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Mathematical Model of the Translational Motion of Friable Materials in a Rotating Drum

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Abstract

This work developed a mathematical model of the translational motion of granular carbon materials in the rotary kiln drum and in the cooler drum. The resulting model will be updated while developing the complete mathematical model of a rotary kiln and a cooler drum for synthesizing optimal control algorithms of a rotary kiln for calcining carbon materials in the production of electrode products.

Keywords: *Rotating drum; Mathematical model; Coal; Anthracite.*

1. Introduction

This work deals with the translational motion of granular materials in the rotary drum. The aim of this study is to optimize the operation of the rotary kiln drum and the drum cooler in the electrode production (for example, in the production of graphitizing electrode blanks [1]). However, the proposed model can be used in the course of studies of other devices with the translational motion of granular materials in rotary drum.

Currently it is used the average speed determined from the loading and unloading of the material instead of any accurate modeling of the translational motion in rotary drums. It is also used the average residence time of the material in the drum based on investigational studies. Taking into account the fact of the material shrinkage and the nonuniformity of motion we will obtain more accurate information about the process at the stage of modeling and synthesis of the control system, and directly at the production [2]. The effect of the nonuniformity of translational motion on product quality is most noticeable in rotary kilns due to uneven temperature distribution.

2. Mathematical modelling

The known computation method for the translational motion speed of granular materials (including carbon) in the rotary kiln drum and in the cooler drum is the Voroshilov formula:

$$w = \frac{4\pi}{3} Dnf\Phi,\ f = \frac{sin^3\left(\frac{\varphi}{2}\right)}{\varphi - sin(\varphi)},\ \Phi = \frac{sin(\alpha)}{\sqrt{sin^2(\beta) - sin^2(\alpha)}},\ \Psi = \frac{\varphi - sin(\varphi)}{2\pi}.$$

The following refinement is used for the fact of furnace charge shrinkage:

$$\Phi = \frac{cos(\gamma)\, sin(\alpha + \alpha')}{\sqrt{sin^2(\beta) - sin^2(\alpha + \alpha')}},\ \alpha' = arcsin(cos(\beta)\, tg(\gamma)).$$

In [3], a number of authors compare the most common computation methods for calculating the translational motion of granular material in a rotary drum. The Voroshilov formula is recognized as the most accurate (according to the experimental evidence).

The movement of carbon inside the cooler drum can be calculated according to the plug flow model due to the significant ratio of length to diameter and due to the "baffles" on the surface of the lining, ensuring the high-quality mixing of carbon. Then the translational motion can be calculated according to the above-mentioned Voroshilov formula with refinement. The motion in the cross section of the granular body will be equated to the ideal mixing model.

The model of material movement will be written in one-dimensional coordinates, where the *l* axis is directed along the axis of the drum rotation (zero of the axis is taken at the load point, a positive direction along the movement of the granular body).

We write down the burden balance in dynamics for the material for the section of length *dl*:

$$G_c(l) - G_c(l+dl) - G_{cc}(l) - G_v(l) - G_w(l) = \frac{\partial m_c(l)}{\partial t}.$$

We express the mass at a section as follows:

$$m_c(l) = S_c(l)\rho_c(l)dl.$$

The change in flow variation during the section passage can be written as follows:

$$G_c(l+dl) = G_c(l) + \frac{\partial G_c(l)}{\partial l}dl.$$

Components $G_{cc}(l)$, $G_v(l)$ $G_w(l)$ we express through the corresponding specifics:

$$G_{cc}(l) = g_{cc}(l)dl,\ G_v(l) = g_v(l)dl,\ G_w(l) = g_w(l)dl.$$

Using Voroshilov formula, we write:

$$G_c(l) = \frac{4\pi}{3}Dn\frac{sin^3\left(\frac{\varphi(l)}{2}\right)}{\varphi(l) - sin(\varphi(l))}\frac{cos(\gamma(l))\,sin\left(\alpha + \alpha'(l)\right)}{\sqrt{sin^2(\beta) - sin^2\left(\alpha + \alpha'(l)\right)}}S_c(l)\rho_c(l)$$

Geometrically:

$$S_c(l) = \frac{D^2}{8}\left(\varphi(l) - sin(\varphi(l))\right),\ tg(\gamma(l)) = \frac{D}{2}\frac{\partial\, cos\left(\frac{\varphi(l)}{2}\right)}{\partial l} = -\frac{D}{4}sin\left(\frac{\varphi(l)}{2}\right)\frac{\partial\varphi(l)}{\partial l}.$$

We introduce the above into the equation of the burden balance of the section in dynamics:

$$-\frac{\partial\left(sin^3\left(\frac{\varphi(l)}{2}\right)\frac{sin\left(\alpha+\alpha'(l)\right)}{\sqrt{1+\left(\frac{D}{4}sin\left(\frac{\varphi(l)}{2}\right)\frac{\partial\varphi(l)}{\partial l}\right)^2}\sqrt{sin^2(\beta)-sin^2\left(\alpha+\alpha'(l)\right)}}\rho_c(l)\right)}{\partial l}\frac{4\pi}{3}Dn - -\frac{8}{D^2}\left(g_{cc}(l) + g_v(l) + g_w(l)\right) = \frac{\partial\left(\left(\varphi(l)-sin(\varphi(l))\right)\rho_c(l)\right)}{\partial t},$$

where $\alpha'(l) = -\,arcsin\left(cos(\beta)\frac{D}{4}sin\left(\frac{\varphi(l)}{2}\right)\frac{\partial\varphi(l)}{\partial l}\right)$.

Then, after obvious transformations and simplifications, we get the following:

$$\frac{\left(\left(\frac{\partial\varphi(l)}{\partial l}\right)^2 cos\left(\frac{\varphi(l)}{2}\right)+\frac{\partial^2\varphi(l)}{\partial l^2}2\,sin\left(\frac{\varphi(l)}{2}\right)\right)sin^3\left(\frac{\varphi(l)}{2}\right)\frac{\pi D^2 n}{6}}{\sqrt{\left(1+\left(\frac{D}{4}sin\left(\frac{\varphi(l)}{2}\right)\frac{\partial\varphi(l)}{\partial l}\right)^2\right)}\sqrt{sin^2(\beta)-sin^2\left(\alpha+\alpha'(l)\right)}}\cdot$$

$$\cdot\left(\begin{array}{l}\frac{\partial\varphi(l)}{\partial l}\frac{sin\left(\frac{\varphi(l)}{2}\right)sin\left(\alpha+\alpha'(l)\right)}{\left(1+\left(\frac{D}{4}sin\left(\frac{\varphi(l)}{2}\right)\frac{\partial\varphi(l)}{\partial l}\right)^2\right)}\frac{1}{4}D + \\ + \frac{\left(1+\frac{sin^2\left(\alpha+\alpha'(l)\right)}{sin^2\left(\alpha+\alpha'(l)\right)-sin^2(\beta)}\right)cos\left(\alpha+\alpha'(l)\right)}{\sqrt{\left(1-\left(cos(\beta)\frac{D}{4}sin\left(\frac{\varphi(l)}{2}\right)\frac{\partial\varphi(l)}{\partial l}\right)^2\right)}}cos(\beta)\end{array}\right) -$$

$$-2\frac{\left(\frac{\partial\varphi(l)}{\partial l}cos\left(\frac{\varphi(l)}{2}\right)+\frac{2}{3}\frac{\partial\rho_c(l)}{\partial l}\frac{1}{\rho_c(l)}sin\left(\frac{\varphi(l)}{2}\right)\right)sin^2\left(\frac{\varphi(l)}{2}\right)sin\left(\alpha+\alpha'(l)\right)\pi Dn}{\sqrt{1+\left(\frac{D}{4}sin\left(\frac{\varphi(l)}{2}\right)\frac{\partial\varphi(l)}{\partial l}\right)^2}\sqrt{sin^2(\beta)-sin^2\left(\alpha+\alpha'(l)\right)}} -$$

$$-\frac{8}{D^2\rho_c(l)}\left(g_{cc}(l)+g_v(l)+g_w(l)\right) =$$

$= \frac{\partial \varphi(l)}{\partial t}\left(1 - cos\left(\varphi(l)\right)\right) + \frac{\partial \rho_c(l)}{\partial t}\frac{1}{\rho_c(l)}\left(\varphi(l) - sin\left(\varphi(l)\right)\right),$

or

$$\begin{pmatrix} \frac{\partial \lambda(l)}{\partial l}\left(\frac{\lambda(l)}{\left(1+\left(\lambda(l)\right)^2\right)} + \frac{\left(1+\frac{sin^2\left(\alpha - arcsin\left(cos(\beta)\,\lambda(l)\right)\right)}{sin^2\left(\alpha - arcsin\left(cos(\beta)\,\lambda(l)\right)\right) - sin^2(\beta)}\right)}{tg\left(\alpha - arcsin\left(cos(\beta)\,\lambda(l)\right)\right)\sqrt{\left(1-\left(cos(\beta)\,\lambda(l)\right)^2\right)}}cos(\beta)\right) \\ -\lambda(l)6\frac{cos\left(\frac{\varphi(l)}{2}\right)}{sin^2\left(\frac{\varphi(l)}{2}\right)}D - \frac{\partial \rho_c(l)}{\partial l}\frac{1}{\rho_c(l)} \end{pmatrix}.$$

$$\cdot\, sin^3\left(\frac{\varphi(l)}{2}\right)\frac{4\pi Dn\, sin\left(\alpha - arcsin\left(cos(\beta)\,\lambda(l)\right)\right)}{3\sqrt{\left(1+\left(\lambda(l)\right)^2\right)\left(sin^2(\beta) - sin^2\left(\alpha - arcsin\left(cos(\beta)\,\lambda(l)\right)\right)\right)}}$$

$$-\frac{8}{D^2\rho_c(l)}\left(g_{cc}(l) + g_v(l) + g_w(l)\right) =$$

$= \frac{\partial \varphi(l)}{\partial t}\left(1 - cos\left(\varphi(l)\right)\right) + \frac{\partial \rho_c(l)}{\partial t}\frac{1}{\rho_c(l)}\left(\varphi(l) - sin\left(\varphi(l)\right)\right),$

where $\lambda(l) = \frac{\partial \varphi(l)}{\partial l} sin\left(\frac{\varphi(l)}{2}\right)\frac{D}{4}$.

Thus, the continuity equation was obtained, which is applicable to both the cooler and the kiln.

Considering the fact that the shrinkage, and, accordingly, the angle γ at the entrance of the material into the drum is zero, we define the following limiting conditions:

$$\varphi(0) = \varphi_0 = 2\,arcsin\left(\frac{1}{D}\sqrt[3]{\frac{3}{2\pi n}\frac{G_{c,0}}{\rho_{c,0}}\sqrt{\frac{sin^2(\beta)}{sin^2(\alpha)} - 1}}\right),\ \rho_c(0) = \rho_{c,0}.$$

The obtained equation describes the average motion of particles, in [3] the empirical formula for the difference of translational motion of particles is given, namely the ratio of the residence time of particles of different diameters:

$$\frac{\tau_a}{\tau_r(r)} = \frac{\frac{r}{r_a}}{1.81\frac{r}{r_a} - 0.78}.$$

Then we can write for the speed:

$$w_r(r) = \frac{w_a}{1.81 - 0.78\frac{r_a}{r}},$$

where w_a – average speed (determined from the equation above).

In [4], the calculating method for the packing density of polydisperse spheres with a continuous distribution of radii was proposed:

$$\phi(l) = \min_{r_p}\left(\frac{\omega(r_p)}{1-\left(1-\omega(r_p)\right)\int_0^{r_p}\eta(l,x)g(x,r_p)dx - \int_{r_p}^{\infty}\eta(l,x)f(x,r_p)dx}\right).$$

$\eta(l,r_p)$ is the content of spheres of radius x in the mixture, that is, the probability density of the radii weighted by volume, that is, $\eta(l,r_p) = \frac{\frac{4}{3}\pi r_p{}^3\mu(l,r_p)}{\int_0^{\infty}4\pi x^2\mu(l,x)dx} = \frac{3r_p{}^3\mu(l,r_p)}{\int_0^{\infty}x^2\mu(l,x)dx}$.

In the case of random packing of monodisperse spheres, the packing density should be up to 0.64, so we will take $\omega(r_p) = \omega = 0.64$ for all r_p.

Functions $f(\cdots)$ and $g(\cdots)$, as shown in [5], are well approximated by the following dependencies:

$$g(r_1,r_2) = \left(1 - \frac{r_2}{r_1}\right)^{1.6},\ f(r_1,r_2) = \left(1 - \frac{r_2}{r_1}\right)^{3.1} + 3.1\frac{r_2}{r_1}\left(1 - \frac{r_2}{r_1}\right)^{2.9}.$$

Then:

$$\phi(l) = \min_{r_p}\left(\frac{\omega}{\left(\begin{array}{l}1-(1-\omega)\int_0^{r_p}\eta(l,x)\left(1-\frac{r_p}{x}\right)^{1.6}dx - \\ -\int_{r_p}^{\infty}\eta(l,x)\left(\left(1-\frac{r_p}{x}\right)^{3.1} + 3.1\frac{r_p}{x}\left(1-\frac{r_p}{x}\right)^{2.9}\right)dx\end{array}\right)}\right).$$

The minimum of this function can be found by the known methods.

If we assume that the furnace charge consists of spherical particles, we can express the pour density in terms of the true one:

$\rho_c(l) = \frac{\rho_{pa}(l)}{\phi(l)}$.

Moreover:

$$\rho_{pa}(l) = \int_0^\infty \eta(l, r_p) \frac{\int_0^{r_p} 4\pi (r')^2 \rho_p(r') dr'}{4\pi r_p{}^3} dr_p = \int_0^\infty \eta(l, r_p) \frac{\int_0^{r_p} (r')^2 \rho_p(r') dr'}{r_p{}^3} dr_p.$$

By virtue of the fact that the distribution of the radii varies along the drum and over time due to the influence of chemical reactions and shrinkage of the particles in the radii, it must be calculated. The radii of the furnace charge particles change when moving along the drum: by the same value, independent on the current radius of the particles, due to the influence of chemical reactions that "spend" the substance from the surface of the particle; by the value, depending on the radius of the particle as a result of the shrinkage. We can write:

$$r_p(l,t) = r_p(0, t-\tau(l,t)) - \delta(l,t) - k\left(l, t, r_p(0, t-\tau(l,t))\right).$$

It is worthwhile to say that both of these functions, $\delta(l,t)$ and $k(l,t,r_p)$, increase monotonically when moving along the drum, since the nature of the processes occurring in the drum is such that the particle radii cannot increase, but can only decrease.

We express the probability density of the radii at the point (l,t)as the probability density of the radii at the entrance (at the point $(0, t-\tau(l,t))$) that had to decrease to the current size (at the point (l,t)). That is, we find the probability density of the radius *r* as the probability density of the radius at the entrance where *r* was formed:

$$\mu(l,t,r_p) = \mu\left(0, t-\tau(l,t), r_p(0,t-\tau(l,t)) - \delta(l,t) - k(l,t,r_p)\right) \cdot\cdot \left(1 + \frac{\int_{-\infty}^0 \mu(0,t-\tau(l,t),x-\delta(l,t)-k(l,t,x))dx}{\int_0^\infty \mu(0,t-\tau(l,t),x-\delta(l,t)-k(l,t,x))dx}\right).$$

The component $1 + \frac{\int_{-\infty}^0 \mu(0,t-\tau(l,t),x-\delta(l,t)-k(l,t,x))dx}{\int_0^\infty \mu(0,t-\tau(l,t),x-\delta(l,t)-k(l,t,x))dx}$ is needed for scaling for the condition $\int_{-\infty}^\infty \mu(l,t,x) d\,x = 1$ to be met. This scaling is necessary because the above expression for the probability density is valid only for positive radii r_p, and if $r_p \le 0$ it is necessary to equate to $\mu(l,t,r_p) = 0$.

Note:

With the expression $\mu(l,t,r_p) = \mu\left(0, t-\tau(l,t), r_p(0,t-\tau(l,t)) - \delta(l,t) - k(l,t,r_p)\right)$ we "shifted" the graph of the probability density to the left along the abscissa axis. The part of the graph to the left of the ordinate axis (at $r_p \le 0$) has an surface area$S_1 = \int_{-\infty}^0 \mu(0, t-\tau(l,t), x-\delta(l,t) - k(l,t,x))dx$, and to the right$S_2 = \int_0^\infty \mu(0, t-\tau(l,t), x-\delta(l,t)-k(l,t,x))dx$. Then, for the equality $S_2 q = S_1 + S_2$ to hold (where *q* is the scaling factor), it is necessary that $q = \frac{S_1+S_2}{S_2} = 1 + \frac{S_1}{S_2}$.

Since we are more interested in the weighted probability density, we substitute in there the values $\mu(l,t,r_p)$ and, after certain simplifications, we get:

$$\eta(l,t,r_p) = \begin{cases} \dfrac{3 r_p{}^3(l,t)\mu\begin{pmatrix} 0, t-\tau(l,t), r_p(0,t-\tau(l,t)) - \\ -\delta(l,t) - k(l,t,r_p) \end{pmatrix}}{\int_0^\infty x^2 \mu(0, t-\tau(l,t), x-\delta(l,t)-k(l,t,x))dx} & , r_p(l,t) > 0 \\ 0 & , r_p(l,t) \le 0 \end{cases}$$

We can find the time $\tau(l,t)$ by this means:

$$\tau(l,t) = \int_0^l \frac{dx}{w(t-\tau(x,t),x)} = \frac{3}{4\pi Dn}\int_0^l \frac{\left(\varphi(t-\tau(x,t),x) - \sin(\varphi(t-\tau(x,t),x))\right)\sqrt{1-\lambda^2(t-\tau(x,t),x)}}{\sin^3\left(\frac{\varphi(t-\tau(x,t),x)}{2}\right)\lambda(t-\tau(x,t),x)} dx.$$

Or in the differential form:

$$\frac{\partial\tau(l,t)}{\partial l} = \frac{1}{w(t-\tau(l,t),l)} = \frac{3\left(\varphi(t-\tau(l,t),l) - \sin(\varphi(t-\tau(l,t),l))\right)\sqrt{1-\lambda^2(t-\tau(l,t),l)}}{4\pi Dn \sin^3\left(\frac{\varphi(t-\tau(l,t),l)}{2}\right)\lambda(t-\tau(l,t),l)}.$$

Wherein $\tau(0,t) = 0$.

3. Further research

Currently the research is carrying out on the mathematical modeling of heat transfer and kinetics of chemical reactions occurring in a rotary kiln for calcining carbonaceous materials and in a drum cooler after this kiln. The complete model will allow to research for optimizing the operation of these installations in the electrode production.

4. Conclusions

Was developed a mathematical model of the incoming movement of loose materials in a revolving drum. Taken into account the effect of shrinkage of the charge particles, was proosed a method of forecasting the changes of the probable density of distribution of the radii of particles of the charge in the cross section of the drum. This kind of model is equation of continuity and supplemented by the necessary equations (the kinetics of chemical reactions, kinetics of structural transformations of particles, the model of heat trasfer and the motion model of gases) can be used for theoretical explorations of appropriate devices and development of optimal control algorithms for it.

Nomenclatures

D – drum diameter;
n – drum rotation frequency;
φ – central angle of the granular material segment;
α – inclination angle of the drum;
β – slope angle of the granular material;
Ψ – fill factor;
γ – angle of inclination of granular material to the drum axis, rad.;
$G_c(l)$ – material consumption at the entrance to the section;
$G_c(l+dl)$ – material consumption at the exit from the section;
$G_{cc}(l)$ – material consumption for chemical transformations;
$G_v(l)$ – evaporated volatiles consumption;
$G_w(l)$ – evaporated moisture consumption;
$m(l)$ – mass accumulated at the section;
$S_c(l)$ – cross-sectional area of the material flow;
$\rho_c(l)$ – pour density of the material;
$G_{c,0}$ – flow rate of the material at the entrance to the drum;
$\rho_{c,0}$ – pour density of the material at the entrance to the drum;
τ_a – average residence time of the particles in the drum;
$\tau_r(r)$ – residence time in the drum of particles of radius r;
r_a – arithmetic mean radius of particles.
$w_r(r)$ – velocity of the particle with radius r;
$\omega(r_p)$ – packing density of monodisperse spheres with radius r_p;
$\mu(l,r_p)$ – probability density of the radii;
$\rho_{pa}(l)$ – average true density of the material at a section;
$\rho_p(r)$ – true density of the material;
$\tau(l,t)$ – time required for the particle to travel from the loading point to the point l;
$\delta(l,t)$ – amount that the particle radius has decreased due to the shrinkage in comparison with the radius at the entrance to the drum;
$k\left(l,t,r_p(0,t-\tau(l,t))\right)$ – value that the particle radius has decreased as a result of the chemical reactions in comparison with the radius at the entrance to the drum (to avoid littering, we will write as $k(l,t,r_p)$).

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