

## PREDICTIVE TOOL FOR BOTTOM-HOLE PRESSURE IN MULTIPHASE FLOWING WELLS

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### Abstract

The analytical model for predicting the pressure at any point in a flow string is essential in determining optimum production string dimension and in the design of gas-lift installations. This information is also invaluable in predicting bottom-hole pressure in flowing wells.

A variety of model on bottom-hole pressure in flowing wells have been reported in the literatures. Most of the early models on pressure drop in the flowing wells were based on single phase flowing wells, even the recent investigators treated the multiphase (liquid and gas phase) as a homogenous single phase flow without accounting for dissolved gas in oil.

This paper present a modification of Sukkar and Cornell model for single phase flowing gas wells and the model was adapted to predict the pressure drop in multiphase flowing wells. The key operational and fluid/ pipe parameters which influence the degree of pressure drop in flowing wells are identified through the modification.

**Key words:** *modelling; pressure drop; multiphase flow; flowing well*

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### 1. Introduction

The simultaneous flow of oil, water and gas in vertical pipe is encountered in many engineering installations. In petroleum, chemical process, nuclear engineering and many other industries, problems associated with simultaneous flow of two or more phases through vertical pipe have been of interest for a long time [1-8]. This interest has increased considerably during recent years due to applications to new processes in petroleum production and refining. One prominent example of vertical two phase flow is provided by the gas lift process where oil, water and gas flow simultaneously [1]. If the pressure profile in a gas-lift well can be predicted within reasonably accuracy, it would be possible to get good estimates of the power required to lift the oil, the optimum depth pressure and the rate at which to inject gas [1]. Furthermore, the effect of production rate and tubing sizes on these quantities can be evaluated before any design decision is made on the installations and operation of the flow string.

Studies on multiphase flow in vertical pipe have sought to develop a technique with which the pressure drop can be calculated. Pressure losses in flow of gas and liquid phase (two-phase) are quite different from those encountered in dry gas phase (single-phase) alone [1-2,4-8,11-14]. The variance is function of interface and gas slippage during the simultaneous flow of gas and liquid. The interface may be smooth or have varying degrees of roughness, depending on the flow pattern [9-10,14]. Therefore, transfer of energy from the gaseous phase to the liquid phase may take place while energy is lost from the system through the wetting phase at the pipe wall. Such an energy transfer may be either in the form of heat exchange or of acceleration. Some each phase must flow through a smaller area than if it flowed alone, amazingly high pressure losses occur when compared to single-phase flow [9-14]. It is important to

note that linear pressure gradients or pressure losses are only appropriate when the vertical flow column consists of a single phase fluid such as liquid. When there is more than the one fluid flowing in the vertical column, such as the presence of solution gas in oil, a radical change takes place. Severe investigators such as Poettmann and Carpenter<sup>[1]</sup>, and Tek<sup>[11]</sup>, Orkiszewsk<sup>[2]</sup> and Ros<sup>[9]</sup> have developed model on pressure drop or pressure gradient along the tubing, which might only be approximate solutions. They may not accurately provide information about pressure conditions at the bottom of the well due to the fluid column consisting of two or more fluid phase. Their models treated the liquid and gas as a homogenous single-phase flow without accounting for dissolved gas in oil.

A methodology which uses a single phase flow model to stimulate multiphase fluid flow system and the mixing rule that correspond to the fluid flow pattern is presented. The formulation also presents methods that incorporate the effects of slippage at the gas-liquid interface, the effect of solution gas in the liquid phases and produced gas specific gravity.

This paper presents a model for predicting the bottom hole flowing pressure (BHFP) in multiphase system, where oil/gas or oil/water/gas are flowing together. The model is a modification to Sukkar and Cornnel's model for dry gas. The modification predicts the bottom hole flowing pressure in multiphase system as function of operational and fluid/pipe parameters. It devised a method of extending a single phase model to be applicable for the multiphase flow system.

## 2. METHODOLOGY

The analytical expressions derived in this study are based on the following fundamental and general assumptions<sup>[1-8]</sup>

- 1 Steady-state flow of fluid was considered throughout the process.
- 2 Change in kinetic energy is small and may be neglected
- 3 Temperature of system is assumed constant at some average value
- 4 Friction is also assumed constant over the length of the conduit

### 2.1 THE MODEL

Consider the flow of fluid from one arbitrary point to another in a given system. Assuming an idealized flow equation, the basic energy equation associated with flow of fluid over the length of the conduit can be given as<sup>[6,15]</sup>:

$$\frac{144}{\rho} dp + \frac{udu}{2\alpha gc} du + \frac{g}{gc} dz + \frac{fu^2}{2gcD} DL + Ws = 0 \quad (1)$$

Assuming no mechanical work is done on the fluid or by the fluid and change in kinetic energy is negligible.

Equation (1) can be reduced to:

$$\frac{144}{\rho} dp + \frac{g}{g_c} dz + \frac{fu^2}{2g_c D} dL = 0 \quad (2)$$

The concept of apparent or average multiphase fluid density is quite useful in characterizing a multiphase fluid mixture. The apparent density of a multiphase mixture is defined observing the "mixing rule"<sup>[13]</sup>.

$$\rho_{fp} = \rho_L H_L + \rho_g (1 - H_L) \quad (3)$$

Therefore, density of gas ( $\rho_g$ ) at a point in a vertical pipe at pressure and temperature may be obtained from the definition of the Gas law as<sup>[15,17-18]</sup>:

$$\rho_g = \frac{28.97G_g P}{ZRT} \quad (4)$$

The density of the liquid (oil and water) is obtained as<sup>16</sup>:

$$\rho_L = \rho_o h_o + \rho_w h_w \quad (5)$$

$$\rho_L = \frac{\{62.4G_o + 0.010036G_g R_s\}h_o}{B_o} + \frac{62.4G_w h_w}{B_w} \quad (6)$$

Substituting equations (4) and (6) into (3) to obtain two-phase density

$$\rho_{tp} = \frac{\{(62.4G_o + 0.0136G_g R_s)h_o + \frac{62.4G_w h_w}{B_w}\} H_L + \frac{28.97PG_g(1-H_L)}{ZRT}}{B_o} \tag{7}$$

The velocity of fluid flow at a cross-section of a vertical pipe may be defined as [6]:

$$U_M = \frac{0.4152q_g TZ}{PD^2} + \frac{0.000082735B_o q_L}{D^2} \tag{8}$$

Substituting equation (7) and (8) into equation (2) and converting diameter D (inches) to feet, we have:

$$\left[ \frac{144dp}{\left[ \left( \frac{62.4G_o + 0.0136G_g R_s}{B_o} \right) h_o + \frac{62.4G_w h_w}{B_w} \right] H_L + \frac{28.97PG_g(1-H_L)}{ZRT}} + \left( 1 + \frac{667fq_g^2 T^2 Z^2}{P^2 D^5} + \frac{26571 \times 10^{-5} fq_g q_L TZ B_o}{PD^5} + \frac{26473 \times 10^{-9} fq_L^2 B_o^2}{D^5} \right) dL = 0 \tag{9}$$

Re-arranging equation (9) we have:

$$\frac{0.01875G_g L}{\bar{T}} = \int_{R_1}^{R_2} \frac{\frac{Z}{P} \phi}{\left[ 1 + C1 \left[ \left( \frac{Z}{P} \right)^2 + C11 B_o \left( \frac{Z}{P} \right) + C12 B_o^2 \right] \right] \left[ C41 \left[ C42 + R_s + C43 \left( \frac{B_o}{B_w} \right) \right] \left[ \left( \frac{1}{B_o} \right) \left( \frac{Z}{P} \right) - 1 \right] H_L + 1 \right]} \tag{10}$$

Where;

$$C1 = \frac{667fq_g^2 T^2}{D^5} \quad ; \quad C11 = \frac{38.836}{\bar{T}} \left( \frac{q_L}{q_g} \right) \quad ; \quad C12 = \frac{39.69}{\bar{T}^2} \left( \frac{q_L}{q_g} \right)^2$$

$$C41 = 22.94h_o \bar{T} \quad ; \quad C42 = \frac{G_o}{G_g} \quad ; \quad C43 = \left( \frac{G_w}{G_g} \right) \left( \frac{h_w}{h_o} \right)$$

The detail of the equation (10) is expressed in appendix A

### 3. ANALYSIS OF RESULTS

This paper presents a new methodology of estimating bottom-hole flowing pressure. The method is capable of providing a satisfactory multiphase flow result if the pressure dependent variables are treated as a function of pressure and not a constant.

Figure 1 shows a pure dry gas single phase flow with no effect of liquid holdup. Figures 2 and 3 show a dramatic change in the curve pattern due to the significance effect of liquid holdup.

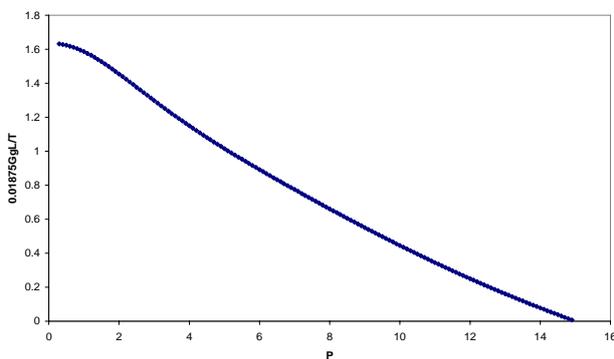


Fig.1: For Gas Only

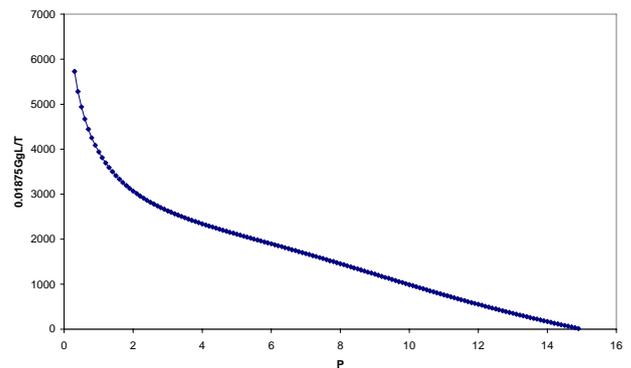


Fig.2: For Oil Only

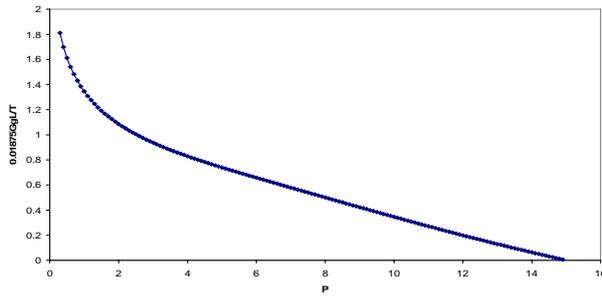


Fig.3 : For Mixture

Better still; the essence of these figures is another technique of estimating the bottom-hole pressure from the wellhead pressure.

The integration is based on two intervals; the tubing head pressure and the flowing well pressure.

The general procedure is based on integrating the two intervals from any arbitrary lower limit, say pressure=0.3

$$\int_{P_{tf}}^{P_{wf}} I(P)dP = \int_{0.3}^{P_{wf}} I(P)dP - \int_{0.3}^{P_{tf}} I(P)dP$$

The tubing head pressure ( $P_{tf}$ ) is known as it is estimated from the wellhead which left the bottom-hole pressure ( $P_{wf}$ ) as unknown which is the point of focus in this regard.

Therefore,

$$\int_{0.3}^{P_{wf}} I(P)dP = \int_{0.3}^{P_{tf}} I(P)dP + \frac{0.01875GgL}{\bar{T}}$$

#### 4. CONCLUSION

- 1) The slippage at the interface between the gas and liquid phase increases as the pressure differential acting on the multiphase fluid decreases.
- 2) The solution gas and liquid holdup were treated as a function of pressure and not as a constant as assumed in some models.
- 3) The model can be used for different phases if the situation warrants.
- 4) Formation volume factor, specific gravity and fraction of oil and water were properly accounted for as a function of pressure as neglected in some previous models.
- 5) The flow rate of gas increases as the pressure differential increases, while those of oil and water decreases as pressure differential increases.
- 6) The friction factor and average temperature were assumed to be constant over a length of a conduit.

#### RECOMMENDATION

The model was perfectly accurate as compared with Sukkar and Cornnel's model for single phase flow. Accountability of interdependence variables must be thoroughly done for the accuracy of the model.

The model will yield an accurate result if the pressure dependent variables are treated as a function of pressure and not a constant. The developed model can be used for single as well as multiphase fluid flow.

#### Subscripts

b-base	o-oil
g-gas	s-solid
L-liquid	w-water
m-mixture	

## NOMENCLATURE

A-cross-sectional area of pipe,ft <sup>2</sup>	PPR-pseudo reduced pressure
API-API gravity, degree	q-volumetric flow rate , $\frac{ft^3}{sec}$
B-formation volume factor, $\frac{bbl}{stb}$	R- gas constant, $10.73 \frac{ft^3 psia}{lb - mole^{\circ}R}$
D-inside diameter of the pipe, ft	T- temperature, °R
f-moody friction factor, dimensionless	TPR-pseudo reduced temp.
g-acceleration due to gravity, $\frac{ft}{sec^2}$	U-average velocity of the fluid, $\frac{ft}{sec}$
gc-conversion factor, $32.17 \frac{lbmft}{lbf s}$	V- specific volume of fluid, $\frac{ft^3}{lbm}$
G-specific gravity, dimensionless	W <sub>s</sub> -mechanical work done on or by the gas(w <sub>s</sub> =0)
h-volume fraction in the liquid	z-gas compressibility factor, dimensionless
H-liquid holdup	dZ-incremental depth
L-length of the flowstring, ft(for a vertical flowstring, L=Z)	$\frac{u du}{2\alpha g_c}$ -pressure drop due to kinetic energy
M-molecular weigh of air, 28.97G	$\frac{fu^2 dl}{2g_c D}$ -pressure drop due to friction effects
P-pressure, psia	$\rho = \text{density} , \frac{lbm}{ft^3}$
dp-pressure differential, $\frac{lb}{ft^3}$	$\alpha$ -correction factor to compensate for the variation of velocity over the tube cross-section

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## APPENDIX A

Define

$$C1 = \frac{667fq_g^2 T^2}{D^5} \quad (10)$$

$$C2 = \frac{26571 \times 10^{-5} fq_g q_L T}{D^5} \quad (11)$$

$$C3 = \frac{26473 \times 10^{-9} fq_L^2}{D^5} \quad (12)$$

$$X = \left[ \left( \frac{62.4G_o + 0.0136G_g R_s}{B_o} \right) h_o + \frac{62.4G_w h_w}{B_w} \right] \quad (13)$$

Substituting equation (10), (11), (12), (13) into equation (9) and integrate, we have

$$\frac{1}{144} \int_0^L dL = \int_{P_1}^{P_2} \frac{dp}{\left[ 1 + C1 \left( \frac{Z}{P} \right)^2 + C2 B_o \left( \frac{Z}{P} \right) + C3 B_o^2 \right] \left[ X H_L + \frac{2.70 P G_g (1 - H_L)}{Z T} \right]} \quad (14)$$

Re-arranging equation (14), we have

$$\frac{0.01875 G_g L}{\bar{T}} = \int_{PPR_1}^{PPR_2} \frac{\frac{Z}{P} dP}{\left[ 1 + C1 \left( \frac{Z}{P} \right)^2 + C2 B_o \left( \frac{Z}{P} \right) + C3 B_o^2 \right] \left[ \frac{X H_L \bar{T}}{2.7 G_g} + \frac{P}{Z} (1 - H_L) \right]} \quad (15)$$

$$\frac{0.01875 G_g L}{\bar{T}} = \int \frac{dP}{\left[ 1 + C1 \left( \frac{Z}{P} \right)^2 + C2 B_o \left( \frac{Z}{P} \right) + C3 B_o^2 \right] \left[ \frac{X H_L \bar{T}}{2.7 G_g} + \frac{P}{Z} (1 - H_L) \right]} \quad (16)$$

Define

$$C4 = \frac{X \bar{T}}{2.7 G_g} \quad (17)$$

$$\left[ C4 - \frac{P}{Z} \right] H_L + \frac{P}{Z} \quad (18)$$

Substituting equation (13) into equation (17)

$$C4 = \left[ \frac{62.4 G_o h_o \bar{T}}{2.7 G_g B_o} + \frac{0.0136 G_g R_s h_o \bar{T}}{2.7 G_g B_o} + \frac{62.4 G_w \bar{T} h_w}{2.7 G_g B_w} \right] \quad (19)$$

Define

$$A1 = \frac{62.4G_o h_o \bar{T}}{2.7G_g} ; A2 = \frac{0.0136h_o \bar{T}}{2.7} ; A3 = \frac{62.4G_w h_w \bar{T}}{2.7G_g B_w}$$

$$C4 = \left[ \left( \frac{A1}{B_o} + A2 \frac{R_s}{B_o} + A3 \right) \left( \frac{Z}{P} \right) - 1 \right] H_L \quad (20)$$

Then;

$$\frac{A1}{A2} = 4588.235 \left( \frac{G_o}{G_g} \right) \quad (21)$$

$$\frac{A3}{A2} = 4588.235 \frac{1}{B_w} \left( \frac{G_w}{G_g} \right) \left( \frac{h_w}{h_o} \right) \quad (22)$$

Equation (20) becomes;

$$C4 = 22.94h_o \bar{T} \left[ \left[ \left( \frac{G_o}{G_g} \right) + R_s + \left( \frac{G_w}{G_g} \right) \left( \frac{h_w}{h_o} \right) \left( \frac{B_o}{B_w} \right) \right] \frac{1}{B_o} \left( \frac{Z}{P} \right) - 1 \right] H_L \quad (23)$$

Recall equation (10), (11), and (12)

$$C1 = \frac{667fq_g^2 \bar{T}^2}{D^5} ; C2 = \frac{26571 \times 10^{-5} fq_g q_L \bar{T}}{D^5} ; C3 = \frac{26473 \times 10^{-9} fq_L^2}{D^5} \quad (24)$$

Therefore;

$$\frac{C2}{C1} = \frac{38.836}{\bar{T}} \left( \frac{q_L}{q_g} \right)^2 \quad (25)$$

$$\frac{C3}{C1} = \frac{39.69}{\bar{T}^2} \left( \frac{q_L}{q_g} \right)^2 \quad (26)$$

In actual sense,

$$\left[ 1 + C1 \left( \frac{Z}{P} \right)^2 + C2 B_o \left( \frac{Z}{P} \right) + C3 B_o^2 \right] = \left[ 1 + C1 \left[ \left( \frac{Z}{P} \right)^2 + \frac{C2}{C1} B_o \left( \frac{Z}{P} \right) + \frac{C3}{C1} B_o^2 \right] \right] \quad (27)$$

Substituting equation (25) and (26) into equation (27), we have

$$\left[ 1 + C1 \left( \frac{Z}{P} \right)^2 + C2 B_o \left( \frac{Z}{P} \right) + C3 B_o^2 \right] = \left[ 1 + C1 \left[ \left( \frac{Z}{P} \right)^2 + \frac{38.836}{\bar{T}} \left( \frac{q_L}{q_g} \right) B_o \left( \frac{Z}{P} \right) + \frac{39.69}{\bar{T}^2} \left( \frac{q_L}{q_g} \right)^2 B_o^2 \right] \right] \quad (28)$$

Substituting equation (23) and (28) into equation (16), we have:

$$\frac{0.01875G_g L}{\bar{T}} = \int_{P_1}^{P_2} \frac{\frac{Z}{P} \phi}{\left[ 1 + C1 \left[ \left( \frac{Z}{P} \right)^2 + C11 B_o \left( \frac{Z}{P} \right) + C12 B_o^2 \right] \right] \left[ C41 \left[ C42 + R_s + C43 \left( \frac{B_o}{B_w} \right) \right] \left( \frac{1}{B_o} \right) \left( \frac{Z}{P} \right) - 1 \right] H_L + 1} \quad (29)$$

Where;

$$C1 = \frac{667fq_g^2 \bar{T}^2}{D^5} ; C11 = \frac{38.836}{\bar{T}} \left( \frac{q_L}{q_g} \right) ; C12 = \frac{39.69}{\bar{T}^2} \left( \frac{q_L}{q_g} \right)^2$$

$$C41 = 22.94h_o \bar{T} ; C42 = \frac{G_o}{G_g} ; C43 = \left( \frac{G_w}{G_g} \right) \left( \frac{h_w}{h_o} \right)$$