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Study of Natural Gas Compressibility Factor (Z) to Improve Experimental Data Prediction Using Peng-Robinson State Equation

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Abstract

Among the essential thermodynamic parameters of behavior, analysis (PVT) in natural gas engineering is the gas compressibility factor (Z). In this work, we improve the results of the compressibility factor (Z) prediction accurately after modifying the cubic equation coefficients for different gas types; the cubic equation of state (EOS) prediction model was tested using 149 experimental data under different conditions. In this study, the gas compressibility factor is calculated by Peng Robinson's equation of state (PR EOS). Additionally, the coefficients were updated to reflect a better representation of natural gas. The results are compared to the predictions of the original PR EOS and other equations of state in the literature. The statistical analysis shows an average absolute deviation percentage (AAD %) of 0,99% and a coefficient of determination (R^2) of 0,9968 at pressure and temperature up to 16632,93 PSI (114,68 MPa) and 426,74K, respectively. The proposed model can effectively calculate (Z) for any type of gas, including condensate gas.

Keywords*: Peng-Robinson; Compressibility factor; Natural gas; Modeling; Statistical analysis.*

1. Introduction

Calculating the reserves of a gas field or determining its performance and developments in production and operations requires knowledge of PVT properties [1]. Besides, accurately predicting these thermodynamic properties remains a difficult problem with great technical significance. However, there are simple laws that have been known for many years for a hypothetical fluid called an ideal gas that describe the behavior of reservoir gases as a function of pressure (P), volume (V) and temperature (T) but not for real gases. Based on that, many studies have been conducted on thermodynamic models in an effort to improve their accuracy and extend their use.

The compressibility factor (Z) is a measure of how much gas can be deflected to its perfect state. It is commonly known as the gas deflection factor (The Gas Deviation Factor) [2-3]. It is a non-dimensional quantity defined by the volume ratio occupied by the gas at a given pressure and temperature compared to the volume occupied by the gas when it is composed as a perfect gas. As a result, $Z = 1$ represents an ideal gas condition. Due to its use in engineering calculations, this property of natural gases is very important compared to other thermodynamic properties [4].

According to the literature, the calculation of (Z) is made by implicit and explicit equations [4-10]. Using the implicit equation requires digital equipment (a computer, an advanced calculator, etc.), such as programmable logic control (PLC) systems used in the gas industry, whereas the explicit equation with high accuracy is solved easily [11].

Cubic equations of state (EOS) provide better accuracy, reliability, simplicity and speed in predicting and correlating fluid properties [12].

Cubic state equations are simple and easy to calculate. While the modification of the energy parameters or coefficients of the equation based on temperature is one of the methods for improving its accuracy [13].

The accuracy of the gas factor (Z) is critical in gas flow estimates, power requirements for gas compression, estimating the amount of gas reservoir calculations, and other gas processing calculations [8,14].

The choice of a correlation for an accurate estimate in a range of pressure, temperature and gas composition is difficult because the results obtained from different correlations of factors (Z), have a large variance for the same sample of gas with the same operating conditions (pressure and temperature).

The error in determining the gas factor can lead to immense errors in calculating natural gas engineering properties like the formation volume factor (Bg), gas compressibility (cg) and original gas in place and other properties. A number of correlations have been reported previously to better estimate gas deflection factors at lower pressures, but their accuracy becomes questionable at higher reservoir pressures [15].

Peng Robinson's cubic equation of state was developed with the most suitable conditions for natural gas. To validate our proposed Peng Robinson's cubic equation, a total of 149 types of gases were used to calculate Z for each pressure. However, statistical error analysis has been carried out on the selected data, confirming the validity of the suggested changes.

The correlations of (Z) available in the literature do not have wide application for higher conditions, and therefore their error may be high in predicting this parameter. On this basis, our study will determine the needs of the gas industry and provide a better understanding of gas mixtures on an industrial scale.

2. Equations of state

2.1. Peng Robinson EOS (PR 1976)

Peng and Robinson [16-17] conducted an extensive study to predict the behavior of natural hydrocarbon systems. The Peng-Robinson equation is recommended for the liquid-vapor equilibrium calculations of hydrocarbons under pressure (petrochemicals and natural gas applications). It is essential and relatively irreplaceable. The Peng and Robinson (PR EOS) formula is as follows:

$$P = \frac{RT}{V_m - b} - \frac{a(T)}{V_m^2 - 2bV_m - b^2} \quad (1)$$

The equation takes the form of a cubic equation in V, and is solved by numerical or analytical techniques when P and T are constants [18]. The critical point has limits leading to the following relationships for PR EOS parameters [17]:

$$a(T) = 0{,}45724 * \frac{R^2 T_c^2}{P_c} \alpha(T) \quad ; \quad b = 0{,}0778 * \frac{RT_c}{P_c} \quad (2)$$

$$\alpha(T) = \left[1 + m(1 - \sqrt{T_r}\right]^2 \quad (3)$$

$$m = 0{,}37464 - 1{,}54226\omega - 0{,}26992\omega^2 \quad (4)$$

Peng Robinson's cubic equation:

$$Z^3 + (B-1)Z^2 + (A - 3B^2 - 2B)Z - (AB - B^2 - B^3) = 0 \quad (5)$$

The following parameters will be used to calculate the factor (Z) of our selected experimental mixtures using the cubic equation,:

$$A = \frac{a_m P}{R^2 T^2} \quad (6)$$

$$B = \frac{b_m P}{RT} \quad (7)$$

$$a_m = \sum_i \sum_j \left[x_i x_j \sqrt{a_i a_j \alpha_i \alpha_j}(1 - k_{ij})\right] \quad (8)$$

$$b_m = \sum_i [x_i b_i] \quad (9)$$

K_{ij} is determined using the Elliot and Daubert approach (1985) [18].

For the prediction of natural hydrocarbon system behaviors, Peng and Robinson [16] do a complete study concerning the evaluation of the use of the state equation of Soave-Redlich-

Kwong [19] (SRK1972). They improved the state equation so that it is capable of determining the density of liquids and other fluid properties, notably in neighboring regions in the critical field. Therefore, there is an opportunity to create a better model.

2.2. Novel ɑ function generalization

Vapor pressure and vapor-liquid equilibrium data can be accurately predicted by an appropriate alpha function of a cubic EoS, while the volume controls the accuracy of the predicted volumetric properties. Alpha functions have no importance or influence on the correlation of liquid molar volumes [20-21].

For complex reservoirs of fluids and natural gases, the alpha functions are defined as the pseudo characteristic constants. Based on the properties of different compounds, universal ɑ functions are proposed for simple liquids, and specific ɑ functions are specially studied for complex liquids and water [22].

According to Hamid [13], the statistical error analysis of the comparison of the adequate form of the new ɑ function gives high performance with an AAD% of 1.42%. It is written as follows:

$$\alpha(T) = exp\left[K_1 T_r + K_2 \ln T_r + K_3(1-\sqrt{T_r})\right] \tag{10}$$

$$K_1 = 0{,}013145\omega + 0{,}003091 \tag{11}$$

$$K_2 = 0{,}482173\omega + 0{,}006487 \tag{12}$$

$$K_3 = 3{,}586161\omega + 0{,}721306 \tag{13}$$

3. Database

A total of 149 data points were used in this work, which were obtained from the experimental analysis of several samples. Table 1 shows the data range used in our proposed model. The model was initially developed using 15 data points from Algeria's Hassi R'Mel field (Table 1). Following that, 134 various types of gas were tested for validation (10 different compositions and pressure, 91 different pressures, 31 different compositions and 2 types of random gas), are presented in Table 2.

Table 1. Compositions of different samples obtained from Hassi R'Mel field, Algeria (15 data).

Compounds	Min	Max	Compounds	Min	Max
C_1	0,7868	0,8012	nC_5	0,0043	0,0048
C_2	0,0735	0,0753	C_6	0,0047	0,0059
C_3	0,0283	0,0296	C_{7+}	0,01	0,0233
iC_4	0,0058	0,0062	N_2	0,0531	0,0542
nC_4	0,0103	0,011	CO_2	0,0017	0,0018
iC_5	0,0033	0,0036	He	0,0017	0,0017
P [Psi]	568,9336	4424,88107			
Z (Exp)	0,878	0,961			
T [R°]	653,67	653,67			

Table 2. Compositions of samples obtained from different types of gas (134 data).

Compounds	Min	Max	Compounds	Min	Max
C_1	0,5	0,98401	C_6	0	0,0059
C_2	0	0,133387	C_{7+}	0	0,0198
C_3	0	0,0685	N_2	0	0,0594
iC_4	0	0,007	CO_2	0,0014	0,5
nC_4	0	0,0133	H_2S	0	0,0053
iC_5	0	0,0035	O_2	0,0002	0,0074
nC_5	0	0,0029			
P [Psi]	14,50377	16632,93			
Z (Exp)	0,848	1,804			
T [R°]	554,67	900			

3.1. Natural gas types

To study a vast collection of compositions of natural gas, 149 types of natural gas with different compositions are presented in Tables A. The types of gases are distributed as follows:
- **Type 1**- A binary gas (50% CH_4 + 50% CO_2), (2 samples), Table A. 1.
- **Type 2**- Different pressures and different compositions (10 samples), Table A. 2.
- **Type 3**- Same composition and different pressures (91 samples), Table A. 3.
- **Type 4**- Same pressure and different compositions (31 samples), Table A. 4.
- **Type 5-**All types of gas (134 samples), Table. 2

4. Material and methods

Empirical correlations replace tables and graphs to determine (Z), some correlations can be used for quick calculations, while others are used for precise calculations under extended pressure and temperature conditions. The creation of the correlation is also influenced by the composition of the gas, non-hydrocarbon components, pressure and temperature range, and the size of the data used. This is why they are adjusted to fit only the data used, which may have additional limitations when other gas samples are used, and provide reassurance of large errors with huge uncertainties [23].

The expression (Eq. 14) is widely used in the petroleum industry, it is proposed by Edmister to estimate the acentric factor of pure fluids and petroleum fractions [24].

$$\omega = \frac{3[log(p_c/14{,}7)]}{7[(T_c/T_b)-1]} - 1 \qquad (14)$$

To calculate the acentric factor we used the equation (Eq. 14) which has the highest coefficient of determination R^2, equal to 0.9998 [25].

The precision of the results (Z_{PR}) provided by the original cubic equation of state PR (Eq. 5) and those calculated by Hamid [13] (Z_H) will be compared and discussed with our proposed modifications to the cubic equation of state PR (Eq. 10).

4.1. Work organization

This study consists in developing new coefficients from data collected in the field of Hassi R'mel, Algeria, by performing the following steps:

Step 1: detecting the problem

We start the calculation of the factor (Z_{PR}) with the cubic equation of state PR (Eq. 5) then the regression analysis to detect the problem and the variables influencing (Z).

Defining the problem is the most important step in regression analysis. So, the Z values of 15 samples from the Hassi R'Mel field were calculated using the cubic equation of state PR (Eq. 5) and compared with experimental Z values.

Step 2: Suggest mathematical equations

Many mathematical modifications for obtaining the coefficient of equation (Eq. 5) have been proposed to establish the closest relationship between experimental and predicted data [13]. In order to achieve that objective, we modify the coefficients of the Peng-Robinson equation (Eq. 5) using the adequate curve approach and the coefficient of determination (R^2).

The explicit equation proposed to predict the Z factor is as follows:

$$Z^3 + A'Z^2 + B'Z - C' = 0 \qquad (15)$$

where:

$$A' = (1-B)+\xi_1 \qquad (16)$$

$$B' = (A-2B-3B^2)+\xi_2 \qquad (17)$$

$$C' = (AB-B^2-B^3)+\xi_3 \qquad (18)$$

The coefficients of correctness ξ_1, ξ_2, and ξ_3 are mentioned in Table 3 and were determined by linear regression for a pressure margin where they are not different and constant, each pressure margin is 725,18 psi (50 bar).

4.2. Evaluation of the correlations

In this study, the proposed equation (Eq. 15) is based on the new coefficients ξ_1, ξ_2, and ξ_3 of the three adjustable parameters A′, B′ and C′. The values of A′, B′ and C′ are specific to the compound and must be optimized for each type of gas. This equation is developed from several types of gas (149 types), starting with the data obtained from the field of Hassi R'Mel, Algeria, as a starting point, and then validating the equation on 134 types of gas. The quantitative evaluation methods used were statistical error analysis (AAD %) and coefficient of determination (R^2). In addition, performance curves will be used as a qualitative evaluation method, i.e, a graph showing predicted properties against experimental data with a 45° reference line. The best correlation is one that has good performance and represents a right of 45° of slope [26].

4.3. Parameter fitting

Adjustable parameters for the cubic equation of state of PR (Eq. 5) are adapted to the different types of natural gas in the range of experimental pressures. The adjustment of a calculated variable with respect to another experimental one is obtained by linear regression. For this purpose, a linear correlation coefficient (R^2) is calculated, which indicates the higher values for one correspond "on average" to higher or lower values for the other. The results of the adjustment of the parameters are mentioned in Table 3.

Table 3. Values of the suggested coefficients for the variable parameters A′, B′ and C′.

	P (psi)	ξ_1	ξ_2	ξ_3
1	14.5-725.18	[-0.0005 ;-0.015]	[-0.0005 ;-0.0019]	[-0.0005 ;-0.0025]
2	725.18-1450.37	[-0.02 ;-0.04]	[-0.002 ;-0.005]	[-0.003 ;-0.005]
3	1450.37-2175.56	[-0.02 ;-0.14]	[-0.005 ;-0.017]	[-0.005 ;-0.017]
4	2175.56-2900.75	[-0.025 ;-0.2]	[-0.008 ;-0.017]	[-0.008 ;-0.017]
5	2900.75-3625.94	[-0.14 ;-0.25]	[-0.017 ;-0.02]	[-0.017 ;-0.02]
6	3625.94-4351.13	[-0.2 ;-0.3]	[-0.019 ;-0.032]	[-0.019 ;-0.032]
7	4351.13-5076.32	[-0.22 ;-0.32]	[-0.025 ;-0.035]	[-0.025 ;-0.035]
8	5076.32-5801.51	[-0.25 ;-0.4]	[-0.029 ;-0.035]	[-0.029 ;-0.035]
9	5801.51-6526.7	[-0.3 ;-0.45]	[-0.029 ;-0.035]	[-0.029 ;-0.035]
10	6526.7-7251.88	[-0.3 ;-0.55]	[-0.029 ;-0.035]	[-0.029 ;-0.035]
11	7251.88-7977.076	[-0.35 ;-0.55]	[-0.029 ;-0.04]	[-0.029 ;-0.04]
12	7977.076-8702.26	[-0.35 ;-0.55]	[-0.025 ;-0.035]	[-0.025 ;-0.035]
13	8702.26-9427.45	[-0.4 ;-0.6]	[-0.03 ;-0.035]	[-0.03 ;-0.035]
14	9427.45-10152.64	[-0.35 ;-0.6]	[-0.029 ;-0.035]	[-0.029 ;-0.035]
15	10152.64-10877.83	[-0.4 ;-0.6]	-0.035	-0.035
16	10877.83-11603.02	[-0.35 ;-0.6]	[-0.029 ;-0.04]	[-0.029 ;-0.04]
17	11603.02-12328.21	[-0.4 ;-0.6]	[-0.03 ;-0.035]	[-0.03 ;-0.035]
18	12328.21-13778.59	[-0.3 ;-0.6]	[-0.025 ;-0.035]	[-0.025 ;-0.035]
19	13778.59-14503.77	[-0.3 ;-0.5]	[-0.025 ;-0.03]	[-0.025 ;-0.03]
20	≥14503.77	-0.3	-0.025	-0.025

Furthermore, to validate the calculated results, the (Z) values obtained from the PR EOS (Z_{PR}), Hamid [13] (Z_H) (under the generalised form for the parameters of the α function), and the modified PR EOS (Z_m) are listed in Tables A. 5-9 of the supplementary data associated with this article.

Statistical error analysis is based on the mean absolute deviation percentage (AAD%) defined by the following equation:

$$AAD\% = \frac{1}{n}\sum_{i=1}^{n}\left(\left|\frac{Z_{\text{Calculated}}-Z_{\text{Experimental}}}{Z_{\text{Experimental}}}\right|_i\right)*100 \quad (19)$$

On the other hand, the coefficient of determination (R²) was used, which determines the validity of the results and illustrates how effectively the regression analysis succeeded in reducing the standard deviation. This could be tested by calculating the adequacy between the obtained results from the proposed model and the experimental data (or to what extent the regression equation is adapted to describe the distribution of points), which is defined as follows:

$$R^2 = 1 - \left[\sum_{i=1}^{n}\left(Z_{\text{Calculated}} - Z_{\text{Experimental}}\right)_i^2 / \sum_{i=1}^{n}\left(Z_{\text{Calculated}} - Z_{avg}\right)_i^2\right] \tag{20}$$

$$Z_{avg} = \left(\sum_{i=1}^{n} Z_{\text{Experimental}}\right)_i / n \tag{21}$$

5. Results and discussions

From Figure 1.a, it can be seen that the values of factor (Z) calculated by the suggested modifications are approximate to the experimental data. Moreover, a comparison between Z_{exp} and Z_m shows a high value of the coefficient of determination R^2 equal to 0,9877 as can be seen in Figure 1. b.

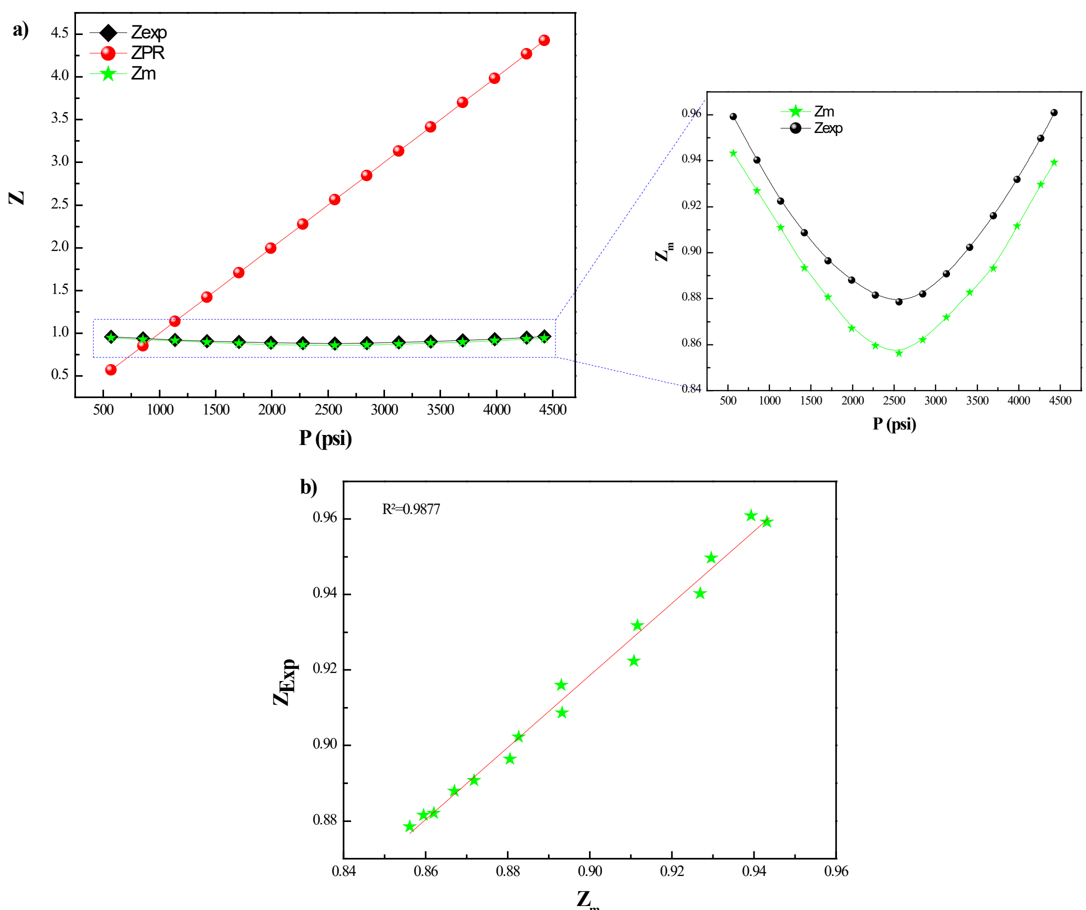

Fig. 1. a) Evolution of Z_{exp}, Z_{PR} and Z_m with respect to pressure. b) Experimental data (Z_{exp}) vs. data Predicted (Z_m). Hassi R'Mel field, Algeria.

For each type of gas, a statistical analysis was calculated and listed in Tables A. 6-9, followed by an overall analysis for all the samples.

As clearly shown in Figures 2. a-c the developed Z_m in this study presented better regularity in the estimation of Z with the highest coefficient of determination (R^2 = 0,9968) compared to the evolution of the experimental data (Z_{exp}) as a function of Z_{PR} and Z_H.

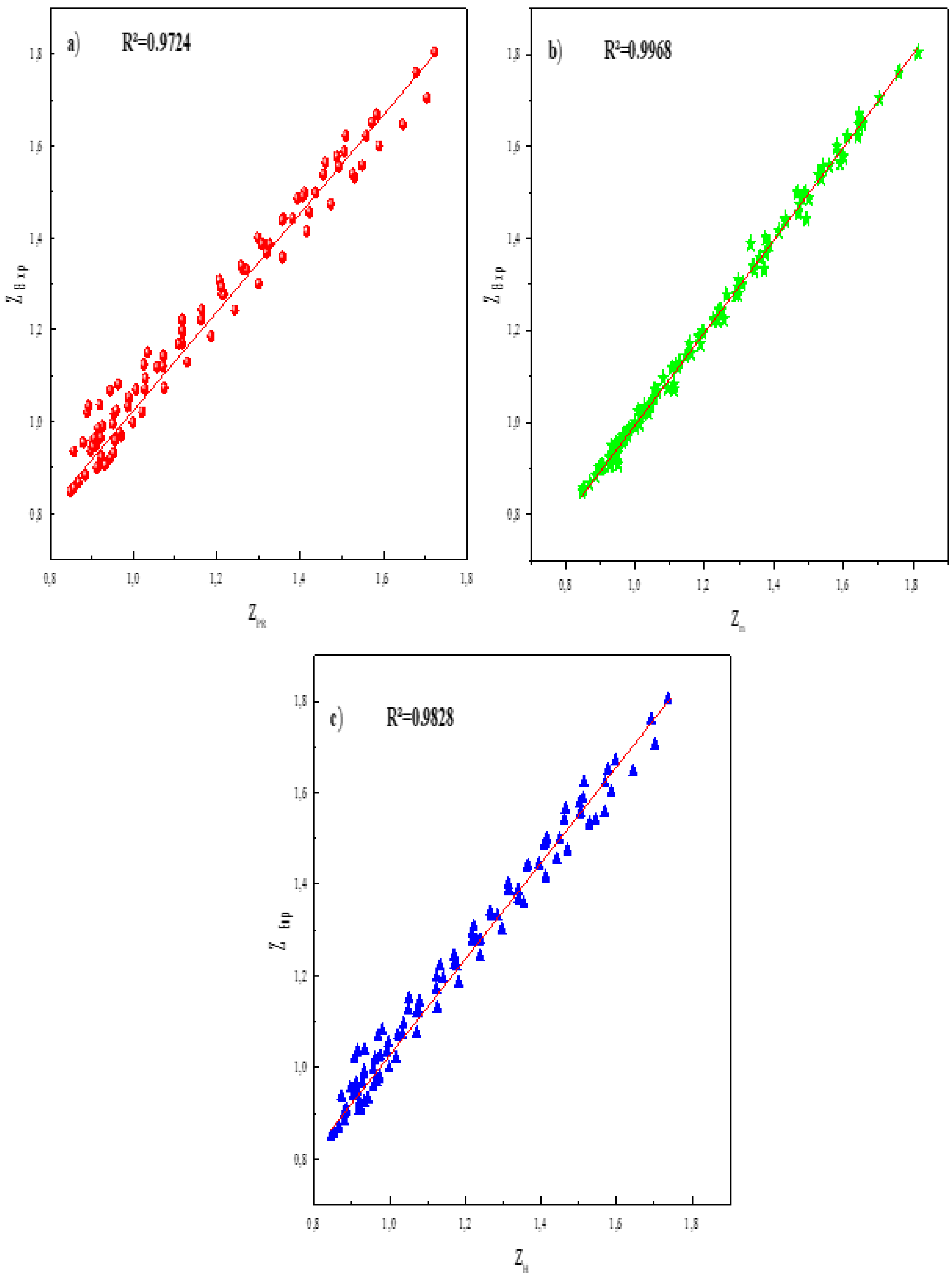

Figure 2. a) Experimental data (Z_{exp}) vs. data Predicted (Z_{PR}). All Types of gas. b) Experimental data (Z_{exp}) vs. data Predicted (Z_m). All Types of gas. c) Experimental data (Z_{exp}) vs. data Predicted (Z_H). All Types of gas.

According to the figure 3, it can be seen that The values of Z_m were found to be significantly smaller than Z_{PR} and Z_H, the absolute error is minimal for all types of gas (type 2 ; type 3 and type 5) except type 1 because it is a binary type (synthesis gas) and type 4, gases of the same pressure and different composition (rarely exists in the gas processing industry).

From the values of AAD% and the cumulative frequency analysis listed in Figure 3 conducted for all types of gas, the proposed modifications in this study give the smallest average absolute error (AAD% = 0,992%).

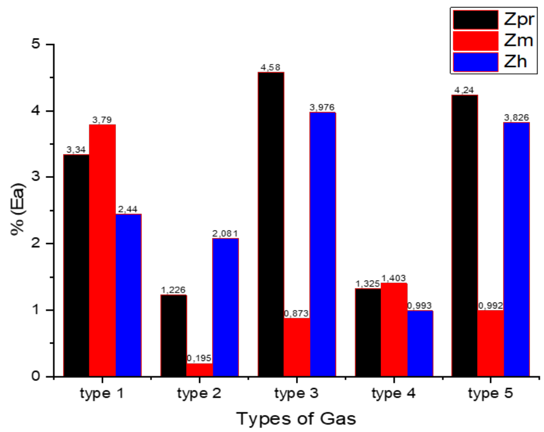

Figure 3. The average absolute deviation percent (AAD %) of different types of gas.

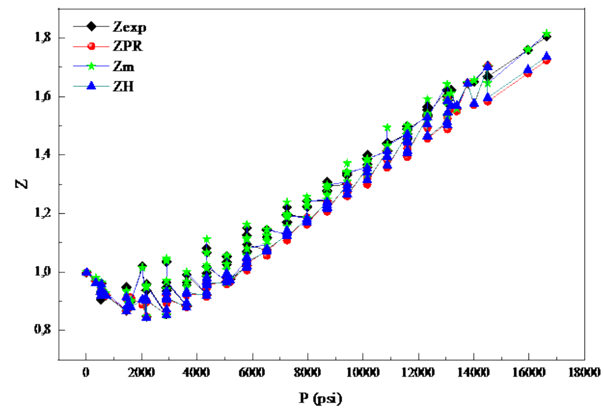

Figure 4. Chart present comparison between Z_{exp}, Z_{PR}, Z_m and Z_H of all Types of gas.

Figure. 4 shows the performance charts of Z versus pressure of the different equations (i.e. Z_{exp}, Z_{PR}, Z_m, and Z_H) in this study compared to experimental data. The Z_m developed in this study shows an improvement in the obtained (Z) values and good performance, which provides a more reliable ability to predict the factor (Z) in a wide range of pressure conditions.

6. Conclusion

This study aimed to identify the gas compressibility factor (Z) and model it using the Peng Robinson cubic state equation, which is widely used in the oil sector. It also aims to evaluate the accuracy of the equation for its applicability in the field of compressibility factor prediction of natural gas.

The development of this equation of state is based on the compressibility factor (Z) data extracted from the different types of gas with different groups. For this, the study has the following steps:

- A study of the statistical errors of the application of Peng Robinson's original cubic equation of state on the Hassi R'Mel field to discover the limits of its use.
- Modifying the Peng Robinson coefficients with what conforms to the Algerian gas Hassi R'Mel field.
- Applying the proposed Peng Robinson equation to different types of gas, accompanied by a statistical errors analysis of the obtained results.

Using statistical evaluations, the new proposed equation is compared to commonly used equations. The results obtained indicate that the new equation is closer to experimental results than other equations. In another form, the equation suggested has a minimum average absolute error percentage and a maximum correlation coefficient compared to those of the other equations studied. From this study, we concluded that:

- The new equation must be an explicit function of (Z) and does not require any analysis to solve it.
- The Peng-Robinson cubic equation of state is the most reliable among the other equations of state and for this reason; it is widely used in industry, particularly for refining and reservoir simulation.

- According to the results of the statistical parameters, the new modification outperformed the other equations by a better ranking of R^2 = 0, 9968 and a better performance graph compared to the existing empirical data.
- The results proved that we cannot use an equation or correlation over the global pressure and temperature ranges of a gas type to calculate (Z) for process simulation and/or other analysis, such as material balance and volumetric estimation.

Appendix A. Data

Table A. 1. Binary gas, 2 data).

Compounds	Min	Max
C_1	0,5	0,5
CO_2	0.5	0.50
P (psi)	725.1887	875.3054
Z (Exp)	0.8864	0.92
T [R°]	716.67	900

Table A. 2. Different pressures and different composition (Hassi R'Mel field, 10 data).

Compounds	Min	Max	Compounds	Min	Max
C_1	0.8327	0.8429	nC_5	0.0009	0.0028
C_2	0.0516	0.0721	C_6	0.0003	0.0039
C_3	0.0191	0.021	C_{7+}	0.0198	0.0198
iC_4	0.0025	0.0041	N_2	0.0335	0.0594
nC_4	0.0043	0.007	CO_2	0.0021	0.0176
iC_5	0.0008	0.0028	H_2S	0	0.0053
P (psi)	1485.202	5200			
Z (Exp)	0.899	0.976			
T [R°]	593.784	679.67			

Table A. 3. Compositions of studied samples with same composition and different pressures (Field GASSI TOUIL; H.LIU and ALL / Fluid Equilibria 500 (2019) 112256, 91 data).

Compounds	Min	Max
C_1	0.81493	0.98401
C_2	0.00338	0.12042
C_3	0.00018	0.02606
iC_4	0	0.00445
nC_4	0	0.007
iC_5	0	0.0028
nC_5	0	0.0028
C_6	0	0.0039
C_{7+}	0	0.0198
N_2	0.00302	0.0335
CO_2	0.00565	0.01782
H_2S	0	0.0053
P (psi)	14.50377	16632.93
Z (Exp)	0.848	1.804
T [R°]	679.67	787.77

Table A. 4. Compositions of studied samples with same pressure and different compositions (Field Gasi Touil; MLN field (Hassi Berkin), 31 data).

Compounds	Min	Max
C_1	0.7656	0.88605472
C_2	0.07810569	0.13338707
C_3	0.0054	0.0685
iC_4	0.0002	0.007
nC_4	0	0.0133
iC_5	0	0.0035
nC_5	0	0.0029
C_6	0	0.0059
N_2	0.0002	0.02717
CO_2	0.0014	0.0179
O_2	0.0002	0.0074
P (psi)	348.09	537.654
Z (Exp)	0.906	0.970
T [R°]	554.67	557.676

Table A. 5. Compressibility Factor (Z_{exp}), (Z_{PR}) and (Z_m) from the Hassi R'Mel field, Algeria, in the pressure range of 568,934 to 4,424.88 PSI.

	P(psi)	Z_{exp}	Z_{PR}	Z_m
1	568.934	0.9592	0.5689	0.9432
2	853.4	0.9403	0.8534	0.9269
3	1137.87	0.9224	1.1379	0.9108
4	1422.33	0.9086	1.4223	0.8933
5	1706.8	0.8964	1.7068	0.8806
6	1991.27	0.8879	1.9913	0.867
9	2275.73	0.8815	2.2757	0.8595
7	2560.2	0.8785	2.5602	0.8562
8	2844.67	0.882	2.8447	0.862
9	3129.13	0.8907	3.1291	0.8718
10	3413.6	0.9023	3.4136	0.8827
12	3698.07	0.916	3.6981	0.8931
13	3982.54	0.9318	3.9825	0.9116
14	4267	0.9497	4.2670	0.9296
15	4424.88	0.9609	4.4249	0.9393

Table A. 6. Statistical analysis of errors and compressibility factor Z (type 1).

	P(psi)	Z_{exp}	Z_{PR}	Z_m	Z_H
Binary gas	725.1887	0.92	0.92	0.9289	0.9184
	875.3054	0.8864	0.9457	0.9451	0.9282
AAD%			3.34	3.79	2.44

Table A. 7. Statistical analysis of errors and compressibility factor Z (type 2).

	P(psi)	Z_{exp}	Z_{PR}	Z_m	Z_H
01	5200	0.976	0.9658	0.9768	0.9725
02	1613.069	0.900	0.9114	0.9012	0.8797
03	1594.721	0.899	0.9119	0.8956	0.8794
04	1539.25	0.902	0.9132	0.9007	0.8815
05	1519.053	0.903	0.9142	0.9028	0.8816
06	1589.316	0.902	0.9134	0.8994	0.8816
07	1574.951	0.902	0.9131	0.8994	0.8820
08	1574.097	0.902	0.9131	0.8995	0.8818
09	1485.202	0.905	0.9162	0.9067	0.8854
10	1494.589	0.904	0.9155	0.9053	0.8842
AAD%			1.226	0.195	2.081

Table A. 8. Statistical analysis of errors and compressibility factor Z (type 3).

	P(psi)	Zexp	ZPR	Zm	ZH
1	14503.77	1.704	1.7040	1.7039	1.7009
2	13778.59	1.646	1.6464	1.6464	1.6433
3	13053.4	1.6	1.5888	1.5812	1.5855
4	12328.21	1.531	1.5311	1.5362	1.5277
5	11603.02	1.473	1.4733	1.4733	1.4699
6	10877.83	1.415	1.4155	1.4137	1.4120
7	10152.64	1.358	1.3578	1.3585	1.3542
8	9427.453	1.3	1.3002	1.3089	1.2965
9	8702.264	1.243	1.2428	1.2478	1.2391
10	7977.076	1.186	1.1858	1.1839	1.1820
11	7251.887	1.13	1.1295	1.1281	1.1256
12	6526.698	1.074	1.0743	1.0938	1.0702
13	5801.51	1.021	1.0207	1.0395	1.0165
14	5076.321	0.97	0.9698	0.9639	0.9654
15	4351.132	0.923	0.9233	0.9259	0.9187
16	3625.943	0.884	0.8839	0.8864	0.8793
17	2900.755	0.856	0.8566	0.8522	0.8520
18	2175.566	0.848	0.8485	0.8479	0.8443
19	1450.377	0.868	0.8681	0.8693	0.8650
20	725.1887	0.92	0.9200	0.9289	0.9184
21	14.50377	0.998	0.9981	0.9997	0.9981
22	13385.53	1.5571	1.5494	1.5591	1.5684
23	13053.4	1.5385	1.5256	1.5342	1.5448
24	11603.02	1.4549	1.4222	1.471	1.4421
25	10152.64	1.3669	1.3192	1.3753	1.3400
26	8702.264	1.2776	1.2174	1.2975	1.2391
27	7251.887	1.194	1.1182	1.1966	1.1410
28	5801.51	1.1247	1.0248	1.1173	1.0486
29	4351.132	1.0676	0.9441	1.0654	0.9683

	P(psi)	Zexp	ZPR	Zm	ZH
30	2900.755	1.035	0.8924	1.039	0.9148
31	2030.528	1.021	0.8883	1.0143	0.9069
32	16632.93	1.8044	1.7233	1.8153	1.7360
33	15954.15	1.7598	1.6787	1.7602	1.6915
34	14503.77	1.6686	1.5832	1.6451	1.5965
35	13053.4	1.5767	1.4878	1.6003	1.5016
36	11603.02	1.4857	1.3928	1.5007	1.4071
37	10152.64	1.4001	1.2987	1.3751	1.3134
38	8702.264	1.3081	1.2061	1.2973	1.2215
39	7251.887	1.222	1.1168	1.239	1.1327
40	5801.51	1.1505	1.0338	1.1622	1.0501
41	4351.132	1.0814	0.9634	1.1122	0.9796
42	2900.755	1.0373	0.9192	1.047	0.9336
43	13037.44	1.6221	1.5092	1.6418	1.5140
44	12328.21	1.5636	1.4602	1.5906	1.4650
45	11603.02	1.4992	1.4101	1.4664	1.4150
46	10877.83	1.4416	1.3601	1.4937	1.3651
47	10152.64	1.3878	1.3104	1.3328	1.3155
48	9427.453	1.3325	1.2610	1.3719	1.2662
49	8702.264	1.2775	1.2121	1.2893	1.2175
50	7977.076	1.2238	1.1639	1.2571	1.1694
51	7251.887	1.1703	1.1167	1.1883	1.1223
52	6526.698	1.12	1.0709	1.1242	1.0766
53	5801.51	1.0706	1.0272	1.1121	1.0330
54	5076.321	1.0331	0.9866	1.0268	0.9925
55	4351.132	0.996	0.9505	1.0134	0.9565
56	3625.943	0.965	0.9213	0.9683	0.9271
57	2900.755	0.9463	0.9020	0.9405	0.9074
58	2175.566	0.9368	0.8969	0.9472	0.9016
59	1450.377	0.9483	0.9104	0.9335	0.9139
60	13181.03	1.6216	1.5575	1.6112	1.5692
61	12328.21	1.5549	1.4924	1.5414	1.5044
62	11603.02	1.4983	1.4370	1.4896	1.4493
63	10877.83	1.4424	1.3817	1.4329	1.3942
64	10152.64	1.3867	1.3264	1.3854	1.3393
65	9427.453	1.3315	1.2713	1.3437	1.2845
66	8702.264	1.2766	1.2165	1.2626	1.2300
67	7977.076	1.2223	1.1622	1.2262	1.1761
68	7251.887	1.1694	1.1087	1.1565	1.1230
69	6526.698	1.1185	1.0563	1.1061	1.0710
70	5801.51	1.0697	1.0057	1.056	1.0209
71	5076.321	1.0248	0.9580	1.0071	0.9737
72	4351.132	0.9856	0.9150	0.9823	0.9310

	P(psi)	Zexp	ZPR	Zm	ZH
73	3625.943	0.9547	0.8794	0.9469	0.8955
74	2900.755	0.9359	0.8558	0.9412	0.8715
75	14010.65	1.6503	1.5711	1.6558	1.5764
76	13053.4	1.5873	1.5055	1.5853	1.5110
77	12328.21	1.5383	1.4559	1.5281	1.4615
78	11603.02	1.4894	1.4064	1.4803	1.4121
79	10877.83	1.4385	1.3571	1.4316	1.3628
80	10152.64	1.3868	1.3080	1.3863	1.3139
81	9427.453	1.3397	1.2592	1.3398	1.2652
82	8702.264	1.2933	1.2110	1.2971	1.2171
83	7977.076	1.2436	1.1634	1.2415	1.1697
84	7251.887	1.1967	1.1169	1.1941	1.1233
85	6526.698	1.1437	1.0719	1.1462	1.0784
86	5801.51	1.0944	1.0289	1.081	1.0356
87	5076.321	1.0532	0.9890	1.0561	0.9958
88	4351.132	1.0181	0.9537	1.0251	0.9604
89	3625.943	0.9908	0.9250	1.0006	0.9316
90	2900.755	0.9641	0.9062	0.9693	0.9123
91	2175.566	0.959	0.9011	0.954	0.9063
AAD%			4.58	0.873	3.976

Table A. 9. Statistical analysis of errors and compressibility factor Z (type 4).

	P(psi)	Zexp	ZPR	Zm	ZH
1	348.09	0.97	0.9692	0.9798	0.9620
2	348.09	0.97	0.9698	0.9805	0.9629
3	348.09	0.97	0.9685	0.9791	0.9612
4	348.09	0.97	0.9690	0.9797	0.9619
5	348.09	0.97	0.9692	0.9798	0.9620
6	348.09	0.97	0.9693	0.9799	0.9621
7	348.09	0.97	0.9691	0.9798	0.9620
8	348.09	0.97	0.9692	0.9798	0.9620
9	348.09	0.9694	0.9694	0.9695	0.9623
10	348.09	0.97	0.9698	0.9699	0.9627
11	348.09	0.9701	0.9701	0.9701	0.9631
12	348.09	0.97	0.9697	0.9698	0.9626
13	348.09	0.9702	0.9702	0.9703	0.9632
14	348.09	0.9701	0.9701	0.9702	0.9631
15	348.09	0.9701	0.9701	0.9702	0.9631
16	348.09	0.9701	0.9701	0.9702	0.9631
17	537.654	0.9093	0.9327	0.9278	0.9190
18	537.654	0.9233	0.9451	0.9418	0.9331
19	537.654	0.9084	0.9333	0.951	0.9197
20	537.654	0.9094	0.9353	0.9308	0.9220
21	537.654	0.9314	0.9522	0.9497	0.9412

	P(psi)	Zexp	ZPR	Zm	ZH
22	537.654	0.9098	0.9343	0.9296	0.9208
23	537.654	0.911	0.9354	0.9308	0.9220
24	537.654	0.9108	0.9354	0.9308	0.9220
25	537.654	0.9107	0.9355	0.9309	0.9221
26	537.654	0.9099	0.9345	0.9298	0.9210
27	537.654	0.9085	0.9343	0.9296	0.9207
28	537.654	0.9096	0.9351	0.9305	0.9216
29	537.654	0.906	0.9316	0.9265	0.9177
30	537.654	0.9065	0.9315	0.9263	0.9175
31	537.654	0.906	0.9317	0.9266	0.9178
AAD%			1.325	1.403	0.993

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