have tried to develop a nonlinear model of parabolic type to estimate some parameters such as volumetric capacity and diffusion coefficient of an incompressible fluid under layer diffusion in a homogeneous hydrocarbon reservoir. Initially for this type of fluid, Darcy's law is applied along with a continuity equation to create a nonlinear differential equation of parabolic type under boundary conditions. Via an auxiliary problem and based on Schauder fixed-point theorem and concepts of Banach spaces, we demonstrate the existence and uniqueness of solution for the aforementioned model. Afterward, we assume that the diffusion coefficient of the fluid is a polynomial function of degree n with unknown coefficients, accordingly a numerical algorithm is proposed for estimating the function as well as volumetric capacity of the fluid, so that the given error function becomes minimum.

***Keywords**: Diffusion; Auxiliary problem; Banach space; Fixed point; Pre-compact set; Inverse problem.*

1. Introduction

Classical solutions for a nonlinear diffusion equation, have been catalogued for many important problems [1-2]. Furthermore, this type of equation with several admissible boundary conditions have been employed as inverse nonlinear problem in order to numerically analyze the problems arising from the oil and gas processes. This work has been motivated by a class of important problems caused by the reservoir processes, such as layer diffusion of incompressible fluid in a hydrocarbon reservoir. The diffusion equations of these problems can be characterized by parabolic differential equations along with the governing boundary conditions.

Determining unknown parameter in a parabolic differential equation has been previously treated by some authors [3-8]. Usually an over-specified data at the boundary $x = 0$ is applied in determining the unknown parameter. Such problems typically arise in oil and gas reservoirs [9-10].

As previously mentioned, in this work we consider a diffusion problem which occurs in a hydrocarbon reservoir. We assume that [11]:

1. The desired fluid is incompressible such as oil and water whose density does not change under the pressure change.
2. The problem is considered before well drilling and production operations.
3. Regarding (2), the incompressible fluid can be in layer diffusion in reservoir rock.

Darcy's law will govern the analysis of the fluid motion. It must be noted that we ignore the deposition of some materials such as asphaltene, so the reservoir rock can be considered completely homogeneous in terms of physical properties such as permeability and porosity. If the volumetric capacity and the diffusion coefficient of the fluid, are respectively shown as u and a then, in the most applied problems, a will be a function of u and the governing partial differerential equation will be nonlinear. Considering the boundary conditions and by using an over-specified data at $x=0$, we will prove the existence and uniqueness of solution for this mathematical model in order to estimate the unknown parameters a and u which is impossible in most cases.

2. Mathematical formulation

As the hydrocarbon reservoir has been completely homogeneous, viscosity and porosity degree ϕ are constant. Also, volumetric capacity u is a function of time and place, also $0 \le u \le \phi$.

As mentioned earlier, the equation governs the fluid motion, is Darcy's law which along with the continuity equation are respectively as follows:

$$q = -a(u)grad(u) \qquad 0 \le u \le \phi \tag{1}$$

$$\frac{\partial u}{\partial t} + div(q) = 0 \tag{2}$$

q: volumetric flux of the fluid.

In particular, by combining them we will have a one-dimensional, nonlinear, parabolic equation as below:

$$\frac{\partial u}{\partial t} = div(a(u)\frac{\partial u}{\partial x}) \tag{3}$$

Supposing a time $T>0$ we define: $Q_T = \{(x,t) \in \Re^2 | 0 < x < 1, 0 < t < T\}$

We assume that the volumetric flux of fluid (q) at the beginning and the end of the reservoir is known and respectively denoted by $g(t)$ and $h(t)$, then based on Darcy's law:

$$q(0,t) = -a(u(0,t))\frac{\partial}{\partial x}u(0,t) = g(t) \tag{4}$$

$$q(1,t) = -a(u(1,t))\frac{\partial}{\partial x}u(1,t) = h(t)$$

Assuming that the initial distribution of volumetric capacity is zero:

$$u(x,0) = 0 \qquad 0 \le x \le 1$$

So, we will have the following nonlinear inverse problem:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}(a(u)\frac{\partial u}{\partial x}) \qquad 0 < x < 1 \qquad 0 < t < T \tag{5}$$

$$u(x,0) = 0 \qquad 0 \le x \le 1 \tag{6}$$

$$-a(u(0,t))\frac{\partial}{\partial x}u(0,t) = g(t) \qquad 0 \le t \le T \tag{7}$$

$$-a(u(1,t))\frac{\partial}{\partial x}u(1,t) = h(t) \qquad 0 \le t \le T \tag{8}$$

As we will faced with many errors and problems during calculating diffusion coefficient a, so calculation and definition of a on Q_T is limited to the boundary of this area. Let us consider the following over-specified condition:

$$u(0,t) = f(t) \qquad 0 \le t \le T \tag{9}$$

3. Auxiliary problem

In this section, for the above problem we consider an auxiliary inverse problem. Assume that q at the bottom of the reservoir $(x=1)$ is zero. Therefore:

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}(a(u)\frac{\partial u}{\partial x}) \qquad 0<x<1 \qquad 0<t<T \tag{10}$$

$$u(x,0)=0 \qquad 0\le x\le 1 \tag{11}$$

$$-a(u(0,t))\frac{\partial}{\partial x}u(0,t)=g(t) \qquad 0\le t\le T \tag{12}$$

$$\frac{\partial}{\partial x}u(1,t)=0 \qquad 0\le t\le T \tag{13}$$

$$u(0,t)=f(t) \qquad 0\le t\le T \tag{14}$$

Now, we assume that $0<a_0\le a(s)\le a_1$ and $s>0$ also a_0 and a_1 are constants. Consider the following transformation introduced for the first time by Cannon [12]:

$$C_a(s)=\int_0^s a(z)dz \qquad s\ge 0 \tag{15}$$

Note that $C'_a(s)=a(s)\ge a_0>0$, therefore C_a is strictly increasing and invertible. Per solution $u(x,t)$ of the problem (10) to (14) we define $v(x,t)$:

$$v(x,t)=C_a(u(x,t))=\int_0^{u(x,t)} a(\delta)d\delta \tag{16}$$

Then the auxiliary problem will be as follows:

$$\frac{\partial}{\partial t}v(x,t)=\hat{a}(v)\frac{\partial^2}{\partial x^2}v(x,t) \qquad 0<x<1 \qquad 0<t<T \tag{17}$$

$$v(x,0)=0 \qquad 0\le x\le 1 \tag{18}$$

$$\frac{\partial}{\partial x}v(0,t)=g(t) \qquad 0\le t\le T \tag{19}$$

$$\frac{\partial}{\partial x}v(1,t)=0 \qquad 0\le t\le T \tag{20}$$

$$v(0,t)=C_a(f(t))=\int_0^{f(t)} a(\delta)d\delta=F(t) \qquad 0\le t\le T \tag{21}$$

where: $\hat{a}(v)=a(C_a^{-1}(v))$ (22)

It is clear that determining $\hat{a}(v)$ results in $a(u)$, also in according to equation (16) there is one-to-one correspondence between the original problem solution and the auxiliary problem solution.

4. Existence and uniqueness of solution for model

We consider the auxiliary problem (17) to (21) as well as the following hypothesis:

A. $v\in C^2(Q_T)$.

B. $\frac{\partial^2}{\partial x^2}v(0,t)\le M$ ، $\forall t\in[0,T]$ and $M>0$.

C. $\hat{a}\in C^{-1}[A,B]$ where $A=\min_{Q_T} v(x,t)$ and $B=\max_{Q_T} v(x,t)$.

D. $\forall s\in[A,B]$ $\hat{a}(s)>0$ ،and $\hat{a}$ is true on $[A,B]$ in Lipschitz condition.

E. $\hat{a}(s)$ in (17) to (21), is true.

F. $F, g \in C^1[0,T]$ *and* $F(0) = g(0) = 0$.
G. $F'(t), g'(t) > 0$، $\forall t \in [0,T]$.
Now we define:

$$\|u\|_{0,Q_T} = Sup_{Q_T} |v(x,t)| \tag{23}$$

$$R(v)_{\alpha,Q_T} = Sup_{(x_1,t_1),(x_2,t_2)\in Q_T} \frac{|v(x_1,t_1) - v(x_2,t_2)|}{\left(|x_1 - x_2|^2 + |t_1 - t_2|^2\right)^{\alpha/2}} \tag{24}$$

Also:
C^{α} Banach space of functions v، continuous on Q_T.
$C^{1+\alpha}$ Banach space of functions v، with continuous first partial derivatives on Q_T.
$C^{2+\alpha}$ Banach space of functions v، with continuous second partial derivatives on Q_T.
Above spaces are respectively with the following norms:

$$\|v\|_{\alpha,Q_T} = \|v\|_{0,Q_T} + R(v)_{\alpha,Q_T} \tag{25}$$

$$\|v\|_{1+\alpha,Q_T} = \|v\|_{\alpha,Q_T} + \left\|\frac{\partial v}{\partial x}\right\|_{\alpha,Q_T} \tag{26}$$

$$\|v\|_{2+\alpha,Q_T} = \|v\|_{\alpha,Q_T} + \left\|\frac{\partial v}{\partial x}\right\|_{\alpha,Q_T} + \left\|\frac{\partial^2 v}{\partial x^2}\right\|_{\alpha,Q_T} + \left\|\frac{\partial v}{\partial t}\right\|_{\alpha,Q_T} \tag{27}$$

Now for $x = 0$ we have:

$$\frac{\partial}{\partial x} v(0,t) = \hat{a}(v(0,t)) \frac{\partial^2}{\partial x^2} v(0,t) \tag{28}$$

Regarding the condition (21):

$$F'(t) = \hat{a}(F(t)) \frac{\partial^2}{\partial x^2} v(0,t) \tag{29}$$

Now we set:
$s = F(t)$
As $F'(t) > 0$, so function F is invertible and can be written as:
$t = F^{-1}(s)$, $s \in [F(0), F(T)]$
Hence, according to equation (29) we have:

$$\hat{a}(s) = \left(\frac{F'(F^{-1}(s))}{\frac{\partial^2}{\partial x^2} v(0, F^{-1}(s))}\right) \tag{30}$$

Now, we consider the following required lemma:

Lemma: We assume that $(v, \hat{a})$ is a solution for problem (17) to (20). In this case, $v(x,t)$ is true for equation (21) if and only if $\hat{a}(v)$ is true in the condition (30):

Proof: As $(v, \hat{a})$ has been assumed as a solution for problem (17) to (20), so if $v(x,t)$ is true for (21), (according to the previous process) $\hat{a}(v)$ will be true for (30). Also, if $\hat{a}(v)$ is true for condition (30) and $v(x,t)$ is a solution for problem (17) to (20) then we must show that $v(0,t) = F(t)$.

By subtracting equation (29) from equation (28):

$$\frac{\partial}{\partial t}v(0,t)-F'(t)=\left(\hat{a}(v(0,t))-\hat{a}(F(t))\right)\frac{\partial^2}{\partial x^2}v(0,t) \qquad (31)$$

Now we define the following function:

$$\tau(t)=v(0,t)-F(t) \qquad (32)$$

So, we can write:

$$\left|\tau'(t)\right|=\left|\hat{a}(v(0,t))-\hat{a}(F(t))\right|\left|\frac{\partial^2}{\partial x^2}v(0,t)\right| \qquad (33)$$

On the other hand, according to the assumption D, a positive number $k>1$ can be found:

$$\left|\hat{a}(v(0,t))-\hat{a}(F(t))\right|\le k\left|v(0,t)-F(t)\right| \qquad (34)$$

So, we have:

$$\left|\tau'(t)\right|\le k\left|\tau(t)\right|\left|\frac{\partial^2}{\partial x^2}v(0,t)\right| \qquad (35)$$

However, according to the assumptions D, E and the equation (29) we can conclude that:

$$\frac{\partial^2}{\partial x^2}v(0,t)>0$$

Therefore, equation (35) can be written:

$$\left|\tau'(t)\right|\le kM\left|\tau(t)\right|\le M\left|\tau(t)\right| \qquad (36)$$

Or:

$$\tau'(t)\le M\left|\tau(t)\right|$$

By integrating the above inequality, we have:

$$\tau(t)\le\tau(0)M\int_0^t\left|\tau(z)\right|dz \qquad (37)$$

According to Gronwall inequality, we conclude that:

$$\tau(t)\le\tau(0)e^{M\int_0^t\left|\tau(z)\right|dz} \qquad (38)$$

But, regarding assumption F and equation (18), $\tau(0)=0$ so $\forall t\in[0,T]$ $\tau(t)=0$, in other words

$\forall t\in[0,T]$, $v(0,t)=F(t)$.

We have shown that if $\hat{a}(v)$ is true in equation (30) then v is true in equation (17) to (21). Now, we consider the following problem:

$$\frac{\partial}{\partial t}v(x,t)=\left(\frac{F'(F^{-1}(\omega))}{\frac{\partial^2}{\partial x^2}\omega(0,F^{-1}(\omega))}\right)\frac{\partial^2}{\partial x^2}v(x,t) \qquad \text{in } Q_T \qquad (39)$$

$$v(x,0)=0 \qquad 0\le x\le 1 \qquad (40)$$

$$\frac{\partial}{\partial x}v(0,t)=g(t) \qquad 0\le t\le T \qquad (41)$$

$$\frac{\partial}{\partial x}v(1,t)=0 \qquad 0\le t\le T \qquad (42)$$

$$v(0,t)=F(t) \qquad 0\le t\le T \qquad (43)$$

where $\omega\in C_r$ with norms $\|\ \|_{1+\alpha,Q_T}$ and $\|\ \|_{1+\gamma,Q_T}$ that $0<\alpha<\delta<1$ and we show these norms by symbols $C_r^{1+\alpha}$ and $C_r^{1+\gamma}$. Also, $\omega\in C_r^{1+\alpha}$ is true in the following conditions**:**

1. $\|\omega\|_{1+\alpha,Q_T}$, $r>0$
2. $\omega(x,t)$ in conditions (40) to (43) is true.

Per $\omega \in C_r^{1+\alpha}$, problem (39) to (43) has a unique solution [13]. Now we consider transformation P so that $v = P\omega$ and $\omega \in C_r^{1+\alpha}$, also v is a solution for problem (39) to (43).

Theorem 1: Assume that $g(t)$ and $F(t)$ are true in F and G. Then per $\omega \in C_r^{1+\alpha}$, $v = P\omega$ has a fixed point.

Proof: First, we can mention the Schauder fixed-point theorem. According to this theorem, if Y is a subset of the Banach space X and S is a continuous function on Y and SY is located in Y and pre-compact, then S has a fixed point e.g. $\exists y_0 \in Y$ so that $Sy_0 = y_0$.

It must be noted, a set is a pre-compact set if and only if that is a subset of a compact set. First, we show that $PC_r^{1+\alpha}$ is in $C_r^{1+\alpha}$, then θ function is defined as:

$$\theta = v - \psi \tag{44}$$

where $\psi \in C_r^{1+\alpha}$, so we have:

$$\psi(x,t) = v(x,t) \qquad \text{in } Q_T$$

Now we write:

$$\frac{\partial v}{\partial t} = \frac{\partial \theta}{\partial t} + \frac{\partial \psi}{\partial t}$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \psi}{\partial x^2}$$

Therefore, according to equation (39) we have:

$$\frac{\partial \theta}{\partial t} - \left(\frac{F'(F^{-1}(\omega))}{\frac{\partial^2}{\partial x^2}\omega(0,F^{-1}(\omega))} \right) \frac{\partial^2 \theta}{\partial x^2} = -\frac{\partial \psi}{\partial t} + \left(\frac{F'(F^{-1}(\omega))}{\frac{\partial^2}{\partial x^2}\omega(0,F^{-1}(\omega))} \right) \frac{\partial^2 \omega}{\partial x^2} \qquad \text{in} Q_T \tag{45}$$

Also, it is clear that:

$$\theta(x,0) = 0 \qquad 0 \le x \le 1 \tag{46}$$

$$\theta(0,t) = 0 \qquad 0 \le t \le T \tag{47}$$

$$\theta(1,t) = 0 \qquad 0 \le t \le T \tag{48}$$

According to the theorem, there is a positive integer such as k that for $1 < \alpha$ the following inequality is true [14]:

$$\|\theta\|_{1+\alpha,Q_T} \le k \left\| \frac{\partial \psi}{\partial t} - \left(\frac{F'(F^{-1}(\omega))}{\frac{\partial^2}{\partial x^2}\omega(0,F^{-1}(\omega))} \right) \frac{\partial^2 \omega}{\partial x^2} \right\|_{0,Q_T} \tag{49}$$

Also, in definition of $C_r^{1+\alpha}$ we can choose r so that:

$$r \geq k\left\|\frac{\partial \psi}{\partial t}\right\|_{0,Q_T} + k\left\|\left(\frac{F'(F^{-1}(\omega))}{\frac{\partial^2}{\partial x^2}\omega(0,F^{-1}(\omega))}\right)\right\|_{0,Q_T}\left\|\frac{\partial^2 \psi}{\partial x^2}\right\|_{0,Q_T} + \|\psi\|_{0,Q_T} \tag{50}$$

Accordingly, we will have:

$$\|v\|_{1+\alpha,Q_T} \leq \|\theta\|_{1+\alpha,Q_T} + \|\psi\|_{1+\alpha,Q_T} \leq k\left\|\frac{\partial \psi}{\partial t}\right\|_{0,Q_T} + k\left\|\left(\frac{F'(F^{-1}(\omega))}{\frac{\partial^2}{\partial x^2}\omega(0,F^{-1}(\omega))}\right)\right\|_{0,Q_T}\left\|\frac{\partial^2 \psi}{\partial x^2}\right\|_{0,Q_T} + \|\psi\|_{1+\alpha,Q_T} \tag{51}$$

In other words, P transforms $C_r^{1+\alpha}$ to itself. Similarly, P transforms $C_r^{1+\gamma}$ to itself as well. Now, we can state the following theorem**:**

Theorem 2: Let D is a bounded set of $\Re^n$ space, also $C^{p+\alpha}$ is a set of functions with m^{th} order partial derivatives. These derivatives are locally continuous Holder functions. Also, we assume that $0<\alpha<\beta<1$ and $0\leq p\leq q$. In this case, the bounded subset of $C^{q+\beta}$ is a pre-compact subset of $C^{q+\alpha}$ [15].

By definition, function f in $x=x_0$ is a continuous Holder function of order α, if there are a constant number such as r and a small sized neighborhood of x_0, so that for each x in this neighborhood we will have: $|f(x)-f(x_0)|\leq r|x-x_0|^{\alpha}$ [16].

The space of single-variable functions whose m^{th} derivative is a continuous Holder function of order $0<\alpha\leq 1$ on $[a,b]$, is shown by $C^{m+\alpha}[a,b]$.

In according to this theorem, $PC_r^{1+\gamma} \subseteq C_r^{1+\gamma}$ is a pre-compact subset of $C_r^{p+\alpha}$. At the present, we have to prove the continuity of P .So we put $v=\theta+v_m$ where $v=P\omega$ and $v_m=P\omega_m$ are solutions for problem (45) to (48). So it is clear that θ is true in the following equation:

$$\frac{\partial \theta}{\partial t} - \left(\frac{F'(F^{-1}(\omega_m))}{\frac{\partial^2}{\partial x^2}\omega_m(0,F^{-1}(\omega_m))}\right)\frac{\partial^2 \theta}{\partial x^2} = \left(\frac{F'(F^{-1}(\omega))}{\frac{\partial^2}{\partial x^2}\omega(0,F^{-1}(\omega))} - \frac{F'(F^{-1}(\omega_m))}{\frac{\partial^2}{\partial x^2}\omega_m(0,F^{-1}(\omega_m))}\right)\frac{\partial^2 v}{\partial x^2} \tag{52}$$

So, when $\|\omega_m-\omega\|_{1+\alpha,QT} \to 0$ we have:

$$\varepsilon_m = Sup_{Q_T}\left|\frac{F'(F^{-1}(\omega))}{\frac{\partial^2}{\partial x^2}\omega(0,F^{-1}(\omega))} - \frac{F'(F^{-1}(\omega_m))}{\frac{\partial^2}{\partial x^2}\omega_m(0,F^{-1}(\omega_m))}\right|\frac{\partial^2 v}{\partial x^2} \to 0 \tag{53}$$

On the other hand, according to the theorem [14]:

$$\|v\|_{1+\alpha,Q_T} = \|P\omega_m - P\omega\|_{1+\alpha,Q_T} \le c\varepsilon_m \to 0 \tag{54}$$

So when $\omega_m \to \omega$, we will have:

$$\|P\omega_m - P\omega\|_{1+\alpha,Q_T} \to 0 \tag{55}$$

In other words, $P\omega_m \to P\omega$ that means P is a continuous transformation. So using the Schauder fixed-point theorem we conclude that P has a fixed point.

5. A proposed numerical algorithm

With considering the problem (10) to (14) we can assume that the diffusion coefficient of $a(u)$ is a polynomial of degree n with unknown coefficients of $c_0,...,c_n$:

$$a(u) = c_0 + c_1 u + ... + c_n u^n \qquad n \in N \tag{56}$$

The aim is to find degree and unknown coefficients of the above polynomial for any time. The equation (10) is written in the form of differential as below:

$$\frac{1}{k}\Delta_t u_{i,j} = \frac{1}{2}\left\{\frac{\delta_x^2(a(u_{i,j})u_{i,j}) + \delta_x^2(a(u_{i,j+1})u_{i,j+1})}{h^2}\right\} \tag{57}$$

Now, according to equations (11) to (13) and assuming $\delta_t = 0.01$ for any time, we get the following nonlinear equations system:

$$-2a(u_1)u_1 + (2+2a(u_0))u_0 = 0 \qquad i=0 \tag{58}$$

$$-a(u_{i-1})u_{i-1} + (2+2a(u_i))u_i - a(u_{i+1})u_{i+1} = 0 \qquad i=1,...,9 \tag{59}$$

$$-2a(u_9)u_9 + (2+2a(u_{10}))u_{10} = 0 \qquad i=10 \tag{60}$$

Now we can consider the following steps:

1. In the first stage, we assume that $a(u) = c_0$. According to the over-specified data (14), in the system of equations (58) and (60), $a(u)$ is known and c_0 as unknown , is inserted instead of $a(u_i)$ and then the values of c_0 and u_i for $i=1,...,10$ will be calculated by solving a system with 11 equations and 11 unknowns. Now $a(u)$ is known, so the equations (10) to (13) can be solved for $0 \le t \le T$ and the minimum of function $J_1 = \int_0^T |u(0,t) - F(t)|^2 dt$ can be obtained.
2. In the second step, with regards to c_0 has been obtained at the first step, by assuming $a(u) = c_0 + c_1 u$, coefficient c_1 can be calculated by solving equations (58) to (60) and we calculate the minimum function $J_2 = \int_0^T |u(0,t) - F(t)|^2 dt$. If the value of J_2 is less than J_1 , degree of the polynomial $a(u)$ will increase in the next step, otherwise we will stop this process.

6. Conclusion

The present work has put forward a nonlinear model which can be used to study the behaviour of an incompressible fluid in layer diffusion in a hydrocarbon reservoir. After proving the existence and uniqueness of solution for the problem, we have proposed a numerical algorithm in order to estimate the unknown parameters of the problem as described earlier. At the first, the mentioned steps in the algorithm, result in estimating the values of $a(u)$ at time $t = 0.01$, afterward, we need to repeat the steps to obtain the diffusion coefficient rates at every time level .In these time levels, if the rates are the same then one of them is considered as the diffusion coefficient, otherwise we have to fit a curve to the obtained diffusion rates. This curve is the total diffusion rate. Eventually, it is very interesting to illustrate some numerical results for unknown parameters as a numerical experiment.

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