OPTIMAL SIDETRACK TIME EVALUATION BY SEGMENTED PROXY MODEL

G. I. Egeonu, O. D. Orodu, O. Olabode

Department of Petroleum Engineering, Covenant University, Ota, Nigeria

Received July 18, 2017; Accepted October 19, 2017

Abstract
Sidetrack operations which involves accessing a new zone and commingling production gives rise to uncertainties and proxy models have shown to be able to incorporate uncertainties and mimic real-life scenarios to an acceptable degree of accuracy. In this paper, a time-based proxy model which integrates the uncertainties that emerge as a result of several reservoir parameters was developed using NPV as the objective function. A black oil simulator was used to generate the production profile and NPV computed using certain economic parameters. The Box-Behnken response surface design was used to generate a combination of variables with which the experiment was carried out. Non-linearity of the NPV which was caused by the impact of time was corrected using segmented regression and the split design was based on visual observation of the main effect plot. The segmented proxy models obtained were reasonable to an acceptable degree of accuracy in mimicking the simulation model.

Keywords: Sidetrack; Design of Experiments; Net Present Value; Segmented Regression; Proxy models.

1. Introduction

Oil exploration and production is capital intensive and a high-risk venture. This brings to bear the need for economic analysis to assess and evaluate the viability of projects that will consume the capital invested [1]. Economic evaluations contain uncertainties. These uncertainties are geologic and technical and involve high-risk decision scenarios with no guarantee of success. Due to these uncertainties, companies are continuously faced with decisions regarding the allocation of scarce resources among projects that contain geological and financial risk and uncertainty. Sequel to this, decision models for evaluating the risk involved in any project under consideration must be used to determine the profitability of every option under consideration.

In the exploration of hydrocarbons, after a producible zone is identified, further investigation leads to the discovery of another potential layer within the same reservoir, which can be produced at the same time with the already producing zone and companies must decide whether it is profitable to sidetrack or recomplete a well instead of drilling a new one. Profit is the major motivation for considering sidetrack/recompletion as an option [2]. Thus, several factors are considered as the decision to sidetrack or not presents economic benefits and/or consequences. The common sidetrack operation usually involves accessing a new zone and producing it while abandoning the already depleted zone. However, advancement in technology makes it possible to access new layers while producing older layers in commingled production.

Lerche and Noeth [3] presented a typical sidetrack decision scenario where one layer has been drilled and is producing and from exploration drilling, a shallower (or deeper) layer that is oil-bearing is discovered. However, the newly discovered layer has been undeveloped for a variety of reasons: less recoverable oil than the ultimate recoverable reserves from the producing layer, higher overpressure so that special production techniques are needed relative to the producing layer or a high sulphur content so that the oil is sour relative to the current producer and so brings less profit per barrel.
Other reasons like multiple layers with intra-layer heterogeneity of the reservoirs and its diverse implications in terms of parameters such as porosity, permeability, water saturation, residual oil saturation, capillary pressures etc. and the complexity it brings also contributes to the reason why the newly discovered layer may be uncompleted. Resolving these aforementioned issues through proper reservoir characterization can bring clarity in terms of the best decision to take. Furthermore, as the first layer reaches its economic limit, the company is faced with the question of producing the new layer for profit maximization.

The worth of undertaking a sidetrack well for an already producing oil field is examined in terms of the chances the sidetrack will fail and also the chances the sidetrack, even if successful, will kill already producing wells [2]. Therefore, it becomes necessary to evaluate the risk involved for economic purposes.

Lerche and Noeth [4] brought to light the modelling of production with time. Models of production of oil with time, selling price, costs and their temporal variations was shown to have an influence on the time the decision to sidetrack or recomplete is made. In other words, one has to tie the economic potential of the sidetrack and/or recompletion job to the chances of success and failure. The time value of money was considered as an important concept with respect to the decision making. Thus, the sidetrack time, \( t_{s} \), is of prime concern in developing an oil field to its maximum potential. Orodu et al. [5] showed that in order to understand the critical parameters influencing sidetrack or recompletion time, \( t_{c} \) was optimized for production and injection well simultaneously under uncertainty with respect to the Expected Monetary Value (EMV).

Orodu, Tang and Anawe [5] obtained the optimal sidetrack time through the analytical approach but they pointed out that the challenge lies in its application to dynamic field performance. Also, considering that empirical and analytical models cannot adequately replicate all reservoir mechanisms. In continuation of the work by [5], Ajibola, Orodu and Onyeukwu [6] used a proxy model function to compute the NPV with respect to reservoir parameters and time of sidetrack for primary recovery under natural drive mechanism based on continuous production from Layer B after production from Layer A commenced. The probability of failure of production from Layer-B after sidetrack and secondary recovery was not considered. Furthermore, Ajibola et al. [6] did not consider the issue of non-linearity which the proxy model was incapable of dealing with in itself. Consequently, optimization studies were not carried out.

The focus of this paper is to be able to have a function for NPV with respect to uncertain reservoir parameters and time for secondary recovery (or for a production and injection well for Layer A and B) and to consider the probability of failure of each layer after sidetrack. The probability of success will also be considered and compared to the result gotten by [6]. In this study, the issue of non-linearity will be addressed.

An economic indicator, NPV, as shown in Equation (1), will be used to evaluate the optimal sidetrack (recompletion) time for Layer A and Layer B. With the combination of variables generated from a response surface design, the production profile of the synthetic oil field with which the NPV will be calculated using economic parameters such as CAPEX, OPEX, Oil Price, etc. shall be obtained. Since the production (and injection) from (for) both layers are commingled, they will be evaluated together, as a single unit.

\[
NPV = \sum_{l=1}^{m} \sum_{x=1}^{n} (1 + i)^{-t} (P_o q_{xL} - C_{xL} - I_{F}^{xL} - I_{V}^{xL})
\]  

Design of experiment (DOE) is used to maximize the amount of useful information that can be obtained from a limited set of experiments. It reduces the number of experiment trials required, thus economical as it finds the combination of factors (subsurface variables) at which the response variable is optimized. DOE is used mainly because it reduces the number of experimental trials required. It finds the combination of factors and levels at which the response variable is optimized. It is a method that simultaneously investigates the effects of multiple variables on an output or response where deliberate changes are made to the input variable and results collected [7].
Figure 1. Decision tree schematic of sequential recompletion (sidetrack) of a production well and an injection well. \(P_A P_B, (1-P_A)(1-P_B)\) represents the probability of success and the probability of failure for Layer A and B for the production well. \(P_A' P_B', (1-P_A')(1-P_B')\) represents probabilities of success and failure for the injection well, respectively \(^5\)

Friedmann, Chawathe and Larue \(^8\) in their work used experimental design to quantitatively assess the uncertainties in recovery predictions for primary and waterflood process and found that the results were in fair agreement with the simulation results. While, Manceau et al. \(^9\) used experimental design in making decisions in an uncertain reservoir environment and obtained results that showed that the methodology enables one to quantify the risk associated with the uncertainties and to optimize production. Furthermore, \(^10-12\) applied experimental design in their respective studies and found the method to be sufficient in determining the parameters that have the largest contribution to the response and understanding the effect of uncertainties in the prediction of cumulative production and production profile.

An experimental design methodology is efficient in creating probable production profiles and covers a wide range of the field’s uncertainty for proper economic analysis. Numerical simulation results in long computational times and therefore are expensive. However, a proxy model is a good approach to deal with issues of cost and time brought about by numerical simulations. Proxy model has been defined as replicates of reservoir simulation model that is data driven and competent in achieving the desired results \(^13-15\) ascertained that proxy models can provide accurate interpretation and provide optimization methods for risk analysis in considering the complex relationship between the uncertain parameters.

Zangl et al. \(^15\) pointed out that simulation models, which are derived from complex studies involving a high degree of uncertainty and many parameters, give very high nonlinear results. Consequently, it poses a problem as the rectification of the nonlinearity may require complex computations using the numerical models and this is time consuming and costly. Proxy models are used to solve this problem as it mimics the nonlinear behaviour and provides a simple platform to solve the problem of nonlinearity. Investment of the petroleum industry in rese-
Vor simulation tools is expensive and the proxy models provide a way to cut cost seeing that proxy modelling is inexpensive yet effective in mimicking numerical simulation models.

In generating the proxy model, it was observed that the major cause of non-linearity is time. When plotted against other variables, the NPV showed either a linear trend or little or no effect. Hence, the application of segmented regression analysis to deal with the issue of non-linearity effect. Segmented regression analysis is a modification to the standard regression analysis. It is a recommended approach for analysing data from an interrupted time series study [16]. According to Taljaard et al. [16], in a basic regression analysis, the time period is divided into segments, and separate intercepts and slopes are estimated in each segment.

Lamberson and Firman [17] compared quadratic regression and segmented regression in estimating nutrient requirements for dietary formulations and found that the segmented regression resulted in the closest prediction to the true nutrient requirement in 73 of 100 replicates while the quadratic regression yielded overestimates. Wagner et al. [18] stated that “Segmented regression analysis enables us to evaluate how much an outcome of interest was affected by an intervention and other factors that contributed to the change, if any.” A time series being, according to Wagner et al. [18], a sequence of values of a particular measure taken at regularly spaced intervals over time. They noted that segments in a time series are defined when the sequence of measures is divided into two or more portions at change points. Change points are specific points in time where the values of the time series may exhibit a change from the previously established pattern because of an identifiable event.

2. Methodology

2.1. Numerical simulation of synthetic field

Reservoir simulation is one of the most important tools in oil and gas reservoir development and management. Its major purpose is to forecast future performance of the reservoir and provide ways to maximize recovery [14]. Reservoir simulators provides speed and accuracy when seeking to understand production behaviour of a reservoir based on specific reservoir parameters, due to a robust set of numerical solutions. It can predict production performance for all kinds of reservoirs, regardless of the complexities encountered.

A black oil simulator was used for the reservoir simulation. It was used to generate a production profile using specific reservoir properties. The reservoir fluid properties are oil gravity (30.8°API), gas-oil ratio (336.8 scf/stb), reservoir temperature (167°F) and reservoir reference pressure of 8702 psia. Two oil producing layers is adopted for the upper and lower layer with initial oil saturation of 0.3 (layer-1) and 0.25 (layer-2) respectively.

3. Experimental design

The application of DOE helps to map out uncertainties and develop a range of feasible geologic models and other variables [6]. Its main objective is to develop one or more proxy equations to represent a reservoir model.

The parameters used in this study are permeability, porosity, layer thickness of both layers and time. These parameters are usually associated with the uncertainty in reserve estimation and thus they are used to model the reservoir performance (see Table 1 for the parameters and the associated range of uncertainty).

The Box Behnken response surface design, which will be used, assigns a low (-1), medium (0), and high (+1) level to each input variable [19].

Table 1. Range of values for each parameter

<table>
<thead>
<tr>
<th>LEVEL</th>
<th>H1</th>
<th>Poro(ϕ)</th>
<th>Perm(K)</th>
<th>H2</th>
<th>Poro(ϕ)</th>
<th>Perm(K)</th>
<th>T (Time)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOW</td>
<td>65</td>
<td>0.15</td>
<td>420</td>
<td>120</td>
<td>0.19</td>
<td>950</td>
<td>0</td>
</tr>
<tr>
<td>HIGH</td>
<td>75</td>
<td>0.20</td>
<td>470</td>
<td>135</td>
<td>0.30</td>
<td>1050</td>
<td>10</td>
</tr>
</tbody>
</table>
After the response surface design was created, it gave a total of 62 experimental runs based on the 7 factors inputted into the experimental design software, thus simplifying the experiment when compared to a full factorial design. The combination of values was in turn used to generate 62 production profiles from the black oil simulator for each branch that was considered in this work, which was used to compute the Simulation NPV required to analyse the response surface design. A proxy model will be generated for each branch considered based on its probability of success/failure for production/injection well sidetrack (secondary recovery) and production well sidetrack (primary recovery).

3.1. Segmented regression

The accuracy of the proxy model is dependent on a match between it and the simulation model. It has been observed that, in most cases, depending on the probability of success/failure, there is a mismatch between the simulation results and the proxy model, which makes it impossible to carry out optimization studies.

There seems to be an issue of non-linearity as a result of failure of the production wells depending on the probability of success (POS), which affects the cumulative production. The response surface design [high and low value] does not deal with the issue of non-linearity.

In the case of non-linearity, segmented or piecewise regression analysis can be used to deal with it. Segmented regression analysis can estimate the size of the effect at different time points, as well as changes in the trend of the effect over time [18].

In this work, the split design is based on the main plot effect. A main effect occurs when the mean response changes over the levels of a factor or when different levels of a factor influence the response differently. Main effects plot is a plot of the means at each level of a factor plotted by a statistical software used in this study. It plots the fitted means at each level of the factor and connects them with a line. The plot is used to compare the relative strength of the effects across factors. When the line is horizontal (parallel to the x-axis), then there is no main effect present.

The main plot effect of NPV against time is visually assessed to identify the break point, [for the branches that were affected by the non-linearity] after which the data is split into two time-segments. Experimental design runs are generated for each time segment, which are used for simulation runs and analysis of the response surface. The generated proxy model for each time segment is fitted together and matched with the simulation model.

4. Result and discussion

After 62 production profiles were generated based on the Box Behnken design for each branch of the decision tree, the NPV for each run was calculated using the formula that incorporates certain economic parameters such as oil price, CAPEX (capital expenditure), OPEX (operating expenditure), discount rate etc. Tables 2 and 3 are the cost estimates ad economic parameters applied in evaluating the optimal sidetrack by first computing NPV.

Table 2. CAPEX (Drilling and Completion)

<table>
<thead>
<tr>
<th>Activity/Cost</th>
<th>Layer-A Production</th>
<th>Injection</th>
<th>Layer-B Production</th>
<th>Injection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drilling</td>
<td>$1.5MM</td>
<td>$0.8MM</td>
<td>$1.0MM</td>
<td>$0.6MM</td>
</tr>
<tr>
<td>Completion</td>
<td>$0.4MM</td>
<td>$0.2MM</td>
<td>$0.3MM</td>
<td>$0.4MM</td>
</tr>
</tbody>
</table>

Table 3. Economic parameters

<table>
<thead>
<tr>
<th>Economic Parameters</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Operating</td>
<td>0.25%</td>
</tr>
<tr>
<td>Cost</td>
<td>CAPEX/Month</td>
</tr>
<tr>
<td>Oil Price</td>
<td>$55.95</td>
</tr>
<tr>
<td>Discount Rate</td>
<td>1.005%/Month</td>
</tr>
<tr>
<td>Variable Operating</td>
<td>$5/bbl</td>
</tr>
</tbody>
</table>

The proxy model, on the other hand, was generated which contained the seven parameters. With NPV as the response, the proxy model was used to validate the simulation runs and the results were compared. For certain branches, there was a mismatch but when segmented regression was applied based on a visual observation of the main effect plot, a proxy model for each segmented part was also obtained and it was found to fit the simulation model to a reasonable degree of accuracy.
4.1. Proxy model

Table I shows the range of values used in the experimental design and below is the presentation of results of the proxy models.

**Branch-B1**: From the main effect plot of Branch-B1 (Figure 2), the Porosity of Zone A and Time shows to be the most influential factors controlling the NPV. There is no issue of curvature.

\[
\text{NPV}_{B1} = -34 + 1.63H_1 + 0.316H_2 + 152\phi_1 - 53\phi_2 - 0.012K_1 - 0.086K_2 - 1.85T - 0.01045H_1^2 \\
- 0.00133H_2^2 + 30\phi_1^2 + 95.6\phi_2^2 - 0.000002K_1^2 + 0.000043K_2^2 - 0.01435T^2 \\
+ 0.00081H_1H_2 - 0.08H_1\phi_1 + 0.278H_2\phi_2 + 0.00014H_1K_1 + 0.000125H_1K_2 \\
- 0.03747H_1T + 0.110H_2\phi_1 + 4.264H_2\phi_2 - 0.000056H_2K_2 + 0.01183H_2T + 56\phi_1\phi_2 \\
+ 0.028\phi_1K_1 + 0.009\phi_1K_2 - 15.93\phi_1T - 0.0143\phi_2K_2 + 6.620\phi_2T + 0.00001K_1T \\
- 0.000132K_2T
\]  

(2)

The model summary: R-sq, R-sq(adj) and R-sq(Pred) are 0.9999, 0.9997 and 0.9990

![Figure 2. Main effect plot for Branch-B1](image)

The main effect plot of Branch-B1 (Figure II) shows that time and porosity of the second layer influences the response, NPV more than any other parameter

![Figure 2a. Proxy Model vs. Calculated NPV for Branch-B](image)

Figure 2a shows the plot of NPV against time for the simulation model and proxy model. The graph shows that the proxy model was able to mimic the simulation model perfectly as no issues of non-linearity was encountered.

**Branch-B2**: The B2 model is non-linear as seen from (Figure 3), therefore quantitative split design was applied in order to fit the simulation result.
\[
NPV_{B2} = 88 + 0.5H_1 - 0.4H_2 + 38\phi_1 - 305\phi_2 - 0.19K_1 - 0.11K_2 + 14.17 - 0.0043H_1^2 - 0.0008H_2^2
- 39\phi_1^2 + 30\phi_2^2 + 0.00021K_1^2 + 0.000051K_2^2 - 3.11907^2 + 0.01H_1\phi_2
+ 0.0001H_1K_2 + 4.11H_2\phi_2 - 0.00003H_2K_2 + 0.1168H_1T - 114\phi_1\phi_2 + 0.5P\phi_1T
- 0.014\phi_2K_2 + 60.06\phi_2T + 0.0001K_1T
\]

(3)

Model summary: R-sq, R-sq(adj) and R-sq(pred) are 0.9937, 0.9852, and 0.9552

From (Figure 3), it is seen that there is severe non-linearity caused by the impact of time. This necessitates the use of segmented regression.

After segmenting Branch-B2 into two time segments, Figure 3a shows a little reduction in the non-linearity seen in Figure 3.

The second segment of Branch-B2 shows that the application of segmented regression was able to solve the problem of curvature as seen in the main effect plot above. Equation 3a shows the split design scenario for Branch-B2.
**Petroleum and Coal**

Figure IIIc is the plot of NPV against time before and after the split design was applied. The split design scenario for Branch-B3 is shown in Eqn 3a:

\[
\text{NPV}_{B_2} = 0 \leq T \leq 5 \text{ and } 5 \leq T \leq 10
\]

\[
\text{NPV}_{B_2_{0-5}} = -499 + 4.6H_1 + 4.54H_2 - 448\phi_1 + 764\phi_2 - 0.11K_1 - 0.02K_2 + 29.7T - 0.0321H_1^2
\]
\[-0.0145H_2^2 + 1345\phi_1^2 - 527\phi_2^2 + 0.00012K_1^2 + 0.000013K_2^2 - 11.687T^2
\]
\[-0.0003H_1H_2 + 0.1H_1\phi_1 - 0.03H_2\phi_2 - 0.03H_1T - 2.57H_2\phi_2 - 0.00003K_2^2
\]
\[+ 0.2234H_2T - 114\phi_1\phi_2 + 1.2\phi_1T - 0.023\phi_2K_2 + 114.7\phi_2^2 + 0.0002K_1T
\]
\[+ 0.0003K_1^2 \quad (3a)
\]

\[
\text{NPV}_{B_2_{5-10}} = -664 + 0.05H_1 + 5.09H_2 + 1462\phi_1 + 1\phi_2 - 0.35K_1 + 0.603K_2 - 7.18T
\]
\[-0.0003H_1^2 + 0.00701H_2^2 - 369\phi_1^2 + 107.8\phi_2^2 + 0.000382K_1^2 - 0.0000084K_2^2
\]
\[+ 0.11047T^2 + 0.00002H_1H_2 + 0.03H_1\phi_1 + 0.01H_2\phi_2 - 0.0007H_1T - 4.75H_2\phi_4
\]
\[+ 4.239H_2\phi_2 - 0.002403H_2K_2 + 0.0103H_1T - 0.713K_1K_2 - 0.27\phi_1T - 0.014\phi_2K_1
\]
\[= 0.01\phi_2K_2 + 5.46\phi_2^2T + 0.000016K_1K_2 - 0.00002K_2^2 \quad (3b)
\]

Model summary: R-sq, R-sq(adj) and R-sq(pred) are 0.9975, 0.9941 and 0.9820 respectively

Model Summary: R-sq, R-sq(adj) and R-sq(pred) are 0.9991, 0.9978 and 0.9935 respectively

Figure 3c. Proxy Model vs. Calculated NPV for Branch-B2

Figure IIIc is the plot of NPV against time before and after the split design was applied. B2_Proxy is the model before the split design and it was unable to fit the simulation model. B2_0_5 and B2_5_10 is the proxy model after the split design was applied and it fits the simulation model much more than when segmented regression had not been applied.

**Branch-B3**: Split design was also applied for this branch because of the curvature. The curvature is to the same with that of Branch-B2 and can be seen in the main effect plot of Branch-B2 (Figure 2) since they are similar.

\[
\text{NPV}_{b_3} = 15 + 0.1H_1 - 0.29K_1 + 20\phi_1 + 0.08K_2 - 358\phi_2 + 14.0T + 0.0003H_1^2 + 0.00032K_1^2
\]
\[+ 159\phi_1^2 - 0.0024H_1^2 - 0.000041K_2^2 + 56\phi_2^2 - 3.03417T^2 + 1.0H_1\phi_1 + 0.01H_1\phi_2
\]
\[= 0.018H_1T - 9.7\phi_1T - 0.000004H_2K_2 + 4.12H_2K_2 + 0.1184H_2T + 0.005\phi_2K_2
\]
\[+ 0.0002K_2T + 60.12\phi_2T \quad (4)
\]

Model summary: R-sq, R-sq(adj), and R-sq(pred) are 0.9926, 0.9825 and 0.94770 respectively. The split design scenario for Branch-B3 is shown in Eqn 3a:

\[
\text{NPV}_{B_3_{0-5}} = -538 - 4.2H_1 - 0.30K_1 - 44\phi_1 + 5.22H_2 + 0.63K_2 + 685\phi_2 - 29.6T + 0.0302H_1^2
\]
\[+ 0.00003K_1^2 + 228\phi_1^2 - 0.0178H_1^2 - 0.000311K_2^2 - 471\phi_2^2 - 11.676T^2
\]
\[+ 0.0018H_1K_1 + 1.8H_1\phi_1 - 0.0058K_1H_2 + 0.01H_2\phi_2 - 0.013H_1T
\]
\[+ 0.00117K_1H_2 - 0.01K_2K_1 - 21\phi_1\phi_2 - 10.4\phi_1T - 0.00005K_1K_2 - 2.49H_2\phi_2
\]
\[+ 0.2264H_2T + 0.002\phi_2K_2 + 0.0003K_2T + 114.8\phi_2T \quad (4b)
\]

Model summary: R-sq, R-sq(adj) and R-sq(pred) are 0.9972, 0.9934 and 0.9801 respectively.
\[ NPV_{B3_{5-10}} = -70 - 0.28H_1 - 0.155K_1 + 152\phi_1 + 0.863H_2 + 0.126K_2 - 12\phi_2 - 5.12T \\
+ 0.00336H_1^2 + 0.000174K_1^2 + 155\phi_1^2 - 0.00223H_2^2 - 0.000061K_2^2 + 65.1\phi_2^2 \\
+ 0.1872T^2 + 1.02 H_1 \phi_1 + 0.00001H_1 H_2 - 0.000001H_1 K_2 + 0.015H_2 \phi_2 \\
- 0.0234H_1 T + 0.002K_1 \phi_1 - 0.00004K_1 T - 0.902\phi_1 H_2 - 11.62\phi_1 T - 0.000038H_2 K_2 \\
+ 4.243H_2 \phi_2 + 0.00129H_2^2 + 0.0047\phi_2 K_2 - 0.00002K_2 T + 5.46\phi_2 T \]  

(4c)

Model summary: R-sq, R-sq(adj) and R-sq(Pred) are 0.9998, 0.9996, and 0.9987 respectively. After the split design was applied on Branch-B3, there was a significant reduction in the curvature. This can be seen in Figure 3a and 3b respectively as Branch-B2 and -B3 are similar.

Figure 4. Proxy Model vs. Calculated NPV for Branch-B3

Figure IV shows the proxy model before and after split design. The proxy models from the split design for the simulation model better than the proxy model before split design was applied.

**Branch-B4**: The proxy model for the branch is not affected by non-linearity just like Branch-B1 as shown in Figure 2.

\[ NPV_{B4} = 12 - 0.22H_1 - 0.131K_1 - 63\phi_1 - 0.03H_2 + 0.108K_2 - 137\phi_2 - 2.97T + 0.00184H_1^2 \\
+ 0.00146K_1^2 + 154\phi_1^2 - 0.00086H_2^2 - 0.000058K_2^2 + 199.5\phi_2^2 - 0.206987T^2 \\
+ 0.05H_1 \phi_1 + 0.00001H_1 H_2 - 0.153H_1 \phi_2 + 0.0012H_2 T + 0.00015K_1 T + 0.11\phi_1 H_2 \\
- 0.81\phi_1 T + 0.000094H_1 K_2 + 4.513H_2 \phi_2 + 0.015333H_2 T - 0.0140\phi_2 K_2 \\
- 0.00001K_2 T + 8.813\phi_2 T \]  

(5)

Model Summary: R-sq, R-sq(adj) and R-sq(pred) are 0.9995, 0.9987 and 0.9961 respectively. The Figure (5) shows a perfect fit between the simulation and proxy model for Branch-B4.

Figure 5. Proxy Model vs. Calculated NPV for Branch-B4
**Branch-C**: Branch-C did not have any issues with non-linearity and can be compared to the trend in Branch-B1. This can be seen in Figure 2. The proxy model matched the simulation model.

\[
NPV_c = -103 - 0.97H_1 - 0.120K_1 - 223\phi_1 + 1.393H_2 + 0.266K_2 - 247\phi_2 - 2.39T + 0.0069H_1^2 \\
+ 0.000069 K_1^2 + 70\phi_1^2 - 0.00244H_2^2 - 0.000088 K_2^2 + 87.4\phi_2^2 - 0.1394T^2 \\
+ 0.00001H_1 H_2 - 0.000001 H_1 K_2 + 0.015H_1\phi_2 + 0.414 K_1\phi_1 - 0.064 K_2\phi_2 \\
- 0.00001K_1T + 64\phi_1\phi_2 - 0.000963 H_2K_2 + 5.299 H_2\phi_2 + 0.1302\phi_2K_2 \\
\]  

(6)

Model Summary: R-sq, R-sq(adj) and R-sq(pred) are 0.9998, 0.9995 and 0.9986 respectively

---

The above Figure (6) shows a match between the simulation and proxy model of Branch-C. **Branch-E1**: This branch was also affected by curvature just like Branch-B2 and -B3 as seen from the main effect plot (Figure 3) and split design was also applied using the scenario shown in (Eqn 3a):

The proxy model before the split design is shown in equation [7a]

\[
NPV_{E1} = 32 + 0.1H_1 - 0.34K_1 - 27\phi_1 + 0.06K_2 - 363\phi_2 + 17.17 - 0.0009H_1^2 + 0.00038 K_1^2 \\
+ 150\phi_1^2 - 0.0025 H_2^2 - 0.000027 K_2^2 + 46\phi_2^2 - 3.4485T^2 + 1.0H_1\phi_1 + 0.01H_1\phi_2 \\
+ 0.003H_1T - 2.39\phi_1 T - 0.00004 H_2 K_2 + 4.11 H_2\phi_2 + 0.1197 H_2 T + 0.005\phi_2 K_2 \\
- 0.0001 K_2 T + 61.61\phi_2 T \\
\]  

(7a)

Model summary: R-sq, R-sq(adj) and R-sq(pred) are 0.9940, 0.9859 and 0.9571 respectively.

After the split design:

\[
NPV_{E1a-b} = -544 - 6.5H_1 + 0.02K_1 - 118\phi_1 + 5.81H_2 + 0.58K_2 + 643\phi_2 + 35.8T + 0.0261 H_1^2 \\
+ 0.0002K_1^2 + 299\phi_1^2 - 0.0191 H_2^2 - 0.000288 K_2^2 - 465\phi_2^2 - 13.305T^2 \\
+ 0.0032H_1K_1 + 1.8H_1\phi_1 + 0.0105 H_1 H_2 + 0.01H_1\phi_2 \\
+ 0.029 H_1 T - 0.00210K_1H_2 - 0.01K_1\phi_2 - 21\phi_1\phi_2 + 4.3\phi_1 T - 0.00005 H_2 K_2 - 2.29 H_2\phi_2 \\
+ 0.2290 H_2 T + 0.004\phi_2 K_2 - 0.001K_2 T + 117.7\phi_2 T \\
\]  

(7b)

Model summary: R-sq, R-sq(adj) and R-sq(pred) are 0.9977, 0.9945 and 0.9834 respectively. There was reduction in the curvature after the split design. The trend can be seen in Figure 3a and 3b.

\[
NPV_{E1a-b} = 6 - 0.36 H_1 - 0.107K_1 - 108\phi_1 + 0.185 H_2 + 0.115 K_2 - 45\phi_2 - 6.99T \\
+ 0.00390 H_1^2 - 0.00005 K_1^2 + 111\phi_1^2 - 0.00048 H_2^2 - 0.000056 K_2^2 + 278\phi_2^2 \\
+ 0.2027T^2 + 1.02H_1\phi_1 + 0.00001H_1 H_2 - 0.00001 H_1 K_2 + 0.015 H_1\phi_2 - 0.0234 H_1 T \\
+ 0.404K_1\phi_1 + 0.183 K_2\phi_2 - 0.0004 K_1 T - 183.3\phi_2 T - 0.000038 H_2 K_2 + 4.242 H_2\phi_2 \\
+ 0.01034 H_2 T + 0.00475\phi_2 K_2 - 0.0002 K_2 T + 5.49\phi_2 T \\
\]  

(7c)

Model summary: R-sq, R-sq(adj) and R-sq(pred) are 0.9998, 0.9995 and 0.9986 respectively.

---

ISSN 1337-7027 an open access journal
As in Branch-B2 and -B3, Figure 6c shows that the proxy models from the split design did better in matching the simulation model.

**Branch-E2**: Non-linearity is also seen in this branch just as in Branch-B2, B3 and E1, (see figure 3) and split design was applied. The split design scenario used is shown in (Eqn 3a):

Before split design was applied:

\[
NPV_{E2} = 4 - 0.1H_1 - 0.15K_1 - 46\phi_1 - 0.3H_2 + 0.10K_2 - 359\phi_2 + 14.3T + 0.0010H_1^2
\]
\[
+ 0.00017K_1^2 + 143\phi_1^2 - 0.0013H_2^2 - 0.000047K_2^2 + 39\phi_2^2
\]
\[
- 3.1539T^2 + 0.0000H_1H_2 + 0.01H_1\phi_2 - 0.001H_1T - 0.0000K_1H_2 + 0.0001K_1T
\]
\[
- 0.8\phi_1T - 0.00004H_2K_2 + 4.11H_2\phi_2 + 0.1197H_2T + 0.005\phi_2K_2 - 0.0001K_2T
\]
\[
+ 6.161\phi_2T
\]

**Model summary**: R-sq, R-sq(adj) and R-sq(pred) are 0.9936, 0.9849 and 0.9542 respectively. The severity of the curvature makes the proxy equation unsuitable. However, after split design there was a reduction in the curvature (See Figure 3a and 3b).

\[
NPV_{E2_{\text{split}}} = -614 - 4.5H_1 - 0.04K_1 - 19\phi_1 + 4.86H_2 + 0.71K_2 + 687\phi_2 + 30.0T + 0.0321H_1^2
\]
\[
+ 0.00005K_1^2 + 122\phi_1^2 - 0.0159H_2^2 - 0.000353K_2^2 - 494\phi_2^2 - 11.795T^2
\]
\[
+ 0.1H_1\phi_1 + 0.01H_2\phi_2 - 0.002 H_1T - 0.03K_1\phi_1 - 0.01 K_2\phi_2 + 0.0002 K_1T
\]
\[
- 38 \phi_1\phi_2 - 1.3\phi_1T - 0.00005H_2K_2 - 2.47 H_2\phi_2 + 0.2290 H_2T + 0.004 \phi_2K_2
\]
\[
- 0.0001K_2T + 117.7 \phi_2T
\]

**Model summary**: R-sq, R-sq(adj) and R-sq(pred) are 0.9973, 0.9937 and 0.9810 respectively.

\[
NPV_{E2_{\text{split}}} = -32 - 0.43H_1 - 0.101K_1 - 41\phi_1 + 0.454H_2 + 0.123K_2 - 6.7\phi_2 - 6.68T + 0.00312H_1^2
\]
\[
+ 0.000109K_1^2 + 109\phi_1^2 - 0.00154H_2^2 - 0.000060K_2^2 + 50.9\phi_2^2 + 0.0824T^2
\]
\[
+ 0.03H_1\phi_1 + 0.00001 H_1K_2 - 0.000001 H_1\phi_2 + 0.015H_1\phi_2 - 0.00071 H_1T
\]
\[
+ 0.014K_1\phi_1 + 0.006K_1\phi_2 - 6\phi_1\phi_2 - 0.27\phi_1T - 0.000023H_2K_2 + 4.242 H_2\phi_2
\]
\[
+ 0.01034H_2T + 0.0047 \phi_2K_2 - 0.00002 K_2T + 5.49 \phi_2T
\]

**Model summary**: R-sq, R-sq(adj) and R-sq(pred) are 0.9998, 0.9996 and 0.9889 respectively.
Petroleum and Coal

From the figure above, the proxy models from the split design matched the simulation model better than the proxy model before segmented regression was applied.

**Branch-F:** Just as in Branch-B2, B3, E1 and E2 (see figure 3), it is evident that Branch-F is also non-linear and split design was applied.

\[
NPV_F = 13990 + 23.5H_1 - 33.8K_1 + 2284\phi_1 + 18.9H_2 - 16.35K_2 + 306\phi_2 - 164.8T - 0.168H_1^2 \\
+ 0.0137K_1^2 - 6591\phi_1^2 - 0.076H_2^2 + 0.00332K_2^2 - 1356\phi_2^2 - 4.007T^2 \\
+ 0.1 H\phi_2 + 0.001H_T + 0.0203 K_1 K_2 + 0.2024 K_T T + 0.1\phi_1 H_2 + 0.1\phi_1 T \\
- 0.0001 K_2 \phi_2 + 4.2H_2 \phi_2 + 0.120H_T \phi_2 + 0.014K_2 \phi_2 + 0.1014 K_T \phi_2 + 61.7\phi_2 T \\
\]

(9a)

Model summary: R-sq, R-sq(adj) and R-sq(pred) are 0.9562, 0.8971 and 0.6882 respectively.

After split design was applied, there was a significant improvement in the reduction of the curvature as seen in figure 3a and 3b respectively:

\[
NPV_{F-5} = -634 - 4.3H_1 - 0.06K_1 - 47\phi_1 + 4.63H_2 - 0.68K_2 + 613\phi_2 + 65.5T - 0.0304H_1^2 \\
+ 0.00007 K_1^2 + 149\phi_1^2 - 0.0154H_2^2 - 0.000339 K_2^2 - 464\phi_2^2 - 16.576T^2 \\
+ 0.00001 H K_2 + 0.01H_1 \phi_2 - 0.00005 H_2 K_2 - 2.09 H_2 \phi_2 + 0.2290 H_T \phi_2 + 0.004\phi_2 K_2 \\
- 0.0001 K_T \phi_2 + 117.7\phi_2 T \\
\]

(9b)

Model summary: R-sq, R-sq(adj) and R-sq(pred) are 0.9986, 0.9967 and 0.9899 respectively.

\[
NPV_{F-10} = -397 + 2.49H_1 + 0.66K_1 + 2154\phi_1 + 3.56H_2 + 0.57K_2 + 59\phi_2 - 24.17 - 0.0199H_1^2 \\
- 0.0008K_1^2 + 1952\phi_1^2 + 0.0196H_2^2 - 0.000290 K_2^2 - 138\phi_2^2 + 0.271T^2 \\
+ 0.00035 H K_2 + 0.0012 H_T H_1 + 0.011H_1 \phi_2 + 0.00023 K_1 H_2 - 18.61\phi_1 H_2 - 55.8\phi_1 T \\
+ 0.00008 H K_2 + 3.35 H_2 \phi_2 + 0.1964 H_T \phi_2 + 0.020 K_2 \phi_2 - 0.00002 K_T \phi_2 + 5.49\phi_2 T \\
\]

(9c)

Model summary: R-sq, R-sq(adj) and R-sq(pred) are 0.9907, 0.9781 and 0.9336 respectively.

---

Figure 8. Proxy model vs. calculated NPV for Branch-E2

Figure 9. Proxy Model vs. Calculated NPV for Branch-F
4.2. Proxy model validation

Values that fall between the high and low limit of the 7 factors were used to validate the proxy model (see Table 4 for the data used in validating the accuracy of the proxy models). It was compared with the manual computation of NPV from the simulation runs. The results are shown Figure 10.

Table 4. Parameter values for validating the proxy model

<table>
<thead>
<tr>
<th>T</th>
<th>H1</th>
<th>H2</th>
<th>PORO1</th>
<th>PORO2</th>
<th>PERM1</th>
<th>PERM2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>75</td>
<td>120</td>
<td>0.175</td>
<td>0.245</td>
<td>470</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>75</td>
<td>120</td>
<td>0.175</td>
<td>0.245</td>
<td>470</td>
<td>1000</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
<td>120</td>
<td>0.175</td>
<td>0.245</td>
<td>470</td>
<td>1000</td>
</tr>
<tr>
<td>4</td>
<td>75</td>
<td>120</td>
<td>0.175</td>
<td>0.245</td>
<td>470</td>
<td>1000</td>
</tr>
<tr>
<td>5</td>
<td>75</td>
<td>120</td>
<td>0.175</td>
<td>0.245</td>
<td>470</td>
<td>1000</td>
</tr>
<tr>
<td>6</td>
<td>75</td>
<td>120</td>
<td>0.175</td>
<td>0.245</td>
<td>470</td>
<td>1000</td>
</tr>
<tr>
<td>7</td>
<td>75</td>
<td>120</td>
<td>0.175</td>
<td>0.245</td>
<td>470</td>
<td>1000</td>
</tr>
<tr>
<td>8</td>
<td>75</td>
<td>120</td>
<td>0.175</td>
<td>0.245</td>
<td>470</td>
<td>1000</td>
</tr>
<tr>
<td>9</td>
<td>75</td>
<td>120</td>
<td>0.175</td>
<td>0.245</td>
<td>470</td>
<td>1000</td>
</tr>
<tr>
<td>10</td>
<td>75</td>
<td>120</td>
<td>0.175</td>
<td>0.245</td>
<td>470</td>
<td>1000</td>
</tr>
</tbody>
</table>

The proxy models for Branch-B1, B4 and C, and proxy models from the split design applied to Branch-B2, B3, E1, E2 and F demonstrated the ability to mimic the numerical simulation model to an acceptable degree of accuracy as represented by the model summary of each proxy model of the several branches, the main effect plots and the plot of NPV vs. time of the respective branches as seen in the figures above. Furthermore, the application of segmented regression was able to solve the problem of non-linearity which was majorly a result of the impact of time as seen in the main effect plots.

However, it was observed, even after the split design was applied, that Branches “B2”, “B3”, “E1”, “E2” and “F” still showed curved between time-steps 2 to 4 or 3 to 4. For further studies, split design can be applied for 2 or 3 to 4 segments as the case may be in order to...
totally deal with the issue of non-linearity and to have a better proxy model that can suit any range of values for the input parameters.

5. Conclusion

A time dependent proxy model which incorporates uncertainty of reservoir parameters, using the NPV as the response was developed using experimental design by studying the interaction between parameters and its individual influence on the response. The observed non-linearity due to time was handled by applying segmented regression and the results were quite significant considering the fit with the simulation NPV.

The proxy model was able to replicate reservoir simulation models to an acceptable degree of accuracy. This can be further improved by having more segments in the split design and by also using more than 2 levels in the experimental design.

References


To whom correspondence should be addressed: G. I. Egbonu, Department of Petroleum Engineering, Covenant University, Ota, Nigeria, geraldikennae@gmail.com