

## DETERMINATION OF COLLAPSE AND FRACTURE PRESSURES FOR WELLBORE STABILITY DESIGN BASED ON THE MOGI-COULOMB FAILURE CRITERION

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### **Abstract**

Wellbore stability is one of the crucial issues in oil and gas industries. The issues related to instability of wells, impose significant unwanted costs on drilling operation. Hence, in many oil companies wellbore stability analysis is one of the major activities in the well design stage.

The objective of this paper is to present the 3D wellbore stability prediction models for vertical wellbores. The Mogi-Coulomb strength criterion conjunction with linear poroelastic constitutive model is utilized to develop the models. This models leads to easily computed expression for the critical mud pressure required to maintain wellbore stability. The analytical model is applied to real field case in order to verify the applicability of the developed models.

The results indicate that the increasing the UCS and/or Poisson's ratio will increase the optimum mud pressure window. Furthermore, this window is function of depth.

Such predictions are necessary for providing technical support for well drilling decision-making and predicting the condition at which borehole instability occurs.

**Keywords:** Wellbore stability; Mud pressure; Mogi-Coulomb criterion; Analytical model; Constitutive model.

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### **1. Introduction**

When a well is drilled, the rock surrounding the borehole must take up the load previously supported by the rock that has been removed. This results in the development of a stress concentration at the borehole wall. If the rock is not strong enough, the wall will fail [1].

The integrity of the wellbore plays an important role in petroleum operations. The hole failure problems cost the petroleum industry several billions of dollars each year. Prevention of wellbore failure requires a strong understanding of the interaction between formation strength, in-situ stresses, and drilling practices. As in-situ stress and rock strength cannot be easily controlled, adjusting the drilling practices is the usual way to inhibit wellbore failure [2-3].

During drilling, there are two types of mechanical borehole failure: compressive and tensile failures. Compressive failure occurs when the wellbore pressure is too low compared with the rock strength and the induced stresses. On the other hand, tensile failure occurs when the wellbore pressure is too high [4].

The main aspect of the wellbore stability analysis is to mitigate these drilling problems [5]. In order to avoid borehole failure, drilling engineers should adjust the stress concentration properly through altering the applied mud pressure and the orientation of the borehole with respect to the in-situ stresses. Since borehole allocation (in terms of orientation) is limited, proper adjusting of mud weight (borehole pressure) will play an essential role in prevention of drilling problems [6-7]. The true mud pressure in the borehole depends on the static weight of the mud column increased by the dynamic effect of the flow (known as ECD – Effective Circulating Density), together with occasional fluctuations as the drill string moves (pistoning or suction). In view of these fluctuations, the borehole stability conditions are often borderline [1].

In engineering practice, a linear poroelasticity stress model in combination with a rock strength criterion is commonly used to determine the minimum and maximum mud pressures required for ensuring wellbore stability. Therefore, a main aspect of wellbore stability analysis is the

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selection of an appropriate rock strength criterion. So far, the two most commonly used strength criteria in wellbore stability analysis are the Mohr–Coulomb criterion and the Drucker–Prager criterion [8]. Researchers have found that these two strength criteria can give very different minimum mud pressures. Mohr–Coulomb criterion suffers from two major limitations: (a) it ignores the non-linearity of strength behavior, and (b) the effect of intermediate principal stress is not considered in its conventional form. Thus, the criterion overestimates the minimum mud pressure due to neglecting the effect of the intermediate principal stress. In other hand, the Drucker–Prager criterion underestimates the minimum mud pressure because it exaggerates the intermediate principal stress effect [9].

Zhou [10] introduced a modified Wiebols and Cook [11] criterion and developed a computer program for the wellbore stability analysis. The results indicated the importance of the intermediate principal stress on the stability of wellbores. Ewy developed the Modified-Lade failure criterion and presented the advantages of this new criterion over Mohr–Coulomb and Drucker–Prager [12]. Colmenares and Zoback evaluated seven different rock failure criteria based on polyaxial test data, and they concluded that the Modified Lade and the Modified Wiebols and Cook fit best with polyaxial test data [13]. Aadnoy [14] developed an analytical solution to study the stability of inclined wellbores drilled into rock formations modeled as a transversely isotropic material. He showed that neglecting the anisotropic effects arising from the directional elastic properties can result in errors in the wellbore stability analysis. Al-Ajmi and Zimmerman [15] developed the Mogi–Coulomb failure criterion, according to polyaxial failure data of the variety of rocks. They concluded that Mohr–Coulomb failure criterion is conservative in estimating of collapse pressure during drilling and using Mogi–Coulomb failure criterion can minimize the conservative nature of the mud pressure predictions.

In this paper, the 3D Mogi–Coulomb strength criterion developed by is used to analyze wellbore stability. Furthermore, the analytical models are applied to field data in order to verify the applicability of the developed models.

## 2. Geological background of studied area

A vertical well (called well A) drilled in one of oil fields located in the south west of Iran. This oil field is one of the most important Iranian super giant oil fields, was discovered in 1956 and now has more than 450 producing wells. This oil field has an anticline structure 72 km long and 6 km wide with NW–SE trending symmetrical anticlinal, located in central part of north Dezful region .

The Dezful Embayment is a depressed area within the Zagros Folded Belt. This embayment represents a foreland basin where subsidence at the foot of the uplifting Mountain Front Fault has resulted in the deposition of thick post-Oligocene sediments, including up to 3000 m of Upper Miocene–Pliocene Aghajari and Bakhtiyari formations. In this region, the Miocene evaporites of the Gachsaran formation horizon which forms a very good seal for the Asmari reservoirs, acted as a major detachment. This thick incompetent unit decoupled the folds above it, which are currently exposed at the surface, from the underlying folds, which host the majority of the hydrocarbons in the Iranian sector of the Zagros [16]. The sediments composing the Dezful Embayment are up to 12000 m thick and, except for the Devonian and Carboniferous systems; the section is a nearly continuous, conformable sequence from the Infra-Cambrian to the Pliocene. Sedimentation began with important Infra-Cambrian (Vendian) evaporites, followed by the shallow marine carbonate and clastic deposits of the Lower Paleozoic from the Permian and throughout most of the Mesozoic and up to Lower Miocene; the area was part of a broad, shallow carbonate platform. Subsequently, thick evaporates followed by continental red beds characterize the Mio-Pliocene. Folding accompanied by syn-tectonic and posttectonic molasses took place in Plio-Pleistocene time [17].

Its main reservoir is the Asmari formation and Bangestan Group with the production rate of 1000,000 barrels/day [18]. The Bangestan reservoir is one of the carbonate reservoirs in Southern of Iran, providing approximately 5% of the total production of the southern oil field region. Because of a sufficient amount of oil in place and the good quality of porosity with low

permeability and flowing capacity in some of the production layers, it is a good candidate for a hydraulic fracturing operation [17].

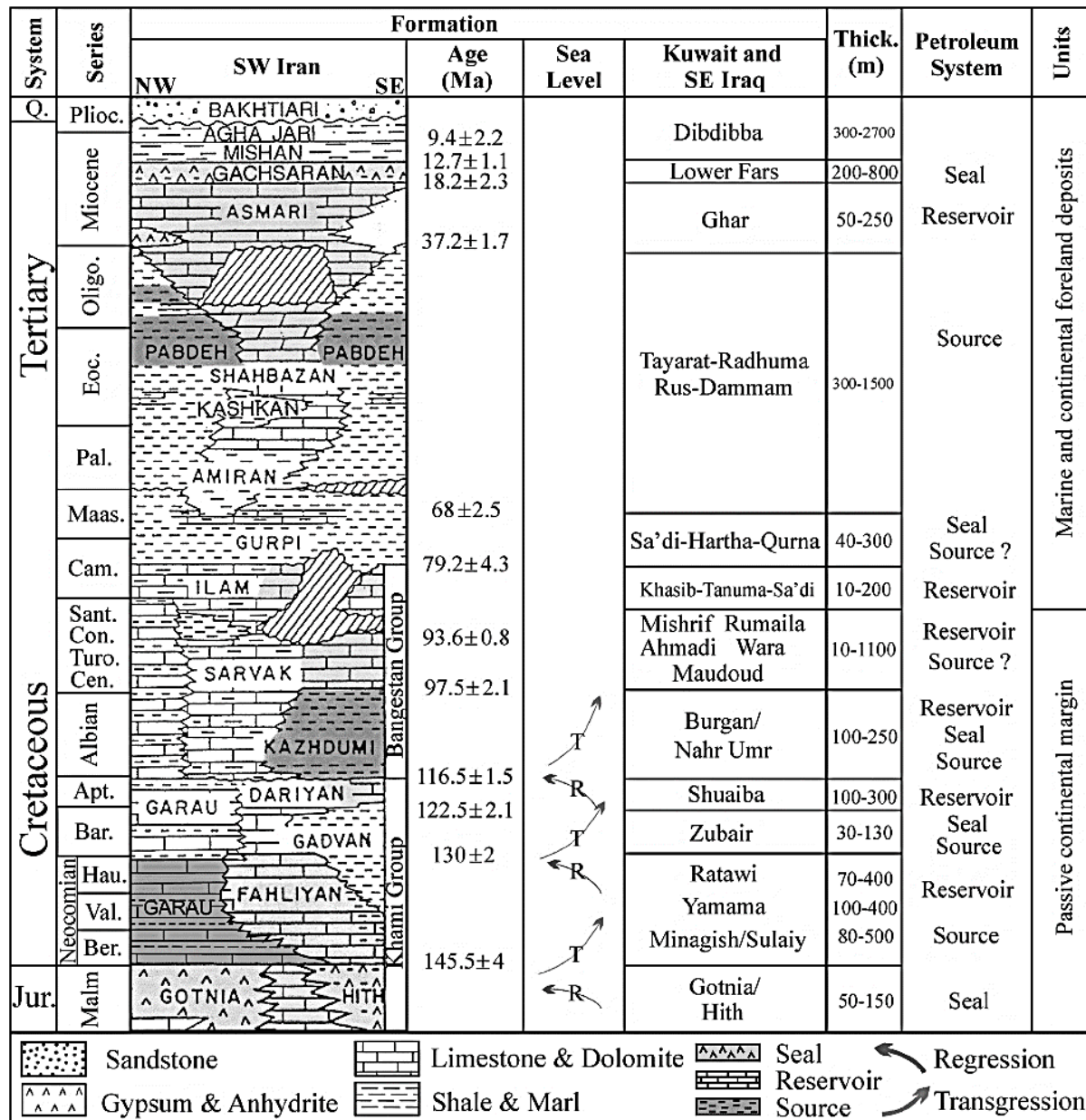


Fig. 1. Schematic lithostratigraphic units, their age, thickness, lateral facies changes in southwest Iran and their time equivalents in southeast Iraq and Kuwait [19]

## 2. Stress concentration around a vertical wellbore at drilling condition

Drilling a borehole will alter the in situ principal stresses, the vertical stress and the maximum and minimum horizontal stresses, in a manner so as to maintain the rock mass in a state of equilibrium. This leads to a stress concentration around the wellbore [20].

The degree of stress concentration depends on the wellbore orientation, the magnitude and orientation of in-situ stresses, and the wellbore pressure. When the elevated stress exceeds the rock strength, the rock will fail resulting in the development of wellbore failure [21]. If excessive, the cavings produced by the spalling of broken materials into the wellbore can

cause drilling problems such as pack-off, over-pulls, stuck-pipe and poor cementing, to name a few [22].

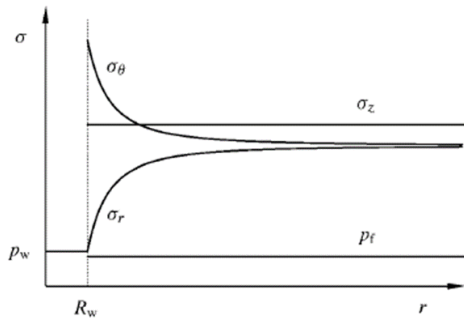


Fig. 2. Stresses around a vertical borehole in a linear elastic formation [26]

To evaluate the stability of a wellbore, a constitutive model is required to compute the stresses around the borehole [23]. Although different constitutive models are available, the linear poroelasticity stress model is commonly used in industry practice [24]. Assuming that the formation behaves like brittle rock, stability analysis in drilling condition, required to compare principal stress around the borehole with an appropriate failure criterion to see if conditions for a wellbore instability will be

fulfilled or not. Based on linear elasticity, maximum stresses, occur in the wellbore wall (Fig. 2). Therefore, borehole instability is expected to initiate at the borehole wall [25].

The stress concentration around a vertical well drilled in an isotropic, elastic medium under anisotropic in-situ stress condition (Maximum and minimum horizontal stresses are different) is described by the Kirsch equations. The general expressions for the effective stresses at the vertical wellbore wall in the drilling situation are [26]:

$$\begin{aligned} \sigma'_r &= P_w - \alpha P_p \\ \sigma'_\theta &= \sigma_H + \sigma_h - 2(\sigma_H - \sigma_h)\cos 2\theta - P_w - \alpha P_p \\ \sigma'_z &= \sigma_v - 2\nu(\sigma_H - \sigma_h)\cos 2\theta - \alpha P_p \end{aligned} \tag{1}$$

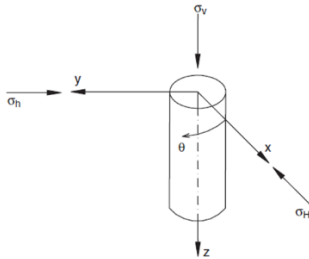


Fig. 3. Coordinate system for a vertical borehole [15]

where  $\sigma_H$  and  $\sigma_h$  are the maximum and minimum horizontal stresses, respectively;  $\sigma_v$  is the vertical stress;  $\alpha$  is Biot's coefficient;  $\nu$  is the Poisson's ratio of the rock;  $\sigma'_r$  is the effective radial stress;  $\sigma'_\theta$  is the effective tangential (hoop) stress;  $\sigma'_z$  is the effective axial stress induced around the wellbore;  $P_w$  is the wellbore pressure;  $P_p$  is the farfield pore pressure and  $\theta$  is the angular position around the wellbore circumference and measured clockwise from the azimuth of maximum horizontal stress (Fig. 3).

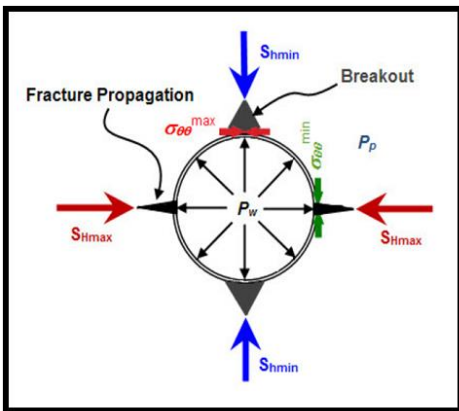


Fig. 4. The location of breakout and tensile fractures on the borehole wall [27]

According to Eqs. (1) the tangential and axial stresses are functions of the angle  $\theta$ . This angle indicates the orientation of the stresses around the wellbore circumference, and varies from  $0^\circ$  to  $360^\circ$ . Consequently, the tangential and axial stresses will vary sinusoidally. The tangential and radial stresses are functions of the well pressure, but the vertical stress is not. Therefore, any change in the mud pressure will only influence the tangential and radial stresses. Inspection of these equations reveals that both tangential and axial stresses reach a maximum value at  $\theta = 90^\circ, 270^\circ$  and a minimum value at  $\theta = 0^\circ, 180^\circ$ . The shear failure known as breakouts is expected to happen at the point of maximum tangential stress where the rock is under maximum compression. Tensile failure known as hydraulic or induced fracture, however, is expected to occur at the point where mini-

imum tangential stress is applied to the rock: an orientation  $90^\circ$  away from the location of shear failures around the wellbore [23] (Fig. 4).

Reduction of mud pressure, corresponding to lower confining pressures, increases the potential for shear failure. On the other hand, increasing the mud pressure above a certain limit causes the tensile failure to happen. This discussion indicates that there is an optimum window for the mud weight to drill the wellbore in a stable condition. The lower limit for this window corresponds to shear failure (breakouts) with its upper limit being the fracture initiation pressure [28, 25]. The magnitudes of three effective principal stresses around the wellbore to analyze the initiation of induced fracture can be obtained as:

$$\begin{aligned}\sigma'_r &= P_w - \alpha P_p \\ \sigma'_\theta &= 3\sigma_h - \sigma_H - P_w - \alpha P_p \\ \sigma'_z &= \sigma_v - 2\nu(\sigma_H - \sigma_h) - \alpha P_p\end{aligned}\quad (2)$$

For shear failure or breakouts to occur the magnitude of effective principal stresses around the wellbore are estimated as

$$\begin{aligned}\sigma'_r &= P_w - \alpha P_p \\ \sigma'_\theta &= 3\sigma_H - \sigma_h - P_w - \alpha P_p \\ \sigma'_z &= \sigma_v + 2\nu(\sigma_H - \sigma_h) - \alpha P_p\end{aligned}\quad (3)$$

For wellbore instability analysis, consequently, stresses at the borehole wall are the ones that should be compared against a failure criterion.

### 3. Mogi-Coulomb Failure Criterion

Al-Ajmi and Zimmerman [25] developed the three-dimensional Mogi-Coulomb failure criterion. This failure criterion has been justified by experimental evidence from triaxial tests as well as polyaxial tests. According to this criterion

$$\tau_{oct} = a + b\sigma'_{m,2} \quad (4)$$

where  $\sigma'_{m,2}$  and  $\tau_{oct}$  are, respectively, the effective mean stress and the octahedral shear stress defined by:

$$\sigma'_{m,2} = \frac{\sigma'_1 + \sigma'_3}{2} \quad (5)$$

$$\tau_{oct} = \frac{1}{3} \sqrt{(\sigma'_1 - \sigma'_2)^2 + (\sigma'_2 - \sigma'_3)^2 + (\sigma'_3 - \sigma'_1)^2} \quad (6)$$

and  $a$  and  $b$  are material constants which are simply related to cohesive strength ( $S_o$ ) and internal friction angle ( $\phi_f$ ) as follows:

$$a = \frac{2\sqrt{2}}{3} S_o \cos\phi_f \quad (7)$$

$$b = \frac{2\sqrt{2}}{3} \sin\phi_f \quad (8)$$

### 4. Field case study

The developed analytical models will be applied to a well (called well A) drilled in Ahwaz oilfield (One of southern Iranian field in the Middle East) for investigation of stability analysis during drilling. This oil field one of the most important Iranian super giant oil fields, was discovered in 1956 and now has more than 450 producing wells. In-situ stresses and pore pressure profiles of Asmari formation are shown in Fig. 5.

Fig. 6 shows the estimated log based geomechanical properties of this well including Poisson's ratio and UCS.

The most commonly observed order of magnitude of stresses around a wellbore in terms of shear failure is  $\sigma'_\theta \geq \sigma'_z \geq \sigma'_r$  and  $\sigma'_r \geq \sigma'_z \geq \sigma'_\theta$  in case of tensile failure [23]. Considering this assumption and the real mud weight that had been used to drill Well A (i.e.  $1.05 \text{ gr/cm}^3$ ), the calculations were carried out to determine the potential for any shear failure (breakouts) ( $P_w(TCYL)$ ) or tensile failure (induced fracture) ( $(P_w(SWBO))$ ).

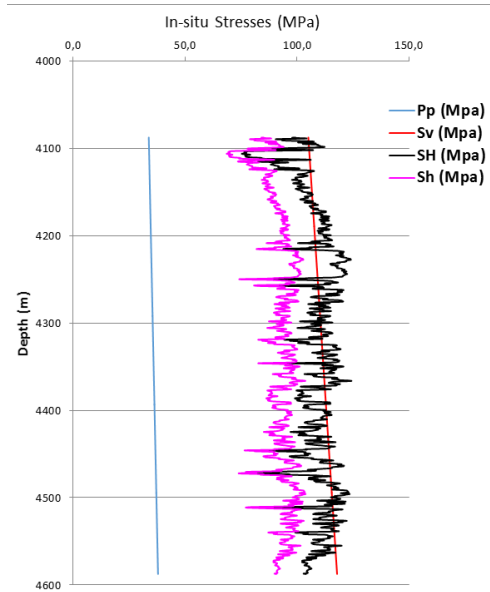


Fig. 5. In-situ stresses and pore pressure profiles of Asmari formation

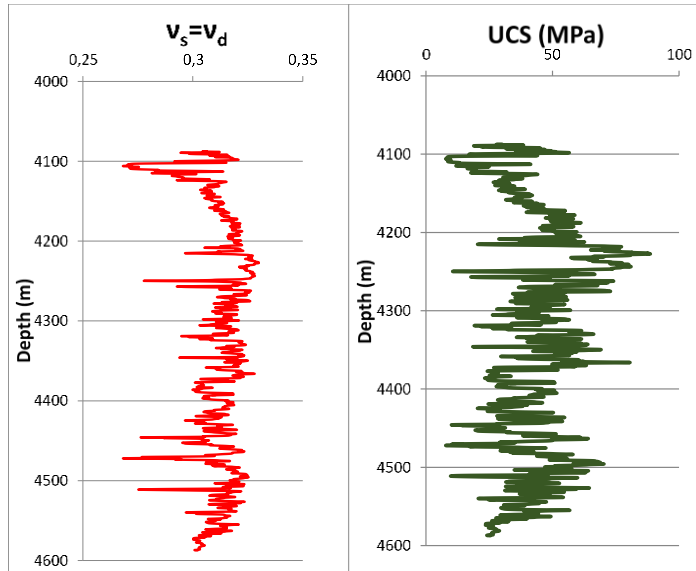


Fig. 6. Geomechanical properties of well A

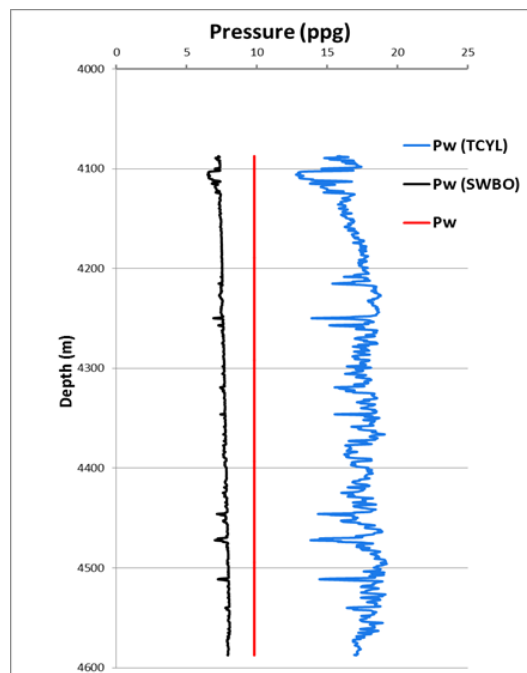


Fig. 7. Determination of minimum and maximum allowable mud pressures for Well A using the 3D Mogi-Coulomb criterion

It can be concluded that the minimum and maximum allowable mud pressures change as function of depth and for well A are varied between the 6.2-9.1 ppg and 13-19.5 ppg, respectively. So, the optimum mud pressure window for this well is 29-43MPa. It can be seen that a good agreement is reached between the results of caliper log and developed model to investigation of the depths related to borehole breakout. As Figs. 6 and 7 depict, optimum mud pressure window is function of geomechanical properties. Therefore, increasing of UCS and/or Poisson's ratio decrease the minimum allowable mud pressure and increase the maximum allowable mud pressure.

## 6. Conclusions

In this paper a new models, based on the 3D Mogi-Coulomb failure criterion and the linear constitutive model for determination of the mud weight window is introduced. The result indicated that optimum mud pressure window increases with increasing the UCS and/or Poisson's ratio. Furthermore, this window is function of depth. Furthermore, a good agreement is reached between the results of caliper log and developed model to investigation of the depths related to borehole breakout.

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