DEVELOPMENT OF GENERALIZED PRESSURE GRADIENT CURVES FOR VERTICAL LIFT PERFORMANCE

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Abstract

This study is a novel approach geared towards the determination of pressure transverse in a vertical well via the generalized pressure transverse curve developed from the modified Hagedorn and Brown correlation. The generalized pressure gradient curve developed in this study eliminates the need for a compatible gradient when taking pressure transverse readings of a particular well. Apart from solving the issue of well-curve compatibility associated with Gilbert and other pressure transverse curves, it is straightforward and gives quick estimation of bottom-hole flowing pressure, well head pressure, bottom-hole flowing pressure for directional wells, equivalent surface depth and equivalent total depth with high degree of accuracy under different well conditions (water-cut, flow rate, GLR, and tubing diameter) and flow regimes which are bubble flow, froth flow, plug flow, and slug flow.

Keywords: Generalized pressure gradient curve; Gilbert curve; Flow regimes; Pressure transverse; Modified Hagedorn; and Brown correlation.

1. Introduction

The most important design method is inevitable when considering an appropriate design for a new project [1-2]. This must be done before investing a huge amount on such a project. However, accurate design, in most cases, requires computer application, but the use of computers is not feasible for field engineers [1]. It is therefore advantageous to construct a set of Pressure Transverse curves for hypothetical values of flow rate, gas-liquid-ratio, well diameter and water cut [5]. These curves can be used to estimate the pressure drop in a well producing under similar conditions [7]. The more closely the curves match the well conditions; the more accurate the result will be. Well-curve compatibility is a major setback to the use of vertical gradient curves. In some situation, it becomes extremely hard to find a matching curve.

This technical work presents the procedure involved in the development of the generalized pressure gradient curve, the method of application, the assumption made as well as its practical applications to some production scenarios.

The generalized pressure gradient curve is a dimensionless semi-log plot of depth against pressure. It was developed from the modified Hagedorn and Brown correlation. The advantages of the generalized curves over the vertical curves are as follows;

- It is not cumbersome, i.e. readings can be taken easily and conveniently.
- It saves space, i.e. limited graphs needed for all works to be done.
- Errors due to unit conversion is eliminated because it deals with dimensionless parameters
- It eliminates well-curve compatibility.
- It involves a simple calculation and does not waste time.

However, it must be noted that the vertical gradient curves give an accurate representation of the vertical lift performance of a well when gradient curve data is compatible with the measured well data. Readings taken from compatible gradient curves, therefore, served as a check for the accuracy of the generalized pressure gradient curves developed in this study.
2. Methodology

The modified Hagedorn and Brown correlation are scaled into dimensionless equation. The Hagedorn and Brown equation in U.S field unit is given below:\(^5\)

\[
144 \frac{dp}{dz} = \frac{m^2}{7.408 \times 10^{-4} \rho p} + \frac{n}{2 \rho g A} \left( \frac{dL}{L} \right) \text{ (lb}_f/\text{ft}^3). 
\] (1)

Scaling up Equation (1) with the acceleration component gives

\[
\frac{dP_D}{dZ_D} = \frac{2gA}{2fM^2 A} + \frac{2.304 \times 10^9 D^5}{9266.112D^2} + 1. \quad (2)
\]

where: \(dP_D = \frac{\rho \Delta u_m^2}{\rho \Delta u_m} \).

\[
dZ_D = \frac{dz}{\Delta Z}. \quad (4)
\]

For bubble-flow regime,

\[
\frac{dP_D}{dZ_D} = \frac{2gA}{2fM^2 A} + \frac{2.304 \times 10^9 D^5}{9266.112D^2} \frac{\rho \Delta u_m^2}{\rho \Delta u_m}. \quad (5)
\]

where: \(\rho = \text{average density of mixture}; \Delta u_m^2 = \text{acceleration parameter.}

The following assumptions were made during the pressure transverse computation:

I. The flow in the tubing is laminar.
II. Temperature is constant along with the tubing at a certain average value.
III. Solution gas-oil-ratio is dependent on the average pressure therefore constant along the tubing length
IV. There is uniform acceleration along the tube.

The improvements made in the generation of the generalized pressure gradient curve are as follows:

I. Compressibility factor was treated as a pressure dependent variable and not as a constant. This had been proved to increase the accuracy of the pressure transverse computation considerably. Hagedorn and Brown used average pressure as well as constant compressibility factor over the entire length of pipe.

II. The superficial gas velocity was computed pressure over the entire pipe length rather than average pressure values as commonly used. This helped in the easy computation of the acceleration component in the Hagedorn and Brown correlation.

III. The spreadsheet was programmed to compute the pressure gradient with Griffith Correlation during bubble flow regime and Hagedorn and Brown for other flow regimes. This considerably increased the pressure transverse calculation.

The generalized gradient curve was generated at the following standard conditions:

average temperature = 150°F; oil API gravity = 35°API; oil viscosity = 1cP; gas gravity = 0.65; interfacial tension = 30dynes/cm; water specific density = 1.07; depth = 30000ft; well head pressure = 200psia.

2.1. Steps involved in the development of the generalized pressure gradient curve

I. Generate a length increment across the entire tubing length.
II. Starting with the well head pressure, select a pressure increment (\(\Delta P\)) to estimate the pressure across the tubing length.
III. Estimate the compressibility factor at each length.
IV. Estimate other parameters at average pressure using appropriate correlation.
V. Calculate the pressure transverse using modified Hagedorn and Brown equation.
VI. \(P_i + \frac{dP}{dl} \Delta l\).
VII. Adjust \(\Delta P\) until the predicted \(P_{wt} \approx \) calculated \(P_{wt}\).
VIII. Estimate the dimensionless pressure transverse by converting the pressure generated in step 2 into dimensionless pressure using \(\frac{9266.112dP}{\rho \Delta u_m^2}\).
IX. Generate the dimensionless depth increment using \(\frac{dz}{Z}\).
X. Calculate the dimensionless pressure transverse using \(P_D + \frac{dP_D}{dZ_D} \Delta Z_D\).
XI. Adjust $\Delta P$ in step 2 until both predicted $P_{wfD} \approx$ calculated $P_{wfD}$ and predicted $P_{wf} = \text{calculated} P_{wf}$.

### 2.2. Correlation for average density and acceleration parameter

For simplicity and easy computation, the average density is expressed as a function of gas-liquid-ratio and water-cut as deduced analytically from the observed trends in the spreadsheet. 

$$\bar{\rho} = -6.80667 \ln GLR + 28.697 f_w + 64.1795.$$  \hspace{1cm} (6)

When $F_w = 0$, then Equation (6) becomes 

$$\bar{\rho} = -7.0465 \ln GLR + 73.804679.$$  \hspace{1cm} (7)

The acceleration parameter is also an important factor in the determination of the dimensionless pressure. It is expressed as a function of flow rate, tubing size, water-cut, and GLR.

At $GLR \geq 200$.

$$\Delta U_m^2 = \frac{0.0000766 q^4}{d^5} (f_w^2 + 0.0001764GLR^2)$$  \hspace{1cm} (8)

At $GLR < 200$.

$$\Delta U_m^2 = \frac{0.0000766 q^4}{d^5} (f_w + 0.00388GLR)^2$$  \hspace{1cm} (9)

Set, $F_w = 0.2$; $3 \leq f_w \leq 1$

### 2.3. The equivalent gas-liquid-ratio

The given GLR cannot be used on the generalized pressure gradient curve directly; it has to be converted to its equivalent form. The equivalent values can then be read directly on the generalized curve.

$$GLR_{eq} = \frac{GLRqf_w^{0.18}}{110.89d^2}.$$  \hspace{1cm} (10)

At zero water-cut,

$$GLR_{eq} = \frac{GLRq}{125.63d^2}.$$  \hspace{1cm} (11)

### 2.4. Steps involved in taking readings from the generalized pressure gradient curve

I. Calculate the dimensionless pressure using the equation below using equation 3
II. Calculate the equivalent GLR using equation 10 or 11 as the case may be.
III. Calculate the dimensionless depth using equation 4.
IV. Note that $\Delta z = 1250ft$
V. Readings can be taken on the generalized curve after the three steps above using the normal procedure employed in gradient curves.

### 3. Results and discussion

The generalized pressure gradient curve is a semi-log plot of dimensionless pressure against dimensionless depth at different equivalent GLR. It has been discovered that the dimensionless pressure gradient curve gives a similar patterned curve regardless of the well conditions (flow rate, water-cut and tubing size, and GLR) and flow regimes. This single feature of the dimensionless plot endows it with the ability to function as a generalized curve which gives accurate readings for a wide range of well conditions and flow regimes. The semi-log plot does not capture some pressure and depth values along the tubing head region because the logarithm graph does not start from zero. The uncaptured sections are plotted on a Cartesian graph.

### 3.1 Curve validation

The Generalized pressure gradient curve was used to determine the bottom-hole flowing pressure, well head pressure, bottom-hole flowing pressure for directional wells, equivalent surface depth, and equivalent total depth of some wells. The results were validated by comparing the results with that generated from compatible gradient curves. The flow regime in...
this study was estimated using the Dos and Runs Map. The absolute value of the percentage error was used to a yardstick to measure the degree of accuracy of the generalized pressure gradient curve. Absolute percentage error = \[ \frac{|GCR - GGCR|}{GCR} \]

where: GCR = Gradient curves reading; GGCR = Generalized gradient curve readings.

Figure 1. The generalized pressure gradient curve

Figure 2. The generalized pressure transverse curve showing the uncaptured surface depth section

3.1.1. Determination of bottom-hole flowing pressure

Data from Well 1; D=2.992 in (3 ½ tubing); H=8000ft; \( P_{wh} = 240 \) psig; \( F_w = 0 \); GLR=500SCF/STB

Table 1. Table showing the bottom-hole flowing pressure as predicted by the generalized pressure gradient curve

<table>
<thead>
<tr>
<th>qL (STB/day)</th>
<th>Equivalent surface depth (ft)</th>
<th>Equivalent bottom-hole depth (ft)</th>
<th>( P_{wf} ) (gradient curve) psig</th>
<th>( P_{wf} ) (generalized curve) psig</th>
<th>Absolute percentage error</th>
<th>Flow regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>8000</td>
<td>800</td>
<td>8800</td>
<td>3040</td>
<td>3058</td>
<td>0.59</td>
<td>Froth flow</td>
</tr>
<tr>
<td>6000</td>
<td>1100</td>
<td>9100</td>
<td>2640</td>
<td>2667</td>
<td>1.02</td>
<td>Froth flow</td>
</tr>
<tr>
<td>4000</td>
<td>1300</td>
<td>9300</td>
<td>2120</td>
<td>2087</td>
<td>1.56</td>
<td>Froth flow</td>
</tr>
<tr>
<td>3000</td>
<td>1900</td>
<td>9900</td>
<td>2000</td>
<td>1996</td>
<td>0.20</td>
<td>Froth flow</td>
</tr>
<tr>
<td>2000</td>
<td>2000</td>
<td>10000</td>
<td>1760</td>
<td>1638</td>
<td>6.93</td>
<td>Froth flow</td>
</tr>
<tr>
<td>1000</td>
<td>3000</td>
<td>11000</td>
<td>1650</td>
<td>1502</td>
<td>8.97</td>
<td>Slug flow</td>
</tr>
<tr>
<td>800</td>
<td>300</td>
<td>11100</td>
<td>1560</td>
<td>1486</td>
<td>4.74</td>
<td>Slug flow</td>
</tr>
<tr>
<td>400</td>
<td>3200</td>
<td>11200</td>
<td>1460</td>
<td>1420</td>
<td>2.74</td>
<td>Bubble flow</td>
</tr>
<tr>
<td>200</td>
<td>3400</td>
<td>11400</td>
<td>1640</td>
<td>1638</td>
<td>0.12</td>
<td>Plug flow</td>
</tr>
</tbody>
</table>

3.1.2. Determination of equivalent total depth and wellhead pressure

Data from Well 2; \( q_L = 500 \) STB/day; \( GOR=800 \) SCF/STB; \( Pr=4000 \) psig; \( F_w = 0.5 \); \( J=5 \) STB/day-psi.

Table 2. Table showing the equivalent total depth and wellhead pressure as predicted by the generalized pressure gradient curve

<table>
<thead>
<tr>
<th>Equivalent total depth (gradient curve) ft</th>
<th>Equivalent total depth (generalized gradient curve) ft</th>
<th>Absolute percentage error</th>
<th>Wellhead pressure (gradient curve)</th>
<th>Wellhead pressure (generalized gradient curve)</th>
<th>Absolute percentage error</th>
<th>Flow regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>14700</td>
<td>14750</td>
<td>0.34</td>
<td>360</td>
<td>364</td>
<td>1.10</td>
<td>bubble</td>
</tr>
</tbody>
</table>
3.1.3 Determination of equivalent surface depth and bottom hole flowing pressure for directional wells

Data from Well 3: \( q_L = 1500 \text{STB/day} \); \( \text{GLR} = 800 \text{scf/STB} \); \( P_{\text{wh}} = 160 \text{ psig} \); \( \text{MD} = 9000 \text{ft} \); \( F_w = 0 \); \( d = 1.995 \text{in} \); \( \text{TVD} = 6000 \text{ft} \).

Table 3. Table showing the equivalent surface depth and bottom-hole flowing pressure for directional wells as predicted by the Generalized Pressure Gradient Curve

<table>
<thead>
<tr>
<th>Equivalent total depth (gradient curve) ft</th>
<th>Equivalent total depth (generalized gradient curve) ft</th>
<th>Absolute percentage error</th>
<th>Wellhead pressure (gradient curve)</th>
<th>Wellhead pressure (generalized gradient curve)</th>
<th>Absolute percentage error</th>
<th>Flow regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>875</td>
<td>9.38</td>
<td>1760</td>
<td>1820</td>
<td>4.09</td>
<td>Froth flow</td>
</tr>
</tbody>
</table>

4. Discussion

The generalized pressure gradient curve was used to determine the bottom-hole flowing pressure in well 1 under four flow regimes, which include froth flow, bubble flow, slug flow, and plug flow. The generalized curve gave a reading with a high degree of accuracy for plug flow, bubble flow, and froth flow while the percentage error recorded for slug flow when the flow rate was 1000STB/day is 8.94%.

From the observed trends in Figure 3, the generalized curve and the gradient curve assume the same path with very minimal deviation. This confirmed the accuracy of the generalized pressure gradient curve. The generalized pressure gradient curve can, therefore, give accurate bottom-hole pressure reading. From Well 2, the generalized pressure gradient curve predicted the equivalent total depth and wellhead pressure with an absolute percentage error of 0.34% and 1.10% respectively. This shows that the generalized pressure gradient curve predicts the equivalent total depth and wellhead pressure with high degree of accuracy. however, the absolute percentage error of 9.38% was obtained when used to predict equivalent surface depth for directional well under froth flow regime.

5. Conclusion

It is evident from the result that the generalized pressure gradient gives accurate and satisfactory results and can be used in-lieu of vertical gradient curves. The generalized pressure gradient curve predicts the bottom-hole flowing pressure, well head pressure, bottom-hole flowing pressure for directional wells, equivalent surface depth and equivalent total depth with a high degree of accuracy under a wide range of well conditions and flow regimes. Some of the concluding observations made during the course of this research work are as follows:

- The acceleration component which is mostly neglected is an important parameter in the generation of dimensionless Hagedorn and Brown Equation. It is, therefore, indispensable in the generation of the generalized pressure gradient curve.
- The modified Hagedorn and Brown correlation is an effective tool for accurate pressure transverse calculation over a wide range of flow regimes and well conditions. This is partly due to the use of Griffith flow correlation during bubble flow.
The pipe diameter and the flow rate have minimal effects on the mixture density, but their effects become remarkable at water cut of zero.

The GLR changes with water cut, flow rate, and pipe diameter.

The generalized pressure gradient curve can be used by field engineers to optimize production capacity quickly and accurately on production sites.

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**Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>Productivity index, STB (psi)/day</td>
</tr>
<tr>
<td>L</td>
<td>Tubing length, ft</td>
</tr>
<tr>
<td>Psig</td>
<td>Pounds per square inch gauge</td>
</tr>
<tr>
<td>Pwh</td>
<td>Wellhead Pressure, psi</td>
</tr>
<tr>
<td>Pwf</td>
<td>Bottom-hole flowing pressure, psi</td>
</tr>
<tr>
<td>PwhD</td>
<td>Dimensionless bottom-hole flowing pressure</td>
</tr>
<tr>
<td>Pr</td>
<td>Reduced pressure</td>
</tr>
<tr>
<td>H</td>
<td>Well depth, ft</td>
</tr>
<tr>
<td>Pd</td>
<td>Dimensionless pressure</td>
</tr>
<tr>
<td>Zo</td>
<td>Dimensionless depth</td>
</tr>
<tr>
<td>lb</td>
<td>Pound force</td>
</tr>
<tr>
<td>f</td>
<td>Fanning friction factor</td>
</tr>
<tr>
<td>U_m</td>
<td>Mixture velocity, ft/sec</td>
</tr>
<tr>
<td>M</td>
<td>Mass flow rate, lb/sec</td>
</tr>
<tr>
<td>M_i</td>
<td>Liquid mass flow rate, Cu-ft/sec</td>
</tr>
<tr>
<td>in</td>
<td>inch</td>
</tr>
<tr>
<td>cP</td>
<td>Centipoise</td>
</tr>
<tr>
<td>ρ</td>
<td>Density, lb/cu-ft</td>
</tr>
<tr>
<td>qL</td>
<td>Oil flow rate, STB/day</td>
</tr>
</tbody>
</table>

**Abbreviations**

| API   | American Petroleum Institute |
| GOR   | Gas oil ratio |
| MD    | Measured depth |
| STB   | Stock tank barrel |
| SCF   | Standard cubic feet |
| TVD   | True vertical depth |
| GLR   | Gas liquid ratio |
| Psig  | Pounds per Square inch absolute |

**References**


