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MODELLING THE EFFECT OF ACCELERATION TERM ON TOTAL PRESSURE DROP IN HORIZONTAL GAS PIPELINES

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Abstract

In this study, the contribution of acceleration term to total pressure drop in horizontal gas pipelines was investigated. This was carried out by developing an analytical model for predicting total pressure drop in horizontal gas pipelines while incorporating the acceleration term in the model. The percentage contribution of pressure drop by acceleration term to total pressure drop (ψ) was then used to measure acceleration term effects. It was deduced that ψ strongly depends on pipe diameter and friction factor. Hence, percentage ratio of pressure drop due to acceleration effect to total pressure drop (ψ) was varied with pipe diameter for different friction factors. As pipe diameter was increasing, percentage ratio of pressure drop due to acceleration to total pressure drop (ψ) was decreasing. And for each pipe diameter, percentage ratios of pressure drop due to acceleration to total pressure drop were decreasing as the friction factor was increasing. In fact, for transmission lines, ψ was as high as 43.86% for a 16 inches pipe with a friction factor of 0.02 and as low as 1.71% for a 48 inches pipe under a friction factor of 0.1. While for distribution lines, ψ was high as 99.5% at a friction factor of 0.02. In the end, it was ascertained that neglecting acceleration term in calculating total pressure drops in horizontal gas pipelines can be misleading, especially for small diameter pipelines.

Keywords: *Pressure drop; Acceleration term; Pipelines; Friction factor; Pipe diameter.*

1. Introduction

Engineering of long distance transportation of natural gas through a pipeline requires enough knowledge of flow correlations for computing capacity and pressure requirements Due to the fact that pressures were low during the early development of the natural gas transmission industry, the equations used for design purposes were simple and suitable.

However, as higher capacity demand became inevitable, pressures were also increased to meet up the demand, whereas new equations were, in turn developed to address the challenge. Presently, the petroleum industry has several equations for computing the flow of gases in pipelines. Probably, Weymouth equation is the most common pipeline flow equation and is preferably known to be for smaller diameter lines (less than or equal to 15 in.). Meanwhile, the Panhandle equation and the Modified Panhandle equation are usually preferred for larger diameter size transmission lines [5].

The general energy equation is usually the starting point for most analytical equations describing numerous systems and scenarios encountered in engineering. Starting with very long and cumbersome equations, a combination of different mathematical operations like differentiation and integration are normally used to arrive at much simpler and easy to use equations we all like. In arriving at the much simpler equations, different levels of assumptions and trade-offs are made. These assumptions are necessary to make the cumbersome general energy equations amenable to mathematical solutions.

But, experience has proven that these assumptions have substantial effects on the accuracy of the resulting; hence, it is pertinent to study the effects of these assumptions. Other analytical equations that measure pressure drop in gas pipelines usually assume acceleration term

to be negligible. And this like other assumptions, affect the reliability of these equations, thereby prompting the need for the effects of this assumption to be studied and evaluated.

2. Model development

In this study, the effects of acceleration term in pressure drop calculations of gas pipelines will be analyzed by considering steady-state flow of dry gas in constant diameter pipelines. The following procedure will be followed:
- Creating a schematic gas pipeline transportation model.
- Starting with the general energy equation, and then deriving a new analytical equation that will incorporate the acceleration term.
- Measuring the contribution of acceleration term to total pressure drop in gas pipelines.

Using general energy equation, energy balance on the whole system between points 1 and 2 in Figure 1 below may be written as

$$U_2 + P_2 V_2 + \frac{mu_2^2}{2g_c} + \frac{mgZ_2}{g_c} = U_1 + P_1 V_1 + \frac{mu_1^2}{2g_c} + \frac{mgZ_1}{g_c} + Q - w - lw \tag{1}$$

Figure 1. A gas transmission line model [5]

where: U = internal energy; pV = energy of compression or expansion; $\frac{mu^2}{2g_c}$ = potential energy; Q = heat energy added to fluid; w = shaft work done by the surrounding on the gas.

Dividing Equ. 1 through by m to obtain an energy per unit mass balance and writing the resulting equation in differential form yields

$$dU + d\left(\frac{p}{\rho}\right) + u\frac{du}{g_c} + g\frac{dz}{g_c} + dQ - dw = 0 \tag{2}$$

Assuming the following:
(a) The flow is the steady state and steady flow.
(b) The flow is isothermal in the pipeline.
(c) The flow is horizontal.
(d) There is no work done by or on the gas during flow across the system.

But,

$$dh = Tds + \frac{dp}{\rho} \text{ and } dU = dh - d\left(\frac{p}{\rho}\right) = Tds + \frac{dp}{\rho} - d\left(\frac{p}{\rho}\right) \tag{3}$$

where: h = enthalpy; s = entropy; T = temperature; ρ = density; p = gas pressure; U = internal energy.

Inserting Equ. (3) into Equation (2),

$$Tds + \frac{dp}{\rho} + u\frac{du}{g_c} + g\frac{dz}{g_c} + dQ - dw = 0 \tag{4}$$

Clausis inequality for an irreversible process states that $ds \geq \frac{-dQ}{T}$

$$Tds = -dQ + d(lw) \tag{5}$$

where: lw = loss work due to irreversibilities.

Substituting Equ. (5) into Equ. (4),

$$\frac{dp}{\rho} + u\frac{du}{g_c} + g\frac{dz}{g_c} + d(lw) - dw = 0 \tag{6}$$

If no work is done by or on the fluid, $dw = 0$ then,

$$\frac{dp}{\rho} + u\frac{du}{g_c} + g\frac{dz}{g_c} + d(lw) = 0 \tag{7}$$
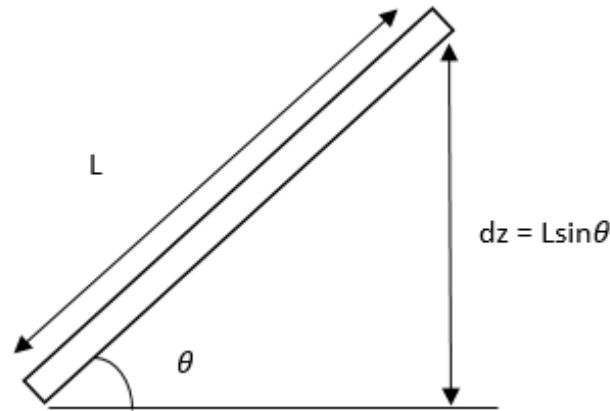
Considering a more general case of an inclined pipe

Figure 2: An inclined pipeline [5]

Therefore, we have

$$\frac{dp}{\rho} + u\frac{du}{g_c} + g\frac{dLsin\theta}{g_c} + d(lw) = 0 \tag{8}$$

Multiplying through by ρ/dL

$$\frac{dp}{dL} + \rho u\frac{du}{g_c dL} + g\frac{\rho sin\theta}{g_c} + \rho\frac{d(lw)}{dL} = 0 \tag{9}$$

Considering pressure drop in the positive direction,

$$\frac{dp}{dL} = \rho u\frac{du}{g_c dL} + g\frac{\rho sin\theta}{g_c} + \rho\frac{fu^2}{2g_c D} \tag{10}$$

where: $\frac{d(lw)}{dL} = \frac{fu^2}{2g_c D}$.

Considering a horizontal pipe,

$$\frac{dp}{dL} = \rho u\frac{du}{g_c dL} + \rho\frac{fu^2}{2g_c D} \tag{11}$$

Recall,

$$u = \left(\frac{q}{86400}\right)\left(\frac{T}{T_b}\right)\left(\frac{P_b}{P}\right)\left(\frac{z}{1.00}\right)\left(\frac{4}{\pi D^2}\right) \tag{12}$$

But, the total surface area of a cylinder = Area of the two circular ends + Area of the curved surface

$A_t = 2\pi r^2 + 2\pi rL$ where L= length of pipe

Rearranging,

$2\pi r^2 = A_t + 2\pi rL;$ $\pi r^2 = \frac{A_t + 2\pi rL}{2};$ $\pi\left(\frac{D}{2}\right)^2 = \frac{A_t + 2\pi rL}{2}$

$$\frac{\pi D^2}{4} = \frac{A_t + \pi DL}{2} \tag{13}$$

Substituting equ. (13) into (12)

$$u = \left(\frac{q}{86400}\right)\left(\frac{T}{T_b}\right)\left(\frac{P_b}{P}\right)\left(\frac{z}{1.00}\right)\left(\frac{2}{A_t + \pi DL}\right) \tag{14}$$

Therefore, $u = B.(A_t + \pi DL)^{-1};$ $u = B.(H)^{-1}$

where $B = \left(\frac{q}{86400}\right)\left(\frac{T}{T_b}\right)\left(\frac{P_b}{P}\right)\left(\frac{z}{1.00}\right)\left(\frac{2}{1}\right); H = A_t + \pi DL$

But $\frac{du}{dL} = \frac{du}{dH}.\frac{dH}{dL}$

But $\frac{du}{dH} = -B.(H)^{-2}$ and $\frac{dH}{dL} = -\pi D$

Hence $\frac{du}{dL} = -B.(H)^{-2} \times -\pi D; = \frac{B\pi D}{H^2}$

Substituting accordingly,

$$\frac{du}{dL} = \left(\frac{q}{86400}\right)\left(\frac{T}{T_b}\right)\left(\frac{P_b}{P}\right)\left(\frac{z}{1.00}\right)\frac{2\pi D}{(A_t + \pi DL)^2} = \frac{u\pi D}{(A_t + \pi DL)} \tag{15}$$

Substituting equ. (14) and (15) into (11) and collecting like terms,

$$\left(\frac{dP}{dL}\right) = \frac{u\rho}{g_c}\left[\frac{fu}{2D} + \frac{du}{dL}\right] \left(\frac{dP}{dL}\right) = \frac{u\rho}{g_c}\left[\frac{fu}{2D} + \frac{u\pi D}{(A_t + \pi DL)}\right] = \frac{u^2\rho}{g_c}\left[\frac{f}{2D} + \frac{\pi D}{(A_t + \pi DL)}\right] \qquad (16)$$

where: $f=$ Moody's friction factor, dimensionless; $u=$ gas velocity, ft/s; $\rho=$ gas density, lbm/ft^3; $D=$ pipe internal diameter, ft; $L=$ pipe length, ft; $g_c=$ conversion factor = 32.17 lbm-ft/lbf-s^2; $A_t=$ total surface area of pipe = $\pi D(r+L)$; $\pi=$ pi = 3.1428571429.

Hence $\frac{\Delta P}{L} = \frac{u^2\rho}{g_c}\left[\frac{f}{2D} + \frac{\pi D}{(A_t + \pi DL)}\right]$

Making u^2 the subject,

$$u^2 = \frac{\Delta P}{L} \cdot \left(\frac{2D(A_t - \pi DL)}{f(A_t - \pi DL) + 2\pi D^2}\right) \cdot \frac{g_c}{\rho} \qquad (17)$$

Recall $u = \left(\frac{Q}{86400}\right)\left(\frac{T}{T_b}\right)\left(\frac{P_b}{P}\right)\left(\frac{z}{1.00}\right)\left(\frac{2}{A_t + \pi DL}\right)$ and substituting for u in equ. (17),

$$\left[\left(\frac{Q}{86400}\right)\left(\frac{T}{T_b}\right)\left(\frac{P_b}{P}\right)\left(\frac{z}{1.00}\right)\left(\frac{2}{A_t - \pi DL}\right)\right]^2 = \frac{\Delta P}{L} \cdot \left(\frac{2D(A_t - \pi DL)}{f(A_t - \pi DL) + 2\pi D^2}\right) \cdot \frac{g_c}{\rho} \left[\left(\frac{Q}{86400}\right)\left(\frac{T}{520}\right)\left(\frac{14.7}{P}\right)\left(\frac{z}{1.00}\right)\left(\frac{2}{A_t - \pi DL}\right)\right]^2 =$$

$$\frac{\Delta P}{L} \cdot \left(\frac{2D(A_t - \pi DL)}{f(A_t - \pi DL) + 2\pi D^2}\right) \cdot \frac{g_c}{\rho}$$

Therefore,

$$\frac{Q^2}{2018525184000000} \cdot \frac{T^2}{1} \cdot \frac{216.09}{P^2} \cdot \frac{z^2}{1} \cdot \frac{4}{(A_t - \pi DL)^2} = \frac{\Delta P}{L} \cdot \left(\frac{2D(A_t - \pi DL)}{f(A_t - \pi DL) + 2\pi D^2}\right) \cdot \frac{g_c}{\rho}$$

Making Q^2 the subject,

$$Q^2 = 2335282965431 \left(\frac{P_{avg}(A_t - \pi DL)}{T_{avg}z_{avg}}\right)^2 \cdot \frac{\Delta P}{L} \cdot \left(\frac{2D(A_t - \pi DL)}{f(A_t - \pi DL) + 2\pi D^2}\right) \cdot \frac{g_c}{\rho}$$

Taking the square root of both sides,

$$Q = 1528163.27 \times \left(\frac{P_{avg}(A_t - \pi DL)}{T_{avg}z_{avg}}\right)^2 \cdot \left[\frac{\Delta P}{L} \cdot \left(\frac{2D(A_t - \pi DL)}{f(A_t - \pi DL) + 2\pi D^2}\right) \cdot \frac{g_c}{\rho}\right]^{0.5}$$

Or

$$Q = 1528163.27 \times \left(\frac{P_{avg}\emptyset}{T_{avg}z_{avg}}\right)^2 \cdot \left[\frac{(P_1 - P_2)g_c}{L\rho} \cdot \left(\frac{2D\emptyset}{f\emptyset + 2\pi D^2}\right)\right]^{0.5}$$

where: $\emptyset = A_t - \pi DL$; $f=$ Moody's friction factor, dimensionless; $u=$ gas velocity, ft/s; $\rho=$ gas density, lbm/ft^3; $D=$ pipe internal diameter, ft; $L=$ pipe length, ft; $g_c=$ conversion factor = 32.17 lbm-ft/lbf-s^2; $A_t=$ total surface area of pipe $= \pi D(r+L)$; $\pi=$ pi = 3.1428571429; $P_{avg}=$ average pressure of the gas, psia; $T_{avg}=$ average temperature of gas, ^{o}R; $z_{avg}=$ average compressibility of gas, dimensionless; $Q=$ gas flow rate (Scfd).

3. Ratio of pressure drop due to acceleration to total pressure drop

The ratio of pressure drop due to acceleration to total pressure drop can now be defined as [2]:

$$\psi = \frac{pressure\ drop\ due\ to\ acceleration}{total\ pressure\ drop}$$

From equation (3.16), recall,

$$\left(\frac{dP}{dL}\right)_{total} = \frac{u^2\rho}{g_c}\left[\frac{f}{2D} + \frac{\pi D}{(A_t + \pi DL)}\right] = \frac{fu^2\rho}{2Dg_c} + \frac{\pi Du^2\rho}{(A_t + \pi DL)g_c}$$

where: $\frac{fu^2\rho}{2Dg_c} =$ pressure drop due to friction; $\frac{\pi Du^2\rho}{(A + \pi DL)g_c} =$ pressure drop due to acceleration.

Therefore,

$$\psi = \frac{\left[\frac{\pi Du^2\rho}{(A_t + \pi DL)g_c}\right]}{\frac{fu^2\rho}{2Dg_c} + \frac{\pi Du^2\rho}{(A_t + \pi DL)g_c}} = \frac{\frac{u^2\rho}{g_c} \cdot \frac{\pi D}{(A_t + \pi DL)}}{\frac{u^2\rho}{g_c} \cdot \left[\frac{f}{2D} + \frac{\pi D}{(A_t + \pi DL)}\right]} = \frac{\frac{\pi D}{(A_t + \pi DL)}}{\left[\frac{f}{2D} + \frac{\pi D}{(A_t + \pi DL)}\right]} = \frac{\varpi}{\left[\frac{f}{2D} + \varpi\right]} = \frac{1}{\left[\frac{f}{2D\varpi} + 1\right]}$$

where $\varpi = \frac{\pi D}{(A + \pi DL)}$

4. Results and discussion

The percentage ratio of pressure drop due to acceleration to total pressure drop (ψ) was varied with pipe diameter at different friction factors. As pipe diameter was increasing, the

percentage ratio of pressure drop due to acceleration to the total pressure drop was also decreasing. Pipeline parameters used in model development are presented in Table 1

Table 4. Pipeline parameters used in model development

Pipeline parameter (symbol)	Value
Base temperature (T_b)	520 °R
Base pressure (P_b)	14.7 psia
Inlet pressure (P_1)	200 psia
Outlet pressure (P_2)	30 psia
Gas specific gravity (γ_g)	0.75
Gas temperature (T)	545°R
Pseudo-critical pressure (P_{pc})	661 psia
Pseudo-critical temperature (T_{pc})	411 °R
Inclination angle	0 degree

These trends were depicted graphically in Figure 3. From Figure 3, the percentage contribution of pressure drop due to acceleration to total pressure drop was large at low pipe diameters. This is because for a fixed flow rate, reducing pipe diameter increases velocity and acceleration, hence increasing pressure drop due to acceleration. But, for bigger diameter pipes, there is normally a larger surface area for fluid flow, and the major contributor to total pressure drop is normally friction between fluid layers and friction along the pipe wall. Showing that neglecting pressure drops due to acceleration can be misleading, especially for small diameter pipes.
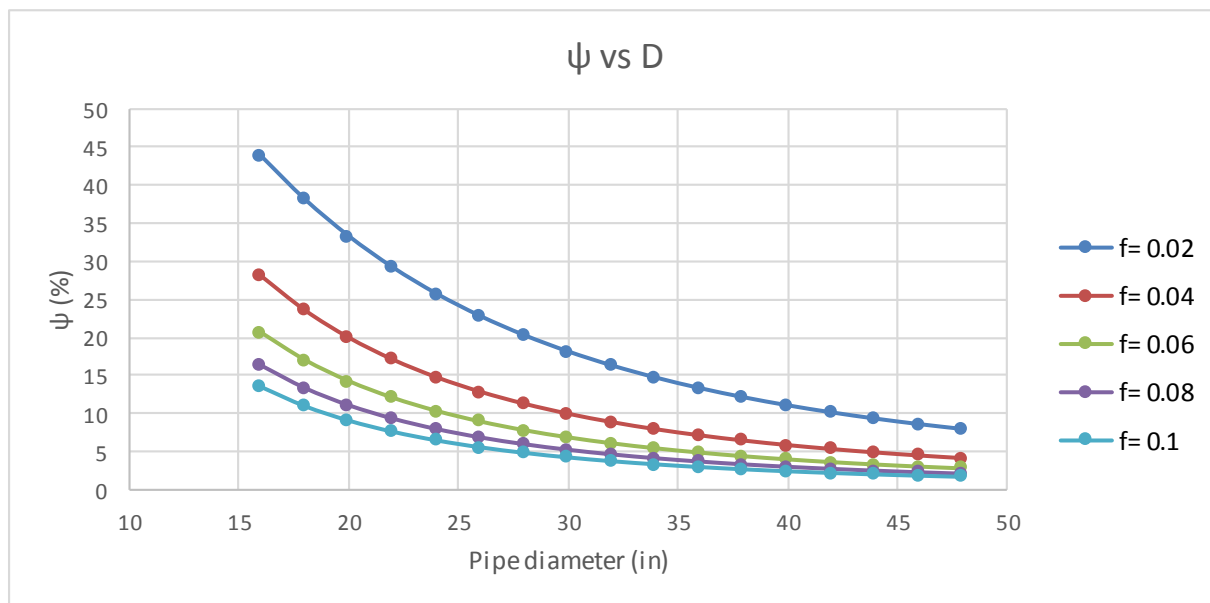

Figure 3. Graphical representation of the variation of ψ with pipe diameter at different friction factor values for transmission lines

Also, the variation of percentage ratio of pressure drop due to acceleration to total pressure drop with ϖ (given by $\frac{\pi D}{(A+\pi DL)}$) was investigated. From the figure, it is obvious that there exists a positive correlation between the two parameters. Consequently, as ϖ was increasing, the percentage ratio of pressure drop due to acceleration to the total pressure drop was also increasing. This trend was depicted graphically in Figure 4.
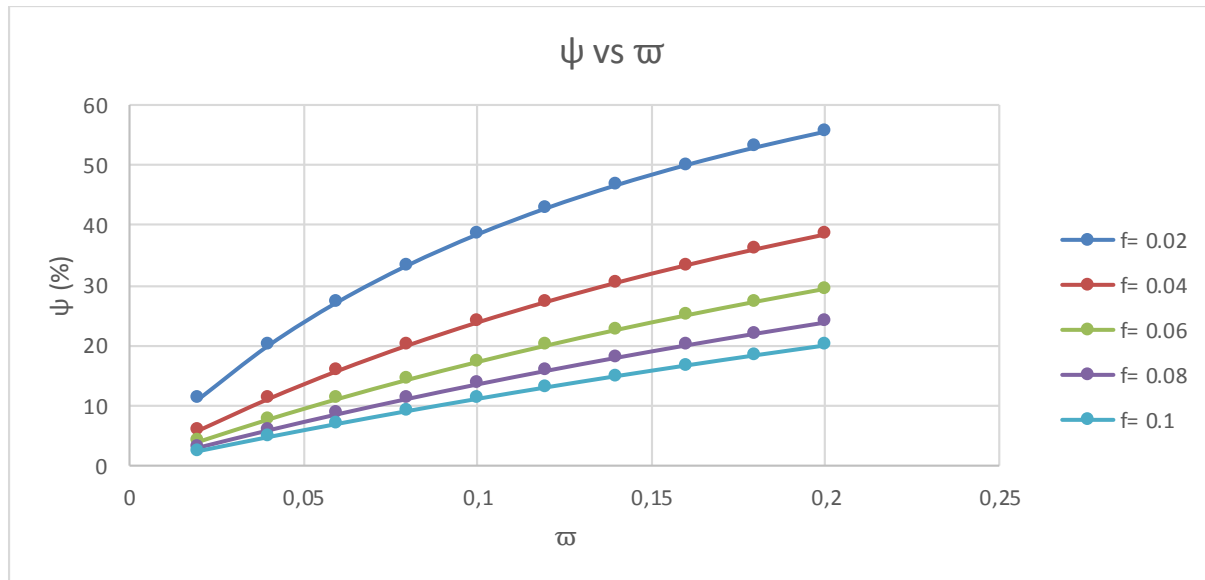
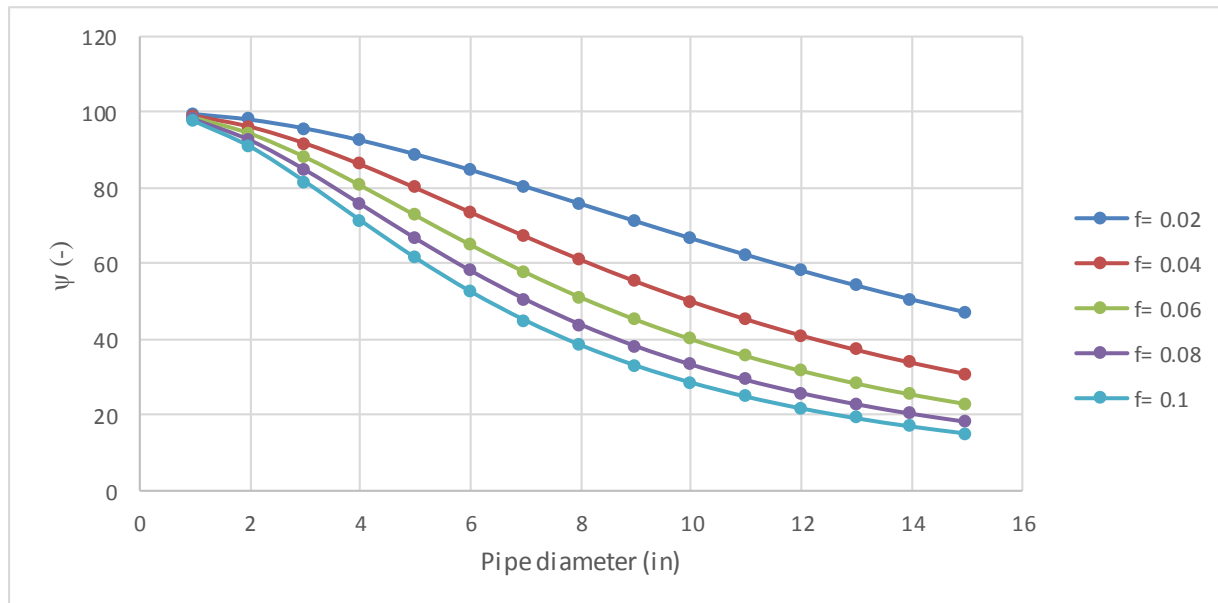Figure 4. Showing the variation of ψ with ϖ for different friction factor values for transmission lines

Figure 5. Graphical representation of variation of ψ with pipe diameter at different friction factor values for distribution lines

Figure 5 above shows the variation of ψ with pipe diameter at different friction factor values for distribution lines. As pipe diameter was increasing, the percentage ratio of pressure drop due to acceleration to the total pressure drop was decreasing. Also, from Figure 5, percentage contribution of pressure drop due to acceleration to total pressure drop is large at low pipe diameters. Like for transmission lines, this can be explained by the fact that for a fixed flow rate, reducing pipe diameter increases velocity and acceleration, hence increasing pressure drop due to acceleration. But, for higher diameter pipes, there is normally a larger surface area for fluid flow, and the major contributor to total pressure drop is normally friction between fluid layers and friction along the pipe wall. Due to smaller pipes, the effects of acceleration term are more pronounced in distribution pipes.

5. Conclusion

From the study, the following conclusions can be made:

a) An analytical model for estimating total pressure drop, including pressure drop due to acceleration in horizontal gas pipelines, was developed.

b) Also, the percentage contribution of pressure drop due to acceleration to total pressure drop in horizontal gas pipelines was investigated.

c) The effects of acceleration term are more pronounced for distribution lines than transmission lines.

d) And it was ascertained that neglecting the acceleration term in predicting total pressure drops in horizontal gas pipelines can be misleading, especially for small diameter pipes.

5. Recommendation

The study can further be extended to gas flows in other conditions like vertical pipe flow; inclined pipe flows etc.

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