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Evaluating the Parameters of a Differential Equating Describing Start-up and Shutdown of a Pumping Unit

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Abstract

The paper presents results of studying various transitional modes of a pumping unit at an oil pumping station, in particular, speed characteristics of its rotor rotation. Input and output variables of this objects are time and angular velocity of the pumping unit rotor rotation, respectively. An ordinary differential equation with constant coefficients is used to describe a dependence linking the variables. The purpose of the study is to illustrate a possibility of application and technique of applying a sensitivity algorithm to the problem.

Keywords: *Differential equation; Sensitivity algorithm; Metrics; Controlled object; Pumping unit.*

1. Introduction

As it is known, e.g., from [1], various classes of differential equations are often and effectively used in order to describe quantitative relations between variables characterizing behavior of dynamic objects in temporal and spatial domain. Selection of a differential equation class in each particular case is determined by existing theoretical understanding of physical, chemical, economic and other processes and mechanisms characteristic of the studied object. Determination of numerical values of the parameters of a selected differential equation class is usually reduced to a problem of minimizing a quadratic or other metric that characterizes a distance between available measured values of the object's output and a solution of the differential equation calculated for the same values of the object's input as were used to obtain the measured output value [2]. Of all the currently known algorithms that may be used to obtain numerical values of differential equation parameters, the most efficient and thus, the most commonly applied is the so-called sensitivity algorithm [3], which is considered below.

The object of research in this paper is velocity performance of a pumping unit rotor [4]. At that, input and output variables of the object are, respectively, time and angular rotational velocity of the pumping unit rotor, while an ordinary differential equation with constant coefficients is used to describe the dependence between the variables [5]. The purpose of the study is to illustrate a possibility of application and technique in applying a sensitivity algorithm to the problem.

First, let us assume that the pumping unit rotor rotational velocity ω as a function of time t may be described with satisfactory accuracy with a differential equation in the form of

$$\frac{d^2\omega}{dt^2} = a_1\frac{d\omega}{dt} + a_2 w + a_3 t + u(t), \qquad (1)$$

where $u(t)$ is a unit function pointing to the moment of the pumping unit start.

Second, we will assume that we have a certain finite number N of measurement pairs in the form of

$$\widetilde{\omega_i} t_i, i = \overline{1, N}, \qquad (2)$$

where ω_i is the value of the pump unit rotational speed at a certain moment of time t_i.

Third, in order to give quantitative assessment of error in description of measurements with equations (1), we are going to use Euclidean metric S, determined by an equation

$$S = \left[\sum_{i=1}^{N} (\widetilde{\omega_i} - \omega(\vec{a}, t_i))^2\right]^{\frac{1}{2}}, \quad (3)$$

where $\widetilde{\omega_i}$ and $\omega(\vec{a}, t_i)$ are, correspondingly, measured and calculated values of the motor rotor revolution rate.

As it is directly seen from this equation, the metric S is a function of assessments $\vec{a}$ of the unknown parameters of the equation (1) and thus its application allows reducing the problem of finding such assessments to solution of the following extremum problem [6]:

$$\vec{a} = \arg\min S \quad (4)$$

Here the Formula 2 symbol (minimum argument) means that in this case such assessments $\vec{a}$ are selected for which the metric S takes the minimum value. Selection of exactly such assessments $\vec{a}$ in the conditions stated above appear justified and efficient.

Fourth, in order to solve the extremum problem (4) let us use a well-known sensitivity algorithm based upon application of sensitivity functions $W_i(\vec{a}, t)$ to assessment of parameters a_i, as defined by equations in the form of

$$W_i(\vec{a}, t) = \frac{\partial \omega(\vec{a}, t)}{\partial a_i}, \quad i = \overline{1,4}. \quad (5)$$

It is an iteration algorithm and at each iteration a finite sequence of computational operations is performed related to forming and solving a system of linear equations with respect to assessments $\vec{a}$. Let us assume that we have already performed $l-1$ iterations and have an assessment vector $\vec{a}^{l-1}$. Let us transform a vector of new parameter assessments $\vec{a}^{l}$ in a form of $\vec{a}^l = \vec{a}^{l-1} + \overrightarrow{\Delta a}^l$, (6)

where $\overrightarrow{\Delta a}^l$ is a correction vector for existing assessments $\vec{a}^{l-1}$.

Let us take a Taylor series expansion of the unknown function $w(\vec{a}^l, t)$ in the vicinity of $\vec{a}^{l-1}$ values and limit ourselves to a linear approximation. As a result, we obtain

$$w(\vec{a}^l, t) \approx w(\vec{a}^{l-1}, t) + \frac{\partial \omega(\vec{a}^{l-1}, t)}{\partial a_1} \Delta a_1 + \ldots + \frac{\partial \omega(\vec{a}^{l-1}, t)}{\partial a_4} \Delta a_4, \ i = \overline{1,4}. \quad (7)$$

Using sensitivity functions, (3) let us transform this equation to

$$w(\vec{a}^l, t) \approx w(\vec{a}^{l-1}, t_i) + W_1(\vec{a}^{l-1}, t_i)\Delta a_1 + \ldots + W_4(\vec{a}^{l-1}, t_i)\Delta a_4, \ i = \overline{1,4}. \quad (8)$$

Analysis of this equation shows that:

1) it is a functional equation which is linear with respect to corrections $\overrightarrow{\Delta a}^l$ for all the values of the argument t;
2) if the function $w(\vec{a}^{l-1}, t)$ and sensitivity functions $W_i(\vec{a}^{l-1}, t_i)$ are known, it allows constructing as many systems of linear equations with respect to corrections $\overrightarrow{\Delta a}^l$ as we wish.

One of such systems of linear equations may be obtained if we use corrections $\overrightarrow{\Delta a}^l$, serving as a solution of the extremum problem (4). To that end, it is evidently sufficient to: 1) substitute in (3) the values of $\omega(\vec{a}, t_i)$ with the right hand side of the equations (6); 2) differentiate the obtained function with respect to corrections $\boldsymbol{a_i^l}, \boldsymbol{i = \overline{1,4}}$ and make the obtained derivatives Formula 4 equal to zero.

Having performed all the necessary calculations, we get a system of linear equations in the form of

$$B\Delta\vec{a}^l = \Delta\omega^l \quad (9)$$

where, Formula 5 is a 4-dimensional column vector and (4×4) is a matrix Formula 6, calculated with the equations

$$\Delta\omega^l = (W_l)^T \Delta\vec{a}^l \quad (10)$$

$$B = (W_l)^T W_l \quad (11)$$

where Formula 9 – Formula 10 is a matrix, elements Formula 11 of which are defined by the following equations:

$$w_{ij} = \frac{\partial \omega(a^{l-1}, t_i)}{\partial a_j}, \quad i = \overline{1, N}, \; \text{j}=\overline{1,4}, \tag{12}$$

i.e., they are the values of the sensitivity functions Formula 13 at the moment Formula 14.

As we know the assessments $\vec{a}^{l-1}$, then, using them to substitute unknown coefficients of the differential equation (1) and setting initial conditions we get a well-known Cauchy problem, which may be solved by any numerical method applicable to solving differential equations, e.g., a Runge-Kutta method. The same method may be used to calculate the sensitivity functions $W_i(\vec{a}^{l-1}, t_i)$, Formula 15 as well, getting all the initial data necessary to form a system of linear equations. Having solved this system of linear equations, we get corrections $\overrightarrow{\Delta a}^l$ and using the equation, we may then calculate new assessments $\vec{a}^l$. This ends the Formula 16-th iteration of solving the problem (3).

Figure 1 shows a rotational velocity characteristic of the pump unit rotor as a function of time and a corresponding assessment obtained with a proposed algorithm; at that coefficients in the equation (1) were set equal to $a_1 = -20.1492$, $a_2 = -297.1612$, $a_3 = 647.2839$, $a_4 = 1499.2696$

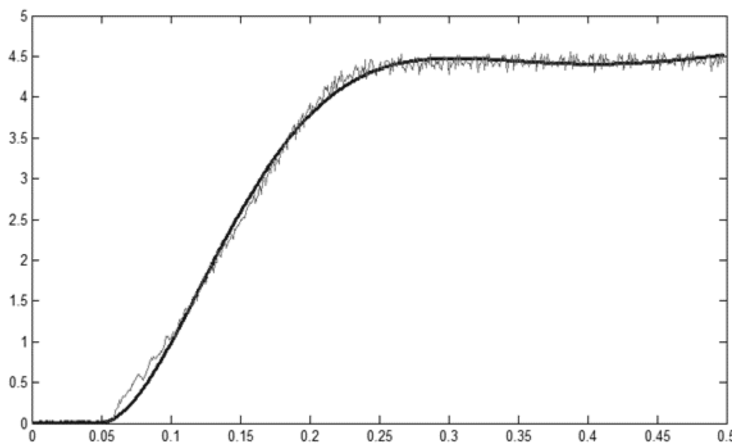

Fig. 1 Experimental results

2. Conclusion

A number of experiments were conducted for various transitional modes of the pumping unit; Figure 1 shows one of them, where it is clearly seen that the proposed mathematical model and the algorithm for determining its coefficients allow obtaining sufficiently accurate assessments of the pumping unit angular velocity. Mean squared deviation of the assessments from true values amounted to 0.0038. Similar accuracy was seen in other experiments [7].

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