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Fluid Flow through Shale Gas Reservoirs: Simplified Conceptual and Mathematical Models

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Abstract

Following the increasing global energy demand and oil prices, gas production has attracted the energy firms significantly. Apart from the natural gas production from conventional reservoirs, production of shale gas from unconventional reservoirs has contributed significantly in meeting the global energy demand despite its environmental impacts. Albeit the focus on hydro, solar, wind, nuclear and geothermal powers, the dependence on fossil fuels in the form of natural or shale gas still remains inevitable in the context of meeting the growing energy demand. However, the estimation and production of shale gas from an unconventional reservoir poses a lot of challenges throughout the production period. In particular, only a particular model is widely used to describe the shale gas adsorption-desorption mechanism irrespective the complex geology associated with the shale gas reservoir. In this context, a sensitive table has been deduced that clearly distinguishes the fundamental differences of gas extraction from a conventional gas reservoir from that of an unconventional shale gas reservoir. Next, a possible list of queries that remain unanswered in the context of field-scale shale gas estimation and production. Third, a detailed list of complexities associated with the development of the conceptual model has been listed. Fourth, a relatively simpler conceptual model along with its associated mathematical model has been deduced to estimate the shale gas concentration. And, finally, a detailed set of various sorption isotherm models have been gathered that will suit the varying nature of a complex shale gas reservoir. Thus, the study is expected to provide field-scale limitations associated with the estimation and production of shale gas from a shale gas reservoir, and it will subsequently aid in deducing better shale gas reservoir management decisions.

Keywords: *Natural gas; Shale gas; Energy demand; Unconventional reservoir; Adsorption isotherm.*

1. Introduction

The rapid decline associated with the production of hydrocarbons from conventional reservoirs with the rapid enhancement in global energy demand has led to the exploration of various forms of alternative energy sources. With the field-scale difficulties associated with the relatively costlier renewable fuels, non-renewable fossil fuel energy source seems to be the only sustainable energy source at least for the coming decade albeit it's environmental impacts that include the contamination of groundwater aquifers resulting from the hydraulic fracturing of a low permeable shale gas reservoir. However, since, there are no significant findings of new conventional oil and gas resources, more attention is being given on the abundant unconventional oil and gas resources. Shale gas is classified as the energy resource extracted from an unconventional reservoir. Unconventional reservoir represents a reservoir that does not allow the mobility of the trapped fluid to get drained with ease due to its extremely low – permeable nature and other associated complex geological features. It should be clearly noted that unconventional resources (fluid properties) such as gas-hydrates are completely different from unconventional reservoirs (rock properties). For example, reservoirs such as shale gas and coal-bed methane reservoirs have conventional resources to be extracted from unconventional reservoirs. The unconventional 'shale gas' with an estimated, technically recoverable amount of nearly 7500 trillion cubic feet, roughly splits over 150 basins (with roughly 700 shale deposits) globally. The fluid flow mechanism associated with a shale gas reservoir typically requires a sound understanding of its associated geo-mechanical aspect, apart from the fundamental principles of reservoir engineering. Since, shale gas reservoir forms both as a source rock as well as a reservoir rock, the fundamental conceptual modelling of a shale gas reservoir itself have many complexities associated with it. Unless one clearly knows the complexities associated with conceptual modelling, it may not be to deduce an appropriate mathematical model. For example, dividing the physical domain into many sub-regions (in and around the Stimulated Reservoir Volume) with gas flow happening in various directions will complicate not only the mathematical model; but also, its associated numerical model as well. In this context, it will be advisable to make use of the existing conceptual models in order to replicate the migration of shale gases within the complex reservoir. Thus, the development of relatively simple conceptual and mathematical models for the estimation and production of shale gases will not only help in making quick reservoir management decisions but also allows to keep updating the model design dynamically as a function of the production period. Further, since, the role of sorption mechanism seems to be very sensitive in a shale gas reservoir, it will be advisable to have all kinds of available sorption isotherms at one place so that any modeller can make of the isotherms of their choice depending on the feasibility and availability of the sorption isotherm parameters. Because, Langmuir/BET isotherm is predominantly used to characterize the sorption mechanism, while, both these techniques can only be applied on a homogeneous surface, which is far away from reality as the pore geometry of the shale matrix is so complicated; and subsequently, the other advantages associated with various other sorption models need to be explored. Thus, the objective of the present manuscript is (a) to clearly bring out the fundamental differences the conventional gas reservoir and unconventional shale gas reservoirs; (b) To deduce a list of complex queries that remain unanswered in the context of estimation and production of Original Shale Gas in Place; and (c) to propose a simplified conceptual model and its associated mathematical model for estimating the shale gas production scenario; (d) to provide a robust list of various adsorption isotherm models that will suit to varying conditions of shale gas reservoirs.

Table 1. Conventional & unconventional gas reservoirs

Conventional gas reservoir	Unconventional shale gas reservoir
Acts as a ‘Reservoir Rock’	Acts as both ‘Source Rock’ as well as ‘Reservoir Rock’
The dominant geological properties include average reservoir porosity and permeability apart from pay zone thickness.	The dominant geological properties include (a) the history of the geological environment; (b) organic geo-chemistry; (c) organic richness; (d) shale-matrix minerology; (e) thermal maturity; (f) in-situ stress regime; and (g) stress history apart from (h) porosity; (i) permeability; & (j) pay zone thickness.
Pay zone thickness varies by a scale factor (10 m – 80 m)	Pay zone thickness varies by orders of magnitude (3 m – 300 m)
Characterized by significantly permeable (1 – 1000 mD) geological formation.	Characterized by extremely low-permeable (0.00001 - 0.000001 mD) geological formation; and hence, requires hydraulic fracturing.
Porosity & Permeability are in general not treated as a function of pressure.	Porosity & Permeability are strong functions of pressure.
Mostly found in a terrestrial environment (with colloidal humic content) pertaining to Type III Kerogen (Terrestrial – Vitrinite)	More than 95% of shales have been deposited in a marine environment pertaining to Type II Kerogen (Aquatic - Amorphous Kerogen)
Gas is produced from granular (relatively larger grain size), significantly porous and permeable geological formation.	Gas is produced from very fine-grained, micro-/nano-porous and relatively impermeable geological formation.

Conventional gas reservoir	Unconventional shale gas reservoir
Conventional source (simple methane gas) from a conventional reservoir.	Conventional source (simple methane gas) from an unconventional reservoir.
The role of Total Organic Carbon (TOC) content remains insignificant in a reservoir rock.	The role of Total Organic Carbon (TOC) content is very sensitive (having greater than 2% indicating good quality kerogen) in a source rock as it controls the extent of hydrocarbon generation as well as the quality of the adsorbed gas.
Average Estimated Ultimate Recovery per well is around 1000 Bcf/well.	Average Estimated Ultimate Recovery per well is around 1 Bcf/well.
Gas can be produced in the absence of hydraulic fracturing as the permeability is not that bad.	Gas can be produced only with hydraulic fracturing as the permeability is really low.
Methane gas is present only as a free gas.	Methane gas is not only present as free gas in inorganic pores and fractures but also in the form of adsorbed gas within the organic matter and on clay minerals.
Simple material balance equation is sufficient in order to estimate the free gas reserves.	Material balance will not be sufficient enough in order to include the presence of adsorbed gases within the organic pore spaces.
Gas is present only as a 'free gas'; and hence, the estimation of OGIP is relatively easier.	Gas is present in the form of 'free gas'; 'adsorbed gas'; and 'dissolved gas'; and hence, the estimation of OGIP is relatively complex.
Free gas accounts for 100% of gas in the absence of any adsorbed gas.	The fraction of both free gas present in fracture and matrix, as well as adsorbed gas (that varies between 10 and 90%), vary widely.
The amount of free gases present within the connected network of pores represents the total quantum of OGIP available.	The amount of free gas present in the fractures are generally several orders of magnitude lesser than that of the amount of free gases present within the shale-matrix; and/or the amount of adsorbed gases. Hence, the amount of free gases present in fractures can generally be ignored while estimating OGIP.
The reservoir can be assumed to have a uniform initial reservoir pressure as it is only associated with free gas.	Since, the reservoir is associated with free gas, adsorbed gas and dissolved gases; it may not be correct to assume the initial reservoir pressure to remain uniform.
Nearly 7000 Tcf of Technically Recoverable Methane gas is available globally.	Nearly 8000 Tcf of Technically Recoverable (wet) shale gas is available globally.
Expected enhancement in the methane gas production rate: 350 Bcf/d in 2015 to 600 Bcf/d in 2040.	Expected enhancement in the shale gas production rate: 35 Bcf/d in 2015 to 200 Bcf/d in 2040.
Vertical drilling is favourable.	Horizontal drilling is favourable as it provides access to a greater portion of the reservoir that is running horizontally over 2 km. (Horizontal wells can cost thrice the amount of vertical wells).
Continuum based Darcy's equation can be applied to describe the methane gas flow in the presence of non-linear quadratic pressure gradient term.	Darcy's equation, based on macroscopic continuum does not accommodate the fluid flow through pore sizes in the range of nano-meters as in this regime the continuum theory fails.
No-slip boundary condition can be applied at the walls.	At the walls, the velocity of the solid and fluid does not remain the same; and the fluid molecules keep slipping; and hence, require Klinkenberg's correction.

Conventional gas reservoir	Unconventional shale gas reservoir
Reservoir permeability generally decreases as the production commences as the reservoir is getting compacted following the depletion.	The apparent permeability in shale matrix gets increased with production resulting from the combined effect of 'slip flow' and the release of 'shale gas' by desorption.
Free molecular diffusion is feasible with the kind of pore sizes.	Free molecular diffusion is not feasible within the nano-pores; and only, either Knudsen diffusion or surface diffusion is possible within the shale matrix.
Gas Recovery Factor does not depend on adsorption/desorption.	Gas Recovery Factor critically depends on the rate at which the adsorbed gas gets desorbed as a function of reservoir pressure.
Gas is stored by the compressibility of the pore fluid.	Gas is not only stored by the compressibility of the pore fluid; but also by the adsorption mechanism due to the associated large specific surface area.

2. Unanswered questions of practical relevance

1. How about the contribution of shale gas with reference to the overall gas recovery? Will it be significant enough to be considered along with the quantum of 'free gas' produced OR the fraction of 'adsorbed gas' contributing to the overall gas recovery can be ignored?
2. Will it be really feasible to measure the 'specific surface area' of the shale matrix from in-situ conditions? Will a greater 'specific surface area' would indicate that the storage of shale gas resulting from adsorption is larger than the storage of shale gas resulting from the pore-fluid compressibility?
3. Will it be feasible to deduce the pore-size distribution associated with the shale-matrix? Is there a way to decide the option between 'Knudsen diffusion' and 'Surface diffusion' – given the 'mean pore size' or 'pore-size distribution'?
4. How exactly the presence of fractures could be delineated in shale-gas reservoir following the hydraulic fracturing? Will it be feasible for us to ensure the hydraulic connectivity between the newly generating fractures resulting from hydraulic fracturing? What will happen to the resultant shale-gas reservoir's permeability, if the 'fracture density' is increased in the absence of its 'hydraulic connectivity'?
5. How exactly to have control over a "coupled horizontal drilling" and "hydraulic fracturing"?
    i. Will this combination really reduce the risk associated with the extremely low permeable shale formation?
    ii. Won't it require an enhanced initial investment?
6. Whether the cheaper production of 'shale gas' has really led to the reduction of gas prices with reference to 'prices of the oil'?
7. Given the complexity associated with the optimization of 'fracture design' associated with 'hydraulic fracturing', will it be really feasible to have better control over the "well performance"?
8. Whether the fracture attributes such as 'fracture length'; 'fracture aperture thickness'; 'fracture spacing'; 'fracture dip'- will have a significant influence on the resulting 'desorption pattern' of shale gases?
9. Desorption of shale gas: Will it be critical during the early or later phases of production? Or will desorption be very sensitive as soon as the reservoir pressure falls below the 'critical desorption pressure'? Or should we wait until the well experiences the 'boundary dominated flow', where desorption commences?
10. Langmuir isotherm, characterized by mono-layer adsorption of shale-gas; won't it lead to under-estimation of the total "adsorbed gas" in the shale matrix?
11. How exactly 'desorption' influences the Shale gas 'recovery factor'?
12. What will be the approximate variation in percentage in the OGIP estimates, 'by considering' and 'by ignoring' the 'desorption' effect?

13. Since ‘adsorption capacity’ is a function of both ‘pressure’ and ‘temperature’, should the reservoir be characterized under non-isothermal conditions?
14. It is known that adsorption and desorption would remain reversible as the associated ‘van Der Waals’ forces are ‘weak’ by nature. But, then, will the ‘rate’ at which the shale gas molecules that ‘get adsorbed’ will be exactly equal to the ‘rate’ at which the shale gas molecules that ‘get desorbed’? In other words; will the sorption be ‘non-linear’ and/or ‘non-equilibrium’? OR ‘sorption kinetics’, where sorption becomes a function of time will be required to characterize the dynamic sorption behavior associated with the shale matrix? If so, how are they expected to influence the resulting gas ‘recovery factor’?
15. Is there a method to delineate the fraction of Shale gas (a) associated with ‘pore-space’; (b) associated with the high-permeable ’natural fractures’; and (c) associated with ‘adsorption’ on to the Shale matrix?
16. What will be the ‘required downhole pressure’ for the ‘release of shale gas’ from its ‘adsorbed state’?
17. How to determine the ‘diffusivity coefficient’ of shale-gas within the reservoir? Will we require ‘porosity’ and ‘tortuosity’ in order to deduce the ‘effective diffusion coefficient’ of shale gas?
18. Considering the behavior of sorption isotherms (using Langmuir or BET) under non-isothermal conditions, would it be significantly varying from that of the isotherms under ‘isothermal conditions’?
19. Whether the estimation of ‘adsorption capacity’ of shale gas of using Langmuir & BET would remain the same?
20. To what extent, the coupling between ‘fluid flow’ and ‘geo-mechanics’ (using in-situ reservoir stresses, and considering those stresses resulting from hydraulic fracturing) would enhance the production forecast of shale gas associated with a shale-gas reservoir?
21. Can a simple ‘material balance method’ be used in order to determine the OGIP associated with a Shale gas reservoir?
22. Since the fraction of gas adsorption (10-90%) can vary significantly in a shale gas reservoir; whether the ‘method of estimation’ of ‘total shale gas’ associated with a shale gas reservoir should remain different for the (a) ‘dominant adsorbed gases’; (b) ‘dominant free gases’; and (c) ‘equally dominant adsorbed and free gases’?
23. Will the ‘selection of adsorption models’ be sensitive to the ‘type of shales’ associated with a shale gas reservoir?
24. Whether the shale gas of interest associated with a typical shale gas reservoir be assumed to have only a single component (such as only methane gas) - given the fact that the shale gas will be a mixture containing multiple components (mostly CO_2 along with methane)?
25. Will it be feasible to estimate the adsorption capacity of shale gas in a shale-gas reservoir as a function of (a) pressure; (b) temperature; (c) porosity; (d) permeability; (e) kerogen content; (f) level of thermal maturity; and (g) the total organic carbon content (TOC)? In other words; a high TOC would always result in an enhanced adsorption capacity in a linear fashion?
26. Whether the ‘reservoir pressure’ will linearly or non-linearly influence the ‘adsorption quantity’? In other words, is there a relation between the ‘binding energy’ of gas molecules on to the shale matrix and the ‘adsorption quantity’?
27. Will it be feasible to find a ‘homogeneous’ shale surface associated with the shale-matrix, and that too, in the absence of any physical or chemical interactions between the adjacent molecules? If not, can Langmuir isotherm be used to characterize the sorption behavior associated with a typical shale gas reservoir? Won’t the number of available sorption sites vary as a function of temperature?
28. Will it be feasible to ignore the concept of ‘competitive adsorption’ between the various gas molecules associated with the ‘shale gas mixture’ that are competitive to get adsorbed on a particular adsorption site?
29. Will, it really feasible to delineate the region with stimulated reservoir fractures and those regions that remains unaffected by the stimulation? It should be noted that based on these

delineations only, most of the shale gas reservoir models have been bifurcated into regions 1 and 2.

30. Is there a way to capture the generated fracture network following the hydraulic fracturing? Unless the connectivity between the tertiary, secondary and primary fractures with that of the horizontal wellbore is ensured, it will be really difficult to estimate OGIP. Also, will pattern and geometry of the generated fracture network following the hydraulic fracturing would remain the same throughout the shale gas production period?

3. Conceptual model of a shale gas reservoir

The modelling of fluid flow through a shale gas reservoir can be carried out using the concept of multi-continuum model. Since the spatial and temporal distribution of the reservoir pressure will not remain smooth and continuous resulting from the heterogeneity of a shale reservoir, the entire shale matrix cannot be conceptualized to be characterized by a single continuum concept, particularly following the hydraulic fracturing. Since new fractures are generated following the hydraulic fracturing, the concept of multi-continuum becomes inevitable in order to describe the fluid mobility within the shale gas reservoir. Fluid flow through a dual-continuum model distinctly varies from a conventional single continuum model in the following aspects:

1. The modelling of transportation of shale gas within the reservoir, in general is a complex process; and the mode of transportation varies as a function space. Thus, migration of shale gas will be characterized by diffusive transport at some places within the shale-matrix, while, it will predominantly be transported by viscous forces as soon as it enters the high permeable fracture. The distribution of the pore sizes at various locations decide the mode of mass transfer.

A. Adsorption of shale gas onto the shale-matrix grains from its free state within the pores
B. Desorption of shale gas from its adsorbed state – back into pores
C. Diffusive mass transfer of shale gas within the low-permeable shale matrix
D. Interface mass transfer of shale gas at the matrix-fracture interface
E. Transportation of shale gas within the high permeable fractures by viscous flow using Darcy's law; and its migration towards the production well.

2. Since the mean pore size associated with a shale gas reservoir generally falls below 1 micron, it is not practically feasible to assume to shale gas to be driven by viscous flow in a shale-matrix. Further, depending on the mean pore size, it may also not feasible to include the diffusion of shale gas by free molecular diffusion as defined by Fick's law, but rather, we have the option of characterizing the diffusion of shales by means of either surface diffusion or Knudsen diffusion only. Hence, conceptualizing the transport of shale gas by a relatively simpler free molecular diffusion may not be correct.
3. The conceptualization of the mass transfer of shale gas between high-permeable fracture and low-permeable shale matrix at the fracture-matrix interface under isothermal and non-isothermal conditions [1-14] will be significantly different; and deserves special attention.
4. When multi-stage hydraulic fracturing is used to enhance the permeability of the shale gas reservoir, it can also be conceptualized with the concept of 'triple continuum' using primary fractures, secondary and tertiary fractures embedded within the shale matrix. The concept of 'Stimulated Reservoir Volume' (SRV) will be very useful in this context.
5. Conceptualization of free gas, adsorbed gas and dissolved gas remains very challenging; and thus, the estimation and production details of total shale gas as a function of time remains very complex.
6. The conceptualization of a shale gas reservoir using the concept of 'Equivalent Porous Medium' (EPM) will provide completely deviating results as a shale gas reservoir has a unique heterogeneity that is different from a conventional sandstone or a fractured reservoir.
7. The conceptualization of the sorption mechanism; and their corresponding adsorption and desorption rates of shale gas associated with a shale gas reservoir using Linear and non-linear sorption isotherms [15-19] will be very sensitive in deducing the resulting OGIP.

8. The porosity and permeability of a shale gas reservoir become a function of space; time; reservoir pressure; and reservoir temperature; and thus, the conceptualization of a shale gas reservoir remains very complex and unique.
9. The conceptualization on the nature of pore systems associated with a shale gas reservoir that includes micro-pores; mesopores; and macro-pores; and its influence on shale gas productivity [20-21] remains very complex as the pore size distribution keeps varying as a function of time, following the hydraulic fracturing.
10. The conceptualization of a shale gas reservoir must couple the fluid flow along with the geo-mechanical module as the resulting flow of shale gases towards the production well will significantly get influenced by both vertical and lateral stresses. The conceptual model should include the details of stress distribution before and after hydraulic fracturing.
11. The conceptual model should explicitly include the spatial and temporal distributions of both porosity and permeability; both before and after hydraulic fracturing. Sometimes, it is also possible that the enhanced permeability by hydraulic fracturing may get lost resulting from the restructuring of the pore-geometry (in particular, it may clog the pores). Thus, the model should explicitly estimate the porosity and permeability distribution before and after hydraulic fracturing.
12. The conceptual model should include a means to include the details of 'specific surface area' of the shale-matrix as a function of time; in particular, before and after hydraulic fracturing. A varying specific surface area as a function of reservoir pressure, reservoir temperature, reservoir porosity, reservoir permeability, thermal maturity, organic geochemistry, organic richness and in-situ stress would be very useful in order to better forecast the OGIP.
13. The conceptual model may exclude the gravity and capillary effects, in general.
14. The conceptual model should include the details of the composition of the associated shale gas; it will mostly be a mixture of gases, predominantly methane gas along with other gases.
15. The conceptual model should include the details of means to estimate the quantum of free gas and adsorbed gas explicitly, apart from the means to compute the total gas associated with the shale gas reservoir.
16. The conceptual model should include the details of estimating the density of the gas as a function of reservoir pressure. It is to be noted that the free gas present within the shale-matrix pores at relatively lower pressure may start condensing at higher reservoir pressure; and in turn, the density of the free gas may reach that of the density of the adsorbed gas. This density variation may lead to an erroneous estimation of total available shale gas within the reservoir.
17. The conceptual model should clearly include the details on (a) the connectivity between the primary fracture and secondary fracture; (b) the connectivity between the secondary and tertiary fracture; (c) the connectivity between the tertiary fracture and the shale-matrix for a better migration of both the free gas as well as adsorbed gases.
18. The conceptual model should clearly delineate the boundary of the 'Stimulated Reservoir Volume' (SRV), where, many tertiary fractures or micro-fractures are created around the horizontal wellbore. Again, it is necessary that the permeability associated with the region of SRV should be estimated both before and after the hydraulic fracturing as it will provide better knowledge of the resulting gas production rate.
19. The conceptual model should ensure that the production rate contributions of hydraulic fractures are estimated individually in the absence of interferences between the adjacent hydraulic fractures, and this individual rate would depend on their respective fracture characteristics.

4. Proposed conceptual and mathematical models

The conceptual model associated with the estimation and production of shale gas from a shale gas reservoir always remained very complex in the absence of simplicity. It is to be noted that when deducing a conceptual model, when the streamlines associated with the flow of fluid has multi-directional fluid flow, the resulting mathematical and numerical models become highly complex. A recent work by [22] makes use of a coupled fracture-matrix system that accommodates the horizontal hydraulic fracture, region with stimulated reservoir volume and region with unstimulated reservoir volume. However, they have not explicitly considered the coupling between the shale gas mass transfer between the horizontal wellbore and region 1; and the shale gas mass transfer between the region 1 and region 2. Instead, they have used an equivalent value for porosity and permeability. Further, it requires a lot of experimental data along with the details on various fracture attributes in order to estimate the gas recovery factor. Also, it involves fluid flow in multiple directions within the physical domain of interest.

Hence, a conceptual model with a unidirectional fluid flow would remain very simple and easy to follow. Also, the Cartesian coordinate system is relatively easier to understand than a radial coordinate system. In addition, in a Cartesian coordinate system, the cross-sectional area normal to the fluid flow remains a constant, while the cross-sectional area in a radial coordinate remains varying. Thus, following a Cartesian coordinate system not only remains simple but also ensures a uniform fluid flow as against the non-uniform fluid flow associated with the radial coordinate. The proposed mathematical model uses the concentration as the primary dependent variable with reduced number of required experimental parameters, unlike the earlier works, which require a larger number of experimental data set. Further, measuring the concentration of shale gas is much easier than measuring the respective fluid velocity at various spatial locations. In this context, a relatively simpler conceptual model is proposed for the estimation of shale gas associated with a shale gas reservoir. For this purpose, the spatial and temporal distribution of the available (not detailed) pressure distribution may be used to deduce the respective velocity distribution using Darcy's law within the horizontal wellbore.

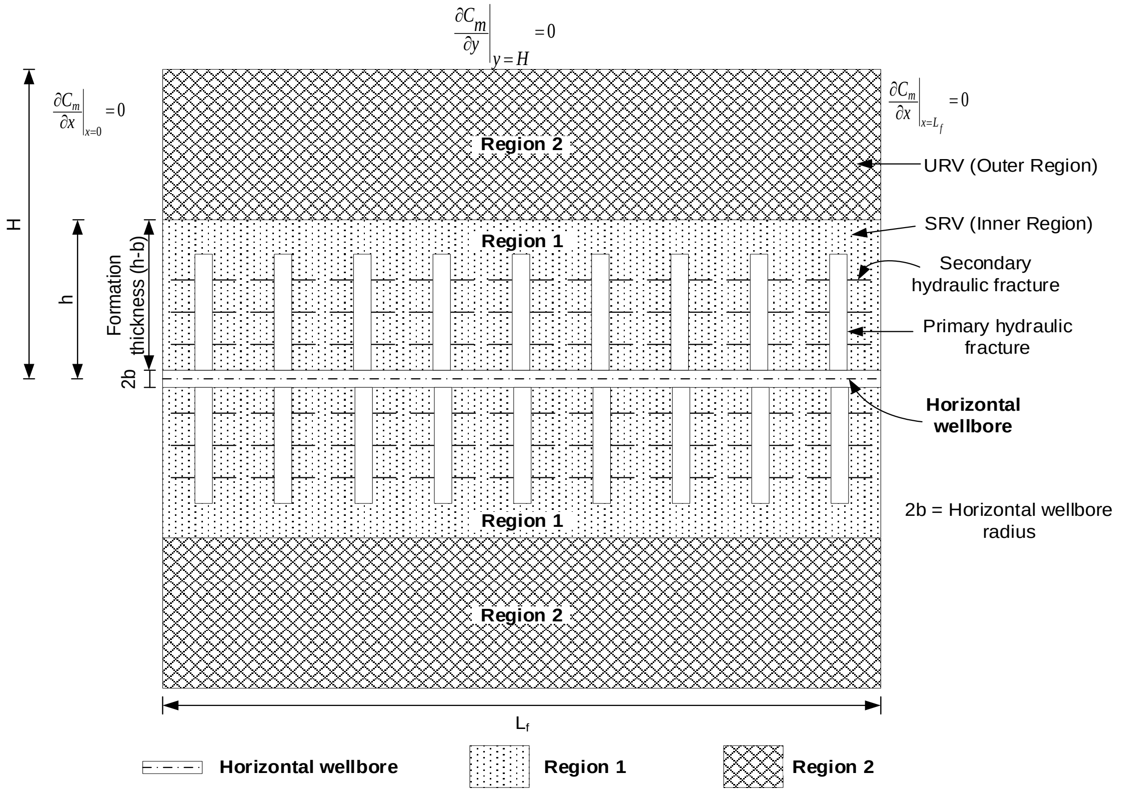

Fig. 1. Horizontal wellbore with SRV (Inner Region) and USRV (Outer Region)

Horizontal wellbore has a length of L_f and it is located below the regions 1 and 2 as shown in fig 1. The rectangular Region 1 pertains to the Stimulated Reservoir Volume (SRV), where both primary and secondary hydraulic fractures have been generated. Primary hydraulic fracture is located perpendicular to the horizontal wellbore, while the secondary hydraulic fracture is located normal to the primary hydraulic fracture. The region 2 pertains to the outer unstimulated region in the absence of any fracture. Neumann type flux boundary conditions have been applied on the three sides of the reservoir as shown, while the mass transfer of shale gas is ensured at the interface of horizontal wellbore and region 2; and the same acts as the boundary condition. The thickness of the horizontal wellbore is '2b', while the thickness of region 1 is (h-b) as shown in fig 1. The mobility of the shale gas from region 1 and 2 eventually reach the horizontal wellbore. The contribution of shale gas from region 2 will be very minimal, while the contribution of shale gas from region 1 will be dominant. In this model, there is no need to consider the details of the various pore sizes associated with region 1 and 2. This model essentially estimates the concentration distribution of shale gas within the reservoir.

Equation (1) provides the spatial and temporal distribution of concentration of shale gas within the high permeable wellbore. The main mechanism associated with the mobility of shale gas within the wellbore are advection and dispersion.

$$\frac{\partial C_{well}}{\partial t} = D_{well}\frac{\partial^2 C_{well}}{\partial x^2} - v_{well}\frac{\partial C_{well}}{\partial x} + \frac{\theta_{R1}D_{R1}}{b}\bigg|_{y=b} \qquad (1)$$

where C_{well} is the concentration of shale gas in wellbore; v_{well} is the velocity of shale gas in the wellbore; D_{well} is the dispersion coefficient of shale gas in the wellbore as a function of shale gas velocity and dispersivity; θ_{R1} is the effective porosity of reservoir pertaining to region 1; D_{R1} is the efficient diffusion coefficient of the reservoir pertaining to region 1.

Equation 2 provides the distribution of shale gas within the region 2.

$$\frac{\partial C_{R1}}{\partial t} = D_{R1}\frac{\partial^2 C_{R1}}{\partial y^2} \qquad (2)$$

$$\frac{\partial C_{R2}}{\partial t} = D_{R2}\frac{\partial^2 C_{R2}}{\partial y^2} \qquad (3)$$

where C_{R1} is the concentration of shale gas in region 1; D_{R1} is the diffusion coefficient resulting from free molecular diffusion associated with relatively larger pores; $C_{R2}$is the concentration of shale gas in region 2; D_{R2} is the diffusion coefficient resulting from coupled effect of surface diffusion and Knudsen diffusion associated with relatively smaller pores.

In eqn (1) the shale gas is driven by viscous flow while in eqns (2) and (3) fluid is driven by concentration gradient.

Initial conditions

$$C_{well}(x,t=0) = C_{R1}(x,y,t=0) = C_{R2}(x,y,t=0) = C_{ini} \qquad (4)$$

where C_{ini} is the initial concentration in the pores.

Boundary conditions

$$C_{well}(x=0,t) = C_o \qquad (5)$$

where C_o is the concentration of shale gas in the production wellbore.

$$C_{well}\left(x=L_f,t\right) = 0 \qquad (6)$$

$$C_{well}(x,t) = C_{R1}(x,y=b,t) \qquad (7)$$

$$\theta_{R1}D_{R1}\frac{\partial C_{R1}(x,y=h,t)}{\partial y} = \theta_{R2}D_{R2}\frac{\partial C_{R2}(x,y=h,t)}{\partial y} \qquad (8a)$$

where θ_{R2} is the effective porosity of reservoir pertaining to region 2.

$$C_{R1}(x,y=h,t) = C_{R2}(x,y=h,t) \qquad (8b)$$

$$\frac{\partial C_{R2}(x,y=H,t)}{\partial y} = 0 \qquad (9)$$

Since, region 1 is directly associated with hydraulic fracturing, the pore sizes and in turn, the pore connectivity will be very good. Hence, random free molecular diffusion of shale gas is assumed within the region 1, dictated by Fick's second law. In region 2, there will not be hydraulic fracturing, and hence, the average pore sizes associated with region 2 will be extremely small. In such cases, random free molecular diffusion cannot be entertained, while the combination of surface and Knudsen diffusion play a crucial role. Thus, eq. (2) has the effective diffusion coefficient resulting from free molecular diffusion, while eq. (3) has an effective diffusion coefficient resulting from the summation of Knudsen and surface diffusive processes. Equation 1 has three terms on its right side. The first term refers to the dispersion of shale gas, while the associated coefficient describes the rate at which the shale gas gets dispersed during its mobility through the horizontal wellbore. The second term represents the advective flow of shale gas within the horizontal wellbore, where the shale gas moves with a finite velocity. The last term on RHS of eq. (1) represents the mass transfer of shale gas from region 1 to the horizontal wellbore. The rate at which this particular shale mass transfer takes place will depend on the effective diffusion coefficient of region 1, effective porosity of region 1 and the radius of the horizontal wellbore. This flux at the interface of the horizontal wellbore and region 1 will be very crucial in the context of estimating the resultant shale gas production. The concentration at the left boundary can be measured from the vertical production bore-well directly. The Neumann type boundary conditions with zero fluid flux is assumed over the three planes, where x=0 (vertical line – left boundary); x=L_f (vertical line – right boundary); and y=H (horizontal line – top boundary) as shown in the fig. 1. The centre of the horizontal wellbore forms the bottom boundary along which the concentration of shale gas within the horizontal wellbore and from the region 1 is assumed to remain the same. This condition is introduced in order to maintain the shale gas fluxes at the interface to remain continuous as the shale gas moves from a region with very low permeability (region 1) to a region with extremely high permeability (horizontal wellbore). Thus, there is a possibility of fluid fluxes getting discontinued at the interface; and in order to avoid that an equal concentration has been assumed at the interface as given in eq. (7). One more boundary condition is required in order to ensure the continuity of the shale gas fluxes between the regions 1 and 2. For this purpose, the concentration of shale gas at the interface between region 1 and 2 is assumed to be equal as given in eq. (8b). It is also possible to consider the fluxes to remain equal at the interfaces between regions 1 and 2 [as given in eq. (8a)], rather than considering it, as the dependent variable. It can be seen that the proposed model does not require the details associated with the region 1, while all the parameters are getting reflected in a single diffusion coefficient as given on the RHS eq. (2). This model will yield quick results as this is a simple linear partial differential equation in the absence of any non-linearity. The model can further be extended in order to include the details of sorption kinetics associated with the regions 1 and 2. Thus, equation 2 and 3 will get modified upon the inclusions of sorption kinetics, while the eq. (1) would remain the same.

Thus, the proposed model provides an easy way to estimate the concentration of shale gas within the horizontal wellbore; and in turn, towards the production well; and subsequently, the quantum of shale gas production can be estimated with ease. Although the proposed model makes use of concentration as the primary dependent variable as against the ‘pressure’, it will still be useful as the respective pressure can be estimated indirectly, having known the fluid velocity and dispersivity. The novelty of this approach is that the shale gas is driven by viscous forces within the horizontal wellbore, and hence, both the measurements on pressure (and in turn, velocity, using Darcy’s law) and concentration can be directly recorded from the wellbore. The porosity and diffusion coefficient are the only required parameters; and this approach does not depend on a large number of experimental parameters that are usually required. Another important aspect to be noted is that the concept of ‘error’ associated with modelling results. Whenever, modelling results are generated using Darcy’s law, we are not supposed to look at any physical / chemical / biological activity; and in turn, their associated parameters of interest at a scale that is lesser than the macroscopic-scale, because, Darcy’s law is based on the macroscopic-based continuum, unlike the Naiver-Stokes Equation, which is based on the microscopic-continuum. This means that the numerical modelling results based on Darcy’s approach only provide the trends or patterns on the distribution of the dependent variables of interest, say, pressure and/or saturation, over the Representative Elementary Volume (REV), which obviously consists of a cluster of solid grains and voids (pore-spaces); and not within a single pore. Thus, the model predictions that require more and more experimental values deduced at the pore-scale will not contribute much, when the approach is Darcy based. Thus, the proposed model uses the least number of parameters, while providing the concentration; and in turn, the pressure estimation of shale gas associated with a shale gas reservoir.

Table 2. Sorption isotherm models

Isotherms	Nomenclature	Remarks
Henry's [23] $q = KC_e$	q = amount of the adsorbate at equilibrium K = adsorption constant C_e = adsorbate equilibrium concentration	Based on Henry's law; one parameter; single-component; low-pressure (< 1MPa); low-coverage adsorption (10%);
Langmuir Adsorption [24] $V(P) = \frac{V_L bP}{1 + bP}$ $b = \frac{1}{P_L}$	V = adsorbed volume V_L = Langmuir volume P_L = Langmuir pressure	Based on Henry's law; two parameters; monolayer; single-component; low pressure; homogeneous surface; no interaction between molecules;
Modified Langmuir [25] $V = (1 - C_a - C_w)\frac{V_L P}{P + P_L}$	C_a = the ash content C_w = the moisture content	For CBM reservoirs
Extended Langmuir [26] $V_i = V_{Li}\frac{\left(\frac{b_i}{\eta_i}\right)P_i^{\frac{1}{q_i}}}{1 + \sum_{i=1}^{N}\left(\frac{b_i}{\eta_i}\right)P_i^{\frac{1}{q_i}}}$	V_i = adsorption capacity b_i, V_{Li} and η_i are isotherm parameters q_i = heterogeneous parameter N = no. of components	Multi-component
Brunauer–Emmett–Teller (BET) [27] $q = \frac{q_s C_{BET} C_e}{(C_s - C_e)\left[1 + (C_{BET} - 1)\left(\frac{C_e}{C_s}\right)\right]}$	q = equilibrium adsorption capacity C_{BET} = BET isotherm constant C_s= adsorbate monolayer saturation concentration q_s = theoretical isotherm saturation capacity	Multilayer; homogeneous surface; no interaction among adsorbed molecules
Frenkel–Halsey–Hill [28] $ln\left(\frac{C_e}{C_s}\right) = -\frac{\alpha}{RT}\left(\frac{q_s}{q_e d}\right)^r$	d and r are isotherm constant	Multilayer; derivation from the potential theory
MacMillan–Teller [29] $q = q_s\left(\frac{k}{ln\left(\frac{C_s}{C_e}\right)}\right)^{\frac{1}{3}}$	where k is an isotherm constant	Multilayer; inclusion of surface tension effects in the BET isotherm
Freundlich Adsorption [30] $q = KC_e^{\frac{1}{n_F}}$	q = adsorbate amount C_e = equilibrium adsorbent concentration K = isotherm constant $1/n_f$ = heterogeneity factor	Monolayer; medium-pressure; low-concentration gas; two parameters; heterogonous surfaces; non-uniform adsorption
Extended Freundlich (FM) [31] $V_1 = \frac{V_{F1}P_1^{n_1+x_1}}{P_1^{x_1} + y_1 P_2^{z_1}}$ $V_2 = \frac{V_{F2}P_2^{n_2+x_2}}{P_2^{x_2} + y_2 P_1^{z_2}}$	V_{F1}, V_{F2} = adsorption capacity of individual components n_1, n_2 and F are isotherm constant x_1, y_1, z_1, x_2, y_2 and z_2 FM experimental data	Two-component systems
Ono Konda [32-33] $\boldsymbol{n = \frac{2n_0\rho_g\left[1 - exp\left(\frac{\varepsilon_s}{k.T}\right)\right]}{\frac{\rho_g\rho_{mc}}{\rho_{mc} - \rho_g} + \rho_{mc}\,exp\left(\frac{\varepsilon_s}{k.T}\right)}}$	n = the adsorption density n_0 = the saturated adsorption density of monolayer ρ_g = free methane molar density ρ_{mc} = maximum adsorbed methane molar density ε_s = methane-micropore interaction energy	Lattice based model; multilayer; can be used for monolayer

Isotherms	Nomenclature	Remarks
Hill-Deboer [34] $ln\left[\frac{C_e(1-\theta)}{\theta}\right] - \frac{\theta}{1-\theta} = -\,ln\,K_1 - \frac{K_2\theta}{RT}$	K_1 = isotherm constant K_2 = energetic constant	Monolayer; three parameters; mobile adsorption; interaction between adsorbed molecules
Fowler-Guggenheim [35] $ln\left[\frac{C_e(1-\theta)}{\theta}\right] = -\,ln\,K + \frac{2w\theta}{RT}$	K = equilibrium constant θ = fractional coverage w = interaction energy between adsorbed molecules	Monolayer; three parameters; Interaction energy (w) = 0 then it becomes Langmuir equation; applicable if $\theta < 0.6$
Dubinin and Astakhov Adsorption [36] $V = V_o\,exp\left[-D\left\{ln\left(\frac{P_o}{P}\right)\right\}^2\right]$	V = amount adsorbed V_o = volume of micropores D = isotherm constant P_o = saturation vapor pressure of the adsorbate P = equilibrium vapor pressure	
Dubinin-Radushkevich [37] $q = q_D \exp\left(-B\varepsilon_D^2\right)$ $\varepsilon_D = RT \ln\left(1 + \frac{1}{C_f}\right)$ $E = \frac{1}{\sqrt{2B}}$	q_D and B are model constant ε_D = the Polanyi potential E = mean adsorption energy	Multilayer; two parameters; based on Gaussian energy distribution for heterogenous surface; medium adsorbate concentrations; single component
Temkin [38] $q = \frac{RT}{b} ln(aC_e)$	a and b are isotherm constant	Monolayer; two parameters; linear decrement of adsorption heat in molecular layering; intermediate ionic concentration
Flory-Huggins [39] $ln\left(\frac{\theta}{C_o}\right) = ln\,K + n\,ln(1-\theta)$ K is used to calculate Gibbs free energy $\Delta G^o = -RT\,ln(K)$	n = number of adsorbates occupying adsorption sites K = equilibrium constant $\theta = \left(1 - \frac{C_f}{C_o}\right)$ = degree of surface coverage	Monolayer; two parameters;
Hill [40] $log\frac{q}{q_H - q} = n\,log(C_e) - log(K)$	K, n, and q_H are isotherm constants	Monolayer; three parameters; homogenous substrate binding of multi species
Halsey [41] $q = \frac{1}{n} I_n K - \frac{1}{n} ln\,C_e$	K and n are isotherm constant	Multilayer; two parameters
Harkin-Jura [42] $\frac{1}{q_e^2} = \frac{B}{A} - \left(\frac{1}{A}\right) log\,C_e$	B and A are isotherm constants	Multilayer; three parameters; heterogeneous surface
Jovanovic [43] $ln\,q = ln\,q_{em}$	q = adsorbate amount at equilibrium q_m = maximum adsorption capacity K = isotherm constant	Multilayer; three parameters; based on Langmuir model
Elovich [44] $ln\frac{q}{C_e} = ln\,K\,q_m - \frac{q}{q_m}$	q_m = maximum adsorption capacity K = isotherm constant	Multilayer; two parameters; exponential site increment with adsorption; chemisorption of gas on solid
Kiselev [45] $\frac{1}{C_e(1-\theta)} = \frac{K_1}{\theta} + K_iK_n$	K_i = equilibrium constant K_n = equilibrium constant θ = surface coverage	Monolayer; three parameters; valid for $\theta > 0.68$

Isotherms	Nomenclature	Remarks
Redlich-Peterson [46] $q = \frac{KC_e}{1+aC_e^{\beta}}$	K = isotherm constant a = model constant β = model exponent C_e = equilibrium adsorbent concentration q = equilibrium adsorbate loading	Monolayer; three parameters; not ideal monolayer adsorption; homogenous or heterogeneous surface; features of both Langmuir and Freundlich isotherms into a single equation
Sips Adsorption [47] $\boldsymbol{q = \frac{KC_e^{\beta}}{1-aC_e^{\beta}}}$	K = isotherm constant a = model constant β = model exponent	Monolayer; three parameters; heterogeneous surfaces; for low adsorbate concentration it behaves like Freundlich; for high adsorbate concentration it behaves like Langmuir
Toth [48] $q = \frac{qe_m}{\left[1+(bC_e)^{\frac{1}{n}}\right]^n}$	b = the Toth model constant n = the Toth model exponent	Monolayer; three parameters; Single component; Langmuir based; heterogeneous surface; for low and high-end boundary adsorbate concentration
Koble-Corrigan [49] $q = \frac{AC_e^n}{1+BC_e^n}$	A, B, and n are isotherm constants	Monolayer; three parameters; Langmuir and Freundlich based; Freundlich for p greater or equal to 1; not applicable for p less than 1
Khan [50] $q = \frac{qe_m}{(1+bC_e)^a}$	a = model exponent b = model constant q_m = maximum adsorption capacity	Monolayer; three parameters; bi-adsorbate adsorption for pure dilute equations solutions
Radke-Prausniiz [51] $q = \frac{q_m KC_e}{(1+KC_e)^m}$	q_m = maximum adsorption capacity K = equilibrium constant m= model exponent	Monolayer; three parameters; Single component; for low adsorbate concentration becomes linear isotherm, for high Freundlich isotherm and if m = 0 then Langmuir isotherm
Langmuir-Freundlich [52] $q = \frac{q_m (KC_e)^m}{1+(KC_e)^m}$	q_m = maximum adsorption capacity K = equilibrium constant for heterogeneous solid m = heterogeneous parameter	Single component; can simulate both Langmuir and Freundlich behaviours; suitable for modeling pH-dependent sorption effects
Jossens [53] $C_e = \frac{q}{H} exp(Fq^p)$	H, p, and F are isotherm constant	Monolayer; three parameters; heterogeneous surface; depends on energy distribution and adsorbate-adsorbent interactions
Fritz-Schlunder [54] $q = \frac{q_m KC_e}{1+q_m C_e^m}$	q_m = maximum adsorption capacity K= equilibrium constant m= model exponent	Monolayer; four parameters; if m =1 then becomes Langmuir isotherm; for high adsorbate concentrations becomes Freundlich model
Baudu [55] $q = \frac{q_m b_o C_e^{1+x+y}}{1+b_o C_e^{1+x+y}}$	q_m = maximum adsorption capacity b_o = equilibrium constant x and Y are isotherm parameter	Monolayer; four parameters; if surface coverage low then becomes Freundlich model

Isotherms	Nomenclature	Remarks
Weber-Van Vliet [56] $C_e = p_1 q^{(p_2 q_e^{p_3} + p_4)}$	C_e = equilibrium adsorbate concentration q = adsorption capacity p_1, p_2, p_3, and p_4 are isotherm parameters	Characterize solution concentration as function of absorbent capacity
Marczewski-Jaroniec [57] $q = q_m \left(\frac{(KC_e)^n}{1 + (KC_e)^n} \right)^{\frac{M}{n}}$	*n* and *M* are parameters that characterize the heterogeneity of the adsorbent surface	Monolayer; four parameters; becomes Langmuir isotherm if n and M = 1
Fritz and Schlunder [54] $q = \frac{q_m K_1 C_e^{\alpha}}{1 + K_2 C_e^{\beta}}$	q_m = maximum adsorption capacity K_1, K_2, α, and β are isotherm parameters	Monolayer; five parameters; becomes Langmuir if α_{FS} and β_{FS} equals 1

5. Conclusion

This manuscript has made an attempt in order to provide an overview of the estimation and production of shale gas associated with a shale gas reservoir. The following conclusions have been deduced from this work.

(i) The shale gas reservoir is unique in nature, and it is completely different from the conventional sandstone as well as fractured carbonate reservoirs. In addition, the mobility of shale gas within the reservoir is very complex and it is again completely different from the conventional natural gas reservoir. As a result, a detailed table has been deduced that distinguishes the fluid flow mechanism associated with a shale gas reservoir from that of the natural gas reservoir.

(ii) Since shale gas reservoir's fluid and rock properties remain very complex, these properties cannot be secured from the conventional PVT cell or core-flooding experiments, and subsequently require a sound understanding of the basic rock and fluid properties and its interaction with the shale matrix. In this context, a list of possible queries that remain unanswered in the context of field-scale estimation and production of shale gas has been deduced. This exercise is expected to initiate sparks among the experimental/field scientists to explore further in this direction.

(iii) A detailed list of complexities associated with the development of the conceptual model and its associated mathematical model has been deduced.

(iv) A relatively simpler conceptual model and its associated mathematical model have been proposed that will make the estimation and production of shale gas in the absence of requiring a complex conceptual, mathematical and numerical model. The results from this study can be used as a quick reference before getting into detailed complex modeling.

(v) A list of various sorption isotherm models have been collected from the existing literature that will suit the varying nature of a complex shale gas reservoir which has been deduced, which will possibly help to include varieties of sorption models that can be used at various time levels during the production of shale gas as against using the same single isotherm throughout the production period.

This study is expected to provide field-scale limitations associated with the estimation and production of shale gas from a shale gas reservoir, and it will subsequently aid in deducing better shale gas reservoir management decisions. The simplified conceptual and mathematical model proposed in this work may further be extended by including the details of complex hydraulic fracturing and dynamic sorption kinetics.

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