

ON THE ASSESSMENT OF PSEUDO STEADY-STATE HORIZONTAL WELL PRODUCTIVITY: APPLICATION IN RECTANGULAR AND SQUARE DRAINAGE AREAS

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Abstract

The productivity index is accounted as one of the main characteristics of well performance, and is of much importance in assessing the production amount/volume of liquids from wells. Furthermore, pseudo skin factor is an effective parameter in calculations related to the productivity index of producing oil/gas wells. In this study, an accurate and reliable method is proposed to estimate the pseudo skin factor as a function of dimensionless length and the ratio of horizontal well length over drainage area in both square and rectangular drainage areas. To measure the accuracy performance of the newly developed method, a smart technique called least square support vector machine algorithm was used as a comparative method. The result demonstrate that the developed method can estimate the pseudo skin factor data with lower average absolute relative deviation compared to the smart technique (9.2 against 11.4%).

Keywords: Productivity performance; Pseudo skin factor; LSSVM model; Drainage area; Horizontal well.

1. Introduction

As a matter of fact, the productivity index (PI) which plays a key role in evaluating the production amount/volume of liquids from wells, is accounted as one of the main characteristics of well performance [1]. In other words, productivity index is required for a large number of calculations related to production and reservoir engineering. As a definition with no considering the type of formation and wellbore, the PI is the amount/volume of possible daily production of reservoir fluids from a gas and/or oil well by 1 psi pressure drop [2]. As a result, several parameters (i.e. the reservoir permeability, geometry of drainage area, the length of horizontal wellbore, the properties of reservoir fluid, etc.) have influences on the value of PI for a drilled well particularly horizontal one. Nowadays, use of horizontal wells has gained considerable attention in petroleum industry because it can increase the production of oil/gas from reservoir, and the ability of injection to reservoirs for enhanced oil recovery (EOR) cases. Therefore, it is of great importance to assess the productivity performance of hydrocarbon reservoirs in case of horizontal wells.

Normally, the producing wells partially penetrates the formation of oil fields, so that if an oil and/or gas well is completed as partially penetrating, the streamline converges and the area for flowing the reservoir liquid reduces in the environs of the wellbore, which results in added resistance (pseudo skin factor) [3]. Subsequently, pseudo skin factor is recognized as an effective parameter in calculations related to the PI and performance evaluation of producing oil/gas wells. Therefore, it is needed to estimate the pseudo skin factor due to partial penetration through analytical and empirical methods. Babu and Odeh [4] proposed a complicated equation for estimating the PI of horizontal wells which needed that the drainage volume be approximately box-shaped, and all the boundaries of the drainage volume be sealed. Li *et al.* [5]

proposed a new method to evaluate the productivity performance of fractured reservoirs. They studied the influences of fracture properties on the PI of the wells. Fokker et al. [6] developed a new method for assessing the PI of complex wells. The results indicated that method proposed could be applied in the finite-conductivity wells, well interference, non-homogenous formations, and hydraulically fractured reservoirs. Bahadori et al. [7] developed an easy-to-utilize empirical correlation for the evaluation of pseudo steady-state PI of horizontal oil wells. To this end, pseudo skin factor was obtained by their new method. In the equation, they considered pseudo skin factor as a function of dimensionless length and the ratio of horizontal well length over drainage area side. Yu et al. [8] conducted a sensitivity analysis using the numerical simulation in order to find the effects of various geometries of multiple transverse hydraulic fractures on gas production.

The aim of present study was to develop a reliable, accurate and applicable method for the evaluation of oil horizontal well productivity through estimation of pseudo skin factor. To this end, the literature data of pseudo skin factor as a function of dimensionless length and the ratio of horizontal well length over drainage area side were collected. Furthermore, the results obtained by the newly proposed method were compared with least square support vector machine (LSSVM). Finally, the influences of dimensionless length and the ratio of horizontal well length over drainage area side on the pseudo skin factor estimated by the newly developed method were evaluated using a sensitivity study technique.

2. Proposing new method

As mentioned earlier, the reservoir properties, geometry of drainage area, the length of horizontal wellbore, and the properties of reservoir fluid, etc., have significant effects on predicting the fluid productivity of wells. Furthermore, pseudo skin factor is accounted as one of the most important parameters in calculations related to well-testing analysis and productivity index. Therefore, a simple method which could rapidly estimate the pseudo skin factor is needed to evaluate the productivity performance of producing wells. To develop such method, the data of pseudo skin factor or shape-related skin factor (S_{CAh}) as a function of dimensionless length (L_D) and the ratio of horizontal well length over drainage area side (U or $L/2xe$) for square and rectangular shapes with ratios of sides 1, 2, and 5, was collected from literature [9-10]. The collected databank collected in this study covers a wide range of pseudo skin factor from 1.412 to 5.86. Additionally, dimensionless length and the ratio of horizontal well length over drainage area side range from 1 to 100 and 0.2 to 1, respectively.

Regarding issues discussed earlier, a simple method with two variables including dimensionless length and the ratio of horizontal well length over drainage area side should be proposed for the determination of pseudo skin factor or shape-related skin factor. During development of the method, average absolute relative deviation (AARD or E_a) was considered as an error function to measure the accuracy of the newly proposed model. Final form of the method is as follow:

$$S_{CAh} = \frac{2U + L_D^{0.3767} + 2.6936}{Ln(2.752 L_D)} \quad (1)$$

where S_{CAh} stands for pseudo skin factor, L_D expresses dimensionless length, and U is the ratio of horizontal well length over drainage area side ($L/2xe$). The equation for calculating dimensionless length has previously been reported as follow:

$$L_D = \frac{L}{2h} \sqrt{\frac{k_v}{k_h}} \quad (2)$$

where L denotes the horizontal well length (ft), h expresses the formation thickness (ft), k_v is vertical permeability (mD), and k_h stands for horizontal permeability (mD). Here, it should be noted that the method proposed (Eq. (1)) is applicable for both rectangular and square drainage areas.

3. Sensitivity study

To illustrate the relevancy of the selected variables (*i.e.* dimensionless length and the ratio of horizontal well length over drainage area side) for the estimation of pseudo skin factor, a sensitivity study was carried out in this study. To this end, the relevancy factor (*r*) approach [11] is employed to measure the influence degree of each variable applied in Eq. (1) for the determination of pseudo skin factor. In this approach, the positive or negative influence of input variables on the pseudo skin factor is however not determined by absolute value of *r*. The *r* values are calculated as follow [12]:

$$r(Inp_k, \mu_g) = \frac{\sum_{i=1}^n (Inp_{k,i} - \overline{Inp_k})(\mu_i - \bar{\mu})}{\sqrt{\sum_{i=1}^n (Inp_{k,i} - \overline{Inp_k})^2 \sum_{i=1}^n (\mu_i - \bar{\mu})^2}} \quad (3)$$

where: $Inp_{k,i}$ stands for *i*th value of the *k*th input variables and Inp_k denotes the average value of the *k*th input variables, μ_i indicates the *i*th value of the pseudo skin factor calculated by Eq. (1), and $\bar{\mu}$ is the average value of the pseudo skin factor calculated by Eq. (1).

4. Results and discussion

As a consequence, the LSSVM approach has successfully been implemented for estimation of the different important properties in petroleum and chemical engineering [13-17]. Therefore, LSSVM methodology [18-19] as a smart predictive technique was used in this study for comparing the results obtained by the newly proposed method (Eq. (1)) for the estimation of pseudo skin factor. In the LSSVM algorithm has two tuning parameters including σ^2 and γ which should be optimized through an optimization technique. Hence, the coupled simulated annealing (CSA) [20-22] as an adjusting strategy was applied in the current study. Using the CSA technique, the tuned values of σ^2 and γ for the developed LSSVM model for estimation of the pseudo skin factor are reported as 0.002228 and 27288.14, respectively. Furthermore, some important error parameters are considered to measure the accuracy of methods developed in this study as shown in Table 1. Table 1 also reports the results obtained by both Eq. (1) and LSSVM model for the estimation of pseudo skin factor. As clear in the table, the average absolute relative deviation obtained for the newly developed method is less than LSSVM model. The AARD values reported for Eq. (1) and LSSVM model are 9.2 and 11.4%, respectively. This clearly shows that Eq. (1) is more accurate than LSSVM model for calculation of pseudo skin factor.

Table 1. Statistics error parameters of developed model for prediction of wax deposition rate

Performance	E _a ^a %	E _r ^b %	SD ^c	RMSE ^d	R ² ^e
LSSVM approach	11.4	-2.7	0.14	0.354	0.90
New method (Eq. (2))	9.2	4.2	0.12	0.382	0.90

$$^a E_a \% = \frac{1}{n} \sum_{i=1}^n |E_i \%| \text{ where } E_i \% = \left[\frac{X_{exp} - X_{rep./pred}}{X_{exp}} \right] \times 100 \Rightarrow i = 1, 2, 3, \dots, n$$

$$^b E_r \% = \frac{1}{n} \sum_{i=1}^n E_i \%$$

$$^c SD = \sqrt{\frac{1}{n-1} \sum_{i=1}^n \left(\frac{X_{i exp} - X_{i rep./pred}}{X_{i exp}} \right)^2}$$

$$^d RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (X_{i exp} - X_{i rep./pred})^2}$$

$$^e R^2 = 1 - \frac{\sum_{i=1}^N (X_{(i) exp} - X_{(i) rep./pred})^2}{\sum_i (X_{(i) rep./pred} - average X_{(i) rep./pred})^2}$$

Figs. 1 and 2 illustrate the pseudo skin factor data calculated by Eq. (1) and LSSVM model in comparison with the literature reported records, respectively. Top vies of the two figures shows scatter diagram, and bottom view is considered for illustrating the distribution of relative error calculated for both Eq. (1) and LSSVM model.

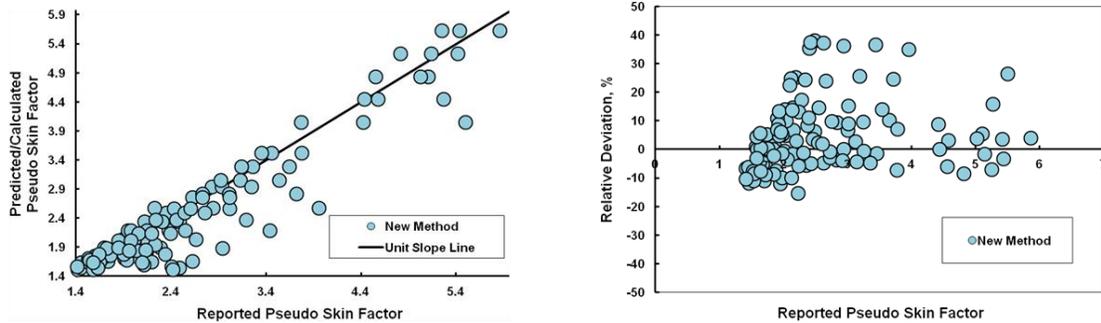


Fig. 1. Graphical error analysis for the proposed method. left: scatter diagram; right: relative error distribution plot

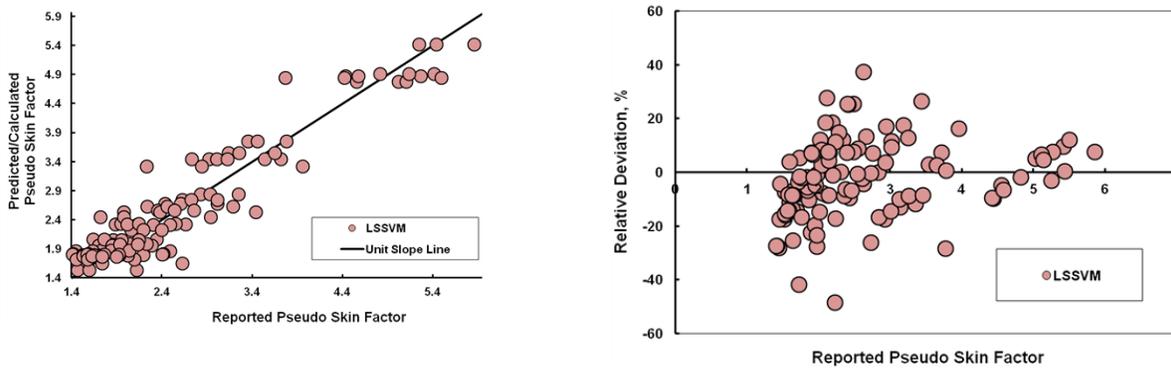


Fig. 2. Graphical error analysis for the developed LSSVM model. Left: scatter diagram; right: relative error distribution plot

As can be seen from the figures, the results obtained by Eq. (1) are in more agreement with the actual data of pseudo skin factor. Additionally, it is clear from the figures that the values obtained by LSSVM model have been more distributed around zero line compared to values calculated by Eq. (1). A further comparison between two methods is displayed in Fig. 3. Fig. 3 indicates the smooth performance of the both Eq. (1) and LSSVM model against actual data of pseudo skin factor. In other words, the figure illustrates the trend plot of pseudo skin factor estimated by Eq. (1) and LSSVM model versus dimensionless length at the ratio of horizontal well length over drainage area side equal to 0.2 ($L/2x_e=0.2$). The figure confirms that the values estimated by Eq. (1) are more matched with the actual data of pseudo skin factor compared to the LSSVM model. This means that the Eq. (1) has an acceptable smoothness trend in estimating pseudo skin factor.

As already mentioned, a relevancy analysis is carried out to see the impacts of dimensionless length and the ratio of horizontal well length over drainage area side on the pseudo skin factor estimated by Eq. (1). Fig. 4 illustrates the results of such sensitivity analysis. This figure shows that the dimensionless length and the ratio of horizontal well length over drainage area side have positive and negative effects on the pseudo skin factor estimated by Eq. (1), respectively. The results obtained in this study demonstrate that the proposed method is capable and reliable for the estimation of pseudo skin factor. Additionally, the method proposed in this study could be applied in reservoir engineering soft wares to estimate the productivity perfor-

mance of horizontal oil wells with rectangular and square drainage areas. Finally. It is worthwhile to mention that the method proposed in this study is a small equation with few number of coefficients which can be re-optimized if more relevant data become available in future.

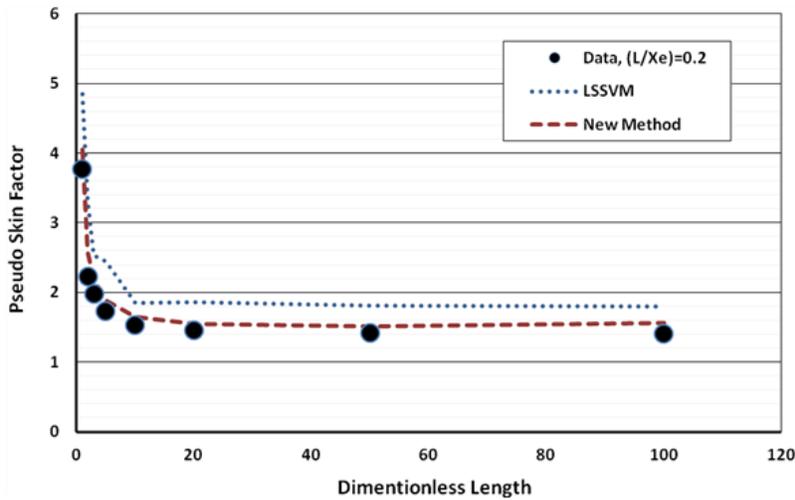


Fig. 3. Smooth performance of the proposed method and LSSVM model in estimating pseudo skin factor at the ratio of horizontal well length over drainage area side equal to 0.2 ($L/2x_e=0.2$)

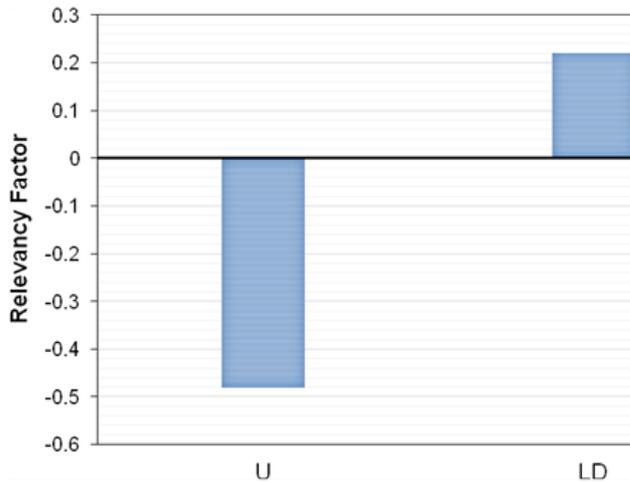


Fig. 4. The impact of variable used in Eq. (1) on pseudo skin factor

5. Conclusions

This study aimed to assess the productivity performance of horizontal oil wells in square and rectangular drainage areas. To this end, an accurate and reliable equation was proposed to estimate the pseudo skin factor. The equation is applicable in both square and rectangular drainage areas. To measure the accuracy performance of the newly developed method, a comparative study was graphically and statistically carried out. Therefore, a smart technique called least square support vector machine algorithm was used to see whether the proposed method is reliable. The result demonstrate that the developed method can estimate the pseudo skin factor data with lower average absolute relative deviation compared to the smart technique (9.2 against 11.4%). Finally, a sensitivity analysis was performed to observe the effects of variables applied in the proposed method. The results indicate that the dimensionless length and the ratio of horizontal well length over drainage area side have positive and

negative effects on the pseudo skin factor estimated by the newly proposed method, respectively.

Nomenclature

<i>EOR</i>	<i>enhance oil recovery</i>
<i>PI</i>	<i>productivity index</i>
<i>LSSVM</i>	<i>least square support vector machine</i>
<i>CSA</i>	<i>coupled simulated annealing</i>
<i>AARD</i>	<i>average absolute relative deviation</i>
<i>R</i>	<i>relevancy factor</i>
<i>SC_{Ah}</i>	<i>pseudo skin factor</i>
<i>L_D</i>	<i>dimensionless length</i>
<i>U</i>	<i>ratio of horizontal well length over drainage area side ($L/2xe$)</i>
<i>L</i>	<i>horizontal well length, ft</i>
<i>h</i>	<i>formation thickness, ft</i>
<i>k_v</i>	<i>vertical permeability, mD</i>
<i>k_h</i>	<i>horizontal permeability, mD</i>

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