

## Optimum Well Trajectory Design And Optimization Based On Numerical Optimization Method PSO Algorithm and Wellbore Stability

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### Abstract

Wellbore trajectory optimization without instability well problems represents the most crucial factor in wells drilling planning and design today. Consequently, this article describes a method for designing and optimizing directional and horizontal well trajectories based on PSO algorithm technique of numerical optimization. The primary purpose of this article is to describe and optimum well trajectory design that at the lowest cost can hit the target objective. The design model relates to an actual well trajectory that has been drilled reference to a field geological data. The selected trajectories are S-type profile and double build horizontal trajectory based on field data analysis. The S-type profile has been design using explicit calculations and the double build horizontal trajectory has been design and optimize using numerical optimization technique (PSO algorithm). This article describes the background theory and wellbore stability models used to develop the study models. A side-tracking well (Well-3) of total vertical depth 10729 ft is drilled in order to reach two targets: Upper and Lower Bahariya sandstone formations at 9956 ft, and 10205 ft. The plan trajectory was vertical profile and due to wellbore instability, the well was side-tracked. The two proposed profiles are modelled and selected for this well based on statistical study analysis of actual well data. Based on the data, it was found that the percentage of hole problems lost time is higher with a maximum of 55% and NPT is the major problem facing drilling processes in zone of well 3. The selected S-shape trajectory results in KOP = 8400 ft, INC = 22.604 degrees, the hold section begins with a constant inclination at MD = 9200 ft, TVD = 9178.92 ft, and HD = 157.6 up to MD = 10100 ft, TVD = 10102.1 ft, and HD = 541.965 ft. Drop section starts at MD = 10262 ft, TVD = 10158.98 ft, and the target is hit at MD = 11052 ft, and TVD = 10929 ft. Using MCM, the optimum survey trajectory for the S-type well profile results at MD = 11052 ft, and azimuth = 43.827 degrees. For the numerical design and optimization of the double build horizontal well, 3 different algorithms are programmed with different input parameters to define a set of optimal solutions. The algorithms are PSO1, PSO2, and PSO3. However, the performance of PSO2 is significantly better than PSO1 and PSO3 based on the statistical test applied using the Wilcoxon ranksum test. Therefore, the parameters of PSO2 are the optimal solutions chosen for the well under study (well-3). The result of the optimized parameters is; KOP<sub>1</sub> = 7182 ft, KOP<sub>2</sub> = 8488 ft, hold angle in degrees = 20.4, build-up rate of upper section = 2.9 deg/100ft, and build-up rate for the lower section = 2.5 deg/100ft.

**Keywords:** Horizontal well; S-shape profile; trajectory optimization; MCM; wellbore Stability; PSO algorithm.

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## 1. Introduction

Optimum well trajectory design is an important factor in drilling process and a key element in achieving a successful drilling operation. Several authors have studied the problem of optimum well trajectory design and its significant importance to the success of the drilling operation. There are a range of drilling techniques each with their own advantages. The term oil well drilling is a wide term that refers to all operations needed to design and drill wells of circular section to reach a target destination located at some distance from the surface.

Due to the lower oil recovery produced by vertical drilled wells, the oil and gas industry is facing the need for more and more directional drilling as drilling operations continue to grow.

Directional drilling is defined as a well-bore deflection from vertical to achieve the purpose of reaching a predetermined target below the earth surface. Directionally drilled wells are an effective way to reach a specific target that are hard to reach with vertically drilled wells. In a successful drilling operation, the main objectives are to design safe and economically efficient wells, the drilling operation's success depends mainly on ideal trajectory selection for drilling. Success, however, also relies on reaching the target. Due to the high-quality demands of the drilling procedure, a successful drilling operation is based on a healthy well plan. Many parameters are involved in well trajectory planning, but the goal is to define the best well path that will successfully reach the target without any instability problems. This means that the design should be based on accurate mathematical calculation models to calculate the well-bore trajectory accurately. The benefits of horizontal and directional drill wells are that they can contact a larger reservoir volume and can more efficiently hit the least thickness pay zone areas than vertical wells, leading to higher exploration and recovery rates, Robert and Stefan [10]. Therefore, drilling optimizations are vital to decrease exploration and growth expenses.

Formulas and mathematics of horizontal and directional trajectories are presented by several authors supported by field cases and example calculations [2-6]. Some of them presented the optimization of these trajectories.

Particle swarm optimization (PSO) is a metaheuristic method of optimization introduced by Kennedy and Eberhart [15]. The PSO is an algorithm based on the population size that generate new solutions from current alternatives making few assumptions about the problem to be optimize with the ability to search for the global optimum solutions. The PSO algorithm utilizes a cooperative optimization search approach using communication between potential candidate solutions known as particles. This is achieved by adjusting the particles position and velocity continuously in the search space using a simplified mathematical formula that determine the position and velocity of each particle as expressed by Onwunalu [16]. The PSO algorithm have been found successful in different applications used by various authors to design and optimized a well-bore trajectory. For example: method for designing the optimal trajectory for drilling a horizontal well, based on particle swarm optimization and analytical hierarchy process by Jun Li [17]; designing and optimizing deviated wellbore trajectories using novel particle swarm algorithm by Amin and David [18]; and optimization of field development using particle swarm optimization and new well pattern descriptions by Onwunalu [16].

The optimum well trajectory design and optimization method used in this article was developed using particles swarm optimization (PSO) and wellbore stability. The primary purpose is to design an optimum well trajectory that at the lowest cost can hit the target objective considering the effect of wellbore stability. The design model in this article is based on a real field drilling report that comprises of (actual well/drilling data, geological data, and reservoir/physical data). The selected trajectories are S-type profile and double build horizontal trajectory. The S-type profile is designed using manual numerical mathematical calculations in excel and the double-build horizontal trajectory is that which is designed, developed and optimized using numerical computer simulation (particle swarm optimization algorithm). This article describes the background theory used to design and develop the model.

The actual selected trajectory for the well under study is S-type profile which was encountered by poor wellbore stability and lead to well side-tracking. The S-type profile has been re-design and updated to hit the target without any instability problem and a new trajectory which is the double-build trajectory has been designed, developed and modelled, which becomes the new proposed well-trajectory for the well under study considering the required field data. The design pass through the objective target. Table 1 describes the main well drilling problems.

The well under study has been investigated based on the drilling problems presented in Table 1. Side-track happens due to poor well-bore stability and drilling fluid properties which causes the inability to drill hard formations successfully. In order to design the optimal well trajectory that will be drilled successfully, the main drilling parameters has been optimized through numerical simulation. Well-bore design planning is needed in order to build a smooth three-dimensional well path that links the surface with the required target.

Table 1. Main well drilling problems

Phase size	Drilling problems	Actions taken
17 1/2 "	Had complete loss	Pumped 100 bbls LCM (100 ppb) cont'd blind drilling to 805 ft, spotted 200 bbls LCM (120 ppb, coarse grains highly concentrated) waiting for soaking, cont'd blind drilling to 1545 ft (casing point).
12 1/4 "	<ul style="list-style-type: none"> <li>Got high torque at 5149 ft</li> <li>Shoe plugged while displacing cement</li> </ul>	<ul style="list-style-type: none"> <li>POOH to change bit.</li> <li>RIH w/5 "OEDP, clean out CMT "tail slurry "to 1500 ft, POOH to surface. And washed down soft cement fm/1500 ft to /5270 ft. Drilled out soft to medium cement fm /5550 ft to 8385 ft.</li> </ul>
8 1/2 "	<ul style="list-style-type: none"> <li>String stucked at 8560 ft</li> <li>Had back off tools stucked at 8414 ft (oil jar depth)</li> <li>Had tight hole at 9700 ft and 8560ft</li> </ul>	<ul style="list-style-type: none"> <li>Kept string under compression and right-hand torque for 30 min's then jarring down for 30 min's, with no progress. Apply Max.torque and slacked down all string weight to 5 Hr's with no progress. Back off string and side-track the well.</li> <li>Worked on till get free</li> <li>Worked on same w /50 klbs max.o. pull and 120 klbs slack off weight, got free.</li> </ul>

In relation to 3D geometric demands, other factors that linked to the drilling process are considered. Such include the drill string mechanical integrity, well-bore stability, casing, cementing and perforating activities, etc. The design criterion to achieve the optimal solution include; the shortest well trajectory length to hit the target, the target hitting accuracy, the lowest drilling cost and the minimum drill-string friction. Several iterations have been carried out before the desired solution is reached.

**2. Horizontal well profiles**

Horizontal well as described by Figure 1, consist of double build sections. A vertical section, followed by the first build-up section, a tangent section followed by the second build-up section and a horizontal section to the target location. The well departure is described as a vertical displacement to the point where the well reaches the horizontal section starting point.

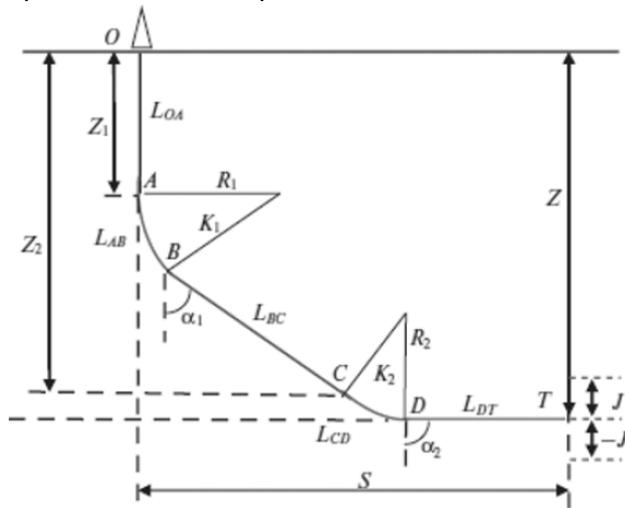


Figure 1. Double build horizontal well profile [17]

Horizontal displacement is the sum of the length and departure of the horizontal segment. The parameters  $Z_1$  and  $Z_2$  are the first and second kick-off points,  $K_1$  and  $K_2$  are the upper and lower build-up rates,  $R_1$  and  $R_2$  are the radius of curvature for the first and second build-up segment, the deviation angle of the hold section is  $\alpha_1$  and  $\alpha_2$  is the deviation angle of the horizontal segment,  $\alpha_2 = 90^\circ$ . The total vertical depth is  $Z$  and  $S$  is the horizontal displacement,  $L$  depicts the well measured depth, and  $J$  represents the offset of the target in the vertical direction. Section AB, BC, CD, DT are the upper build, tangent, lower build, and horizontal sections respectively.

The mathematical equations used in calculating the well trajectory are the same as those used in directional calculations presented by Mitchell [2], Jun Li [17], and Halafawi and Avram [19]. There is an exception, however, that the primary unknown in the combination trajectory plan

is the KOP depth. The problems also result in horizontal turn trajectory, vertical turning calculations, selection of mud weight, 3D profile building and selection of survey methods. The equations for calculating the KOP, horizontal turn trajectory, and vertical turn determination are as follows.

$$\text{Upper build section } L_{AB} = \frac{1719}{K_2} \alpha_1 \frac{\pi}{180} \quad (1)$$

$$\text{Lower build section } L_{CD} = \frac{1719}{K_2} (\alpha_2 - \alpha_1) \frac{\pi}{180} \quad (2)$$

$$\text{Hold section } L_{BC} = \frac{(Z_2 - Z_1) - \frac{1719}{K_1} \sin \alpha_1}{\cos \alpha_1} \quad (3)$$

$$\text{Inclined } MD = R_1 K_1 + L_{BC} + R_2 K_2 \quad (4)$$

$$TMD = Z_1 + L_{AB} + L_{BC} + L_{CD} + L_{DT} \quad (5)$$

$$\text{Horizontal departure to the target } (T) = L_{DT} \sin \alpha_2 \quad (6)$$

$$\text{Change in TVD of horizontal departure} = L_{DT} \cos \alpha_2 \quad (7)$$

The resulting problems during planning a horizontal turn and vertical turn in the horizontal section of the planned wellbore can be solved using Mitchell [2] mathematical equations and programs.

### 2.1. Selection of mud weight for horizontal wellbore

Mud weight selection methods used in vertical well can't be used in directional or horizontal holes. Therefore, Mitchell [2] suggested the following equation for selecting mud densities (ppg) to stabilize drill holes:

$$MW_{horizontal} = MW_{vertical} + (OBW - LOT) \frac{1 - \cos 2\phi}{1.6} \quad (8)$$

where MW = Mud weight, ppg; OBW = Overburden weight (Overburden stress), ppg; LOT = Leak off test value, ppg; and  $\phi$  = Inclination, degrees.

### 3. Three-Dimensional well profile for a wellbore trajectory

A smooth 3D route that connect a surface or subsurface location to the required target or targets is needed for the design scheme. Some factors that relate to the drilling process should also be considered by the designer in addition to 3D geometric requirements. The factors include the mechanical integrity of the drill string, stability of the wellbore, transportation of cuttings, cementing, perforation procedures and casing operations Robert and Stefan [10]. Usually minimum drilling cost and minimum drilling time are used as the criterion for well-path optimization and the process requires few iterations before the design goal is achieved. There are five methods available for the design of a 3D well trajectory. The five methods Robert and Stefan [10] are the average angle method (AAM), Radius of curvature method (RCM), Constant build and turn rate (CBTM), Constant curvature and build rate (CCBM), and minimum curvature method

(MCM). 3D geometric considerations and the methods description are presented by Azar [1], Mitchell [2], Bourgoyne [3], Hossain [5], Rabia [9], and Robert [10]. However, the most commonly used method in the petroleum industry for both well trajectory planning and directional survey evaluation is the minimum curvature method (MCM) Robert and Stefan [10].

#### 3.1. Minimum curvature method (MCM)

Taylor and Mason [12], and Zaremba [14] initially suggested MCM as a way to enhance the analysis of the directional survey. Zaremba used the concept of circular-arc technique and used the vector technique to carry out the development. Sawaryn and Thorogood [11] have recently released helpful algorithms for directional-well planning and orientation of deflection tools. The equations of the MCM are as follows:

$$RF = \frac{\Delta MD}{\beta} \tan \frac{\beta}{2} \quad (9)$$

$$\Delta X = (\sin \phi_1 \cos \vartheta_1 + \sin \phi_2 \cos \vartheta_2) RF \quad (10)$$

$$\Delta Y = (\sin \phi_1 \sin \vartheta_1 + \sin \phi_2 \sin \vartheta_2) RF \quad (11)$$

$$\Delta Z = (\cos \phi_1 + \cos \phi_2) RF \quad (12)$$

where RF = Ratio Factor;  $\beta$  = Dog-leg angle; deg,  $\phi$  = inclination angle; deg.  $\vartheta$  = Azimuth angle, deg.

#### 4. Wellbore instability prediction modelling

There are various models that have been developed by several authors. However, Halafawi and Avram [22] presented most of well-bore stability models with equations used for horizontal and deviated wells based on equations of in-situ stress. In order to meet the goal of running the established model to predict well-bore instability issues and to determine the optimum mud weight for future drilling operations, it is essential to estimate in-situ stress, pore stress, rock failure criteria, rock mechanical characteristics, and rock strength etc. The in-situ stress and pore pressure prediction equations are shown in Table 2.

Table 2. In-situ stresses and pore pressure correlations and equations [22].

Author/Publisher Name	Vertical Stress Formula	
Yi et al. [15], and Zobak [18]	$\sigma_v = \int_0^H \rho_b(h)dh$	(13)
Author/Publisher Name	Min. Horizontal Stress Formula	
Yi et al. [15], Hudson et al. [19], and Biot [20]	$\sigma_h = \frac{\nu}{1-\nu} (\sigma_v - \alpha_b P_p) + \alpha_b P_p$	(14)
Ahmed et al. [21], Cipolla et al. [22], and Iverson [23]	$\sigma_h = \frac{\nu}{1-\nu} (\sigma_v - \alpha_b P_p) + \alpha_b P_p + \sigma_{tec}$	(15)
Mike Mullen Equation [19, 28]	$\sigma_h = \sigma_v \left( \frac{\nu_{fast}}{1-\nu_{fast}} \right) - \alpha P_r \left( 1 - \frac{\nu_{fast}}{1-\nu_{fast}} \right)$	(16)
Blanton and Olson [24]	$\sigma_h = \nu C_1 \varepsilon_{tec} + C_2$ where: $C_1 = \frac{E}{1-\nu^2}$ ; $C_2 = \frac{\nu \sigma_v + (1-2\nu) \alpha P_p + E \alpha t \Delta T}{1-\nu}$ ; $\varepsilon_{tec} = \frac{S_h - C'_2}{\nu C'_1}$	(17)
Harikrishnan and Hareland [25]	$\sigma_h = K_o (\sigma_v - \alpha P_p) + \alpha P_p$ where: $K_o = 1 - \sin \beta$ ; $\beta = \arcsin \left( \frac{S_2 - S_1}{S_2 - S_1 + 4\Delta} \right)$ ; $S_1 = C_o (1 + a_s (P_e - \Delta))^{b_s}$ ; $S_2 = C_o (1 + a_s (P_e + \Delta))^{b_s}$	(18)
Author/Publisher Name	Max. Horizontal Stress Formula	
Barton et al. [27]	$\sigma_H = \frac{C_o + \Delta P_w + 2P_r}{1 - 2\cos 2\theta} - \sigma_h \frac{1 + 2\cos 2\theta}{1 - 2\cos \theta}$	(19)
Mike Mullen Equation [19, 28]	$\sigma_H = \sigma_v \left( \frac{\nu_{slow}}{1-\nu_{slow}} \right) - \alpha P_r \left( 1 - \frac{\nu_{slow}}{1-\nu_{slow}} \right)$	(20)
Addis et al. [26]	$\frac{\sigma_H}{\sigma_h} = \nu \left( 1 + \frac{1}{K_\beta} \right) + \frac{P_p}{\sigma_v} \left[ 1 - \nu \left( 1 + \frac{1}{K_\beta} \right) \right]$ for Normal Fault	(21)
	$\frac{\sigma_H}{\sigma_h} = \nu (1 + K_\beta) + \frac{P_p}{\sigma_v} [1 - \nu (1 + K_\beta)]$ for Thrust Faults	(22)
	where: $K_\beta = \frac{\sin(\theta + 2\beta) + \sin \theta}{\sin(\theta + 2\beta) - \sin \theta}$	
Author/Publisher Name	Pore Pressure Formula	
Eaton [29]	$P_r = \sigma_v - (\sigma_v - P_{rn}) \left( \frac{\Delta t_{norm}}{\Delta t_{log}} \right)^x$	(23)

#### 5. Methodology flow diagrams

Fig. 2, describes the flow diagram or the process steps followed in order to solve the drilling problems (pore wellbore stability) of Well-3 (well under study). The input steps in (C) are those described in PSO algorithm description and steps in this article, and supplementary diagrams present the steps involved in (A) and (B) respectively.

#### 6. PSO algorithm description

The PSO algorithm is a stochastic optimization process that is based on population size, each particle in the swarm moves to a new place in the search space at each iteration and each particle is a potential candidate solution to the optimization problem. Let  $x$  be referred to as a potential solution to a  $n$ -dimensional optimization problem in the search space, therefore the position of particle  $i$ th in iteration  $t$  will be denoted as;  $X_i(t) = \{x_{i,1}(t); \dots; x_{i,n}(t)\}$ , particle  $i$ th best previous solution found so far up to iteration  $t$  can be denoted as  $X_i^{Pbest}(t)$ ,

the global best which belongs to all particles in the search space is denoted as  $X_g(t)$ . The  $(i)$  is the index of particle and  $(X)$  is the vector of position.

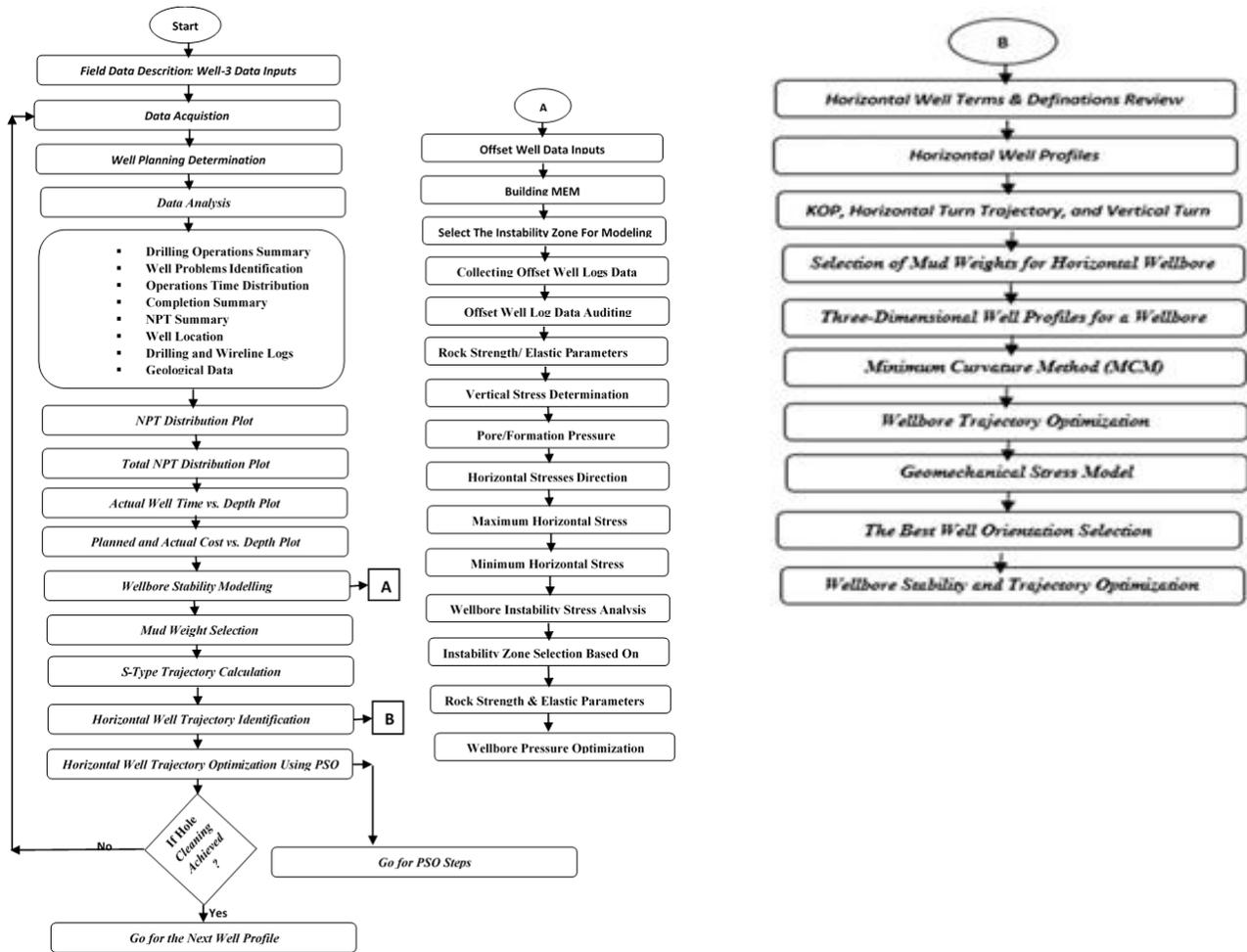


Figure 2. Methodology flow diagrams of Well 3 study

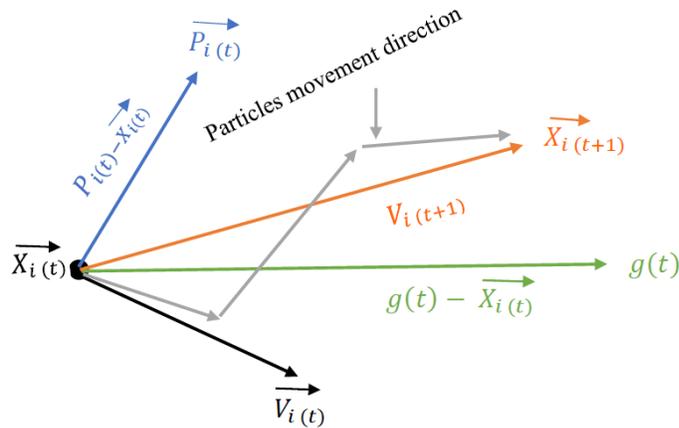
The search space is a set of all real numbers from (0 to 1). The position and velocity of a particle are vectors on the same dimension and the velocity describes the movement of particle  $(i)$  in the sense of direction and in the sense of distance and step size. At every iteration of PSO, the position and velocity of particles are updated. By adding a velocity,  $V_i(t + 1)$ , to the current position  $x_i(t)$ , Kennedy and Eberhard [15], the new position of particle  $i$  in iteration  $t + 1$ ,  $x_i(t + 1)$ , is updated.

$$X_i(t + 1) = X_i(t) + V_i(t + 1) \cdot \Delta t \tag{24}$$

The parameter  $V_i(t + 1) = \{v_{i,1}(t + 1), \dots, v_{i,n}(t + 1)\}$  represents the velocity of particle  $i$  at iteration  $t + 1$ ,  $\Delta t$  is an increment of time usually set to be equal 1 in standard PSO implementation. However, it should be observed that latest research has shown enhanced outcomes using a variable time increment, Martinez and Ganzalo [20]. The velocity vector components are calculated as presented by Engelbrecht [23].

$$\begin{aligned} V_{ij}(t+1) &= \omega * V_{ij}(t) \rightarrow \text{Inertia Term} \\ &+ C_1 * r_1 * (X_{ij}^{pbest}(t) - X_{ij}(t)) \rightarrow \text{Cognitive Components} \\ &+ C_2 * r_2 * (X_g^{best}(t) - X_{ij}(t)) \rightarrow \text{Social Components} \end{aligned} \tag{25}$$

where the parameters  $\omega$ ,  $c_1$ , and  $c_2$  represent the weights  $r_1$  and  $r_2$  are diagonal matrices with uniformly distributed random variables in the range of  $[0,1]$   $j$  is the  $j$ th components of the optimization variables  $\{1, 2, 3, \dots, n\}$ .



Clerc [21] performed numerical experiment and proposed the values of  $\omega = 0.721$  and  $c_1 = c_2 = 1.193$ . Figure 3 demonstrates the velocity calculation and update of the solution in iteration  $(t)$  to iteration  $(t + 1)$ .

Figure 3. PSO velocity and position update for particle  $X_i$  from iteration  $(t)$  to  $(t + 1)$

### 6.1. PSO algorithm steps

The number of population size is set from (50 up to 200) and the results presented by each set value is investigated. The maximum number of iterations is set to (1000). The steps are as follows:

- Step 1: Initializing the PSO parameters  $\omega$ ,  $C_1$ ,  $C_2$ , nPop (number of population size), nVar (number of unknown variables), Varsize (matrix size of decision variables) and maxiter (Max. number of iterations).
- Step 2: Set the index of iteration  $t = 1$ : nPop
- Step 3: Initialize the swarm (particles position),  $x_{i,j}(t)$ , with a set of uniformly distributed random values for  $(1 \leq i \leq 50 \text{ to } 200)$ .
- Step 4: initialize the swarm's (particles velocity),  $v_{i,j}(t)$ , and set to zero
- Step 5: compute the objective function, for this case is to optimize the decision variables for the optimum trajectory design and computes the fitness values functions of the optimized variables (minimum trajectory length to hit objective target, minimum drilling cost, and maximum accuracy of target hitting).
- Step 6: update the particles previous best position
- Step 7: update the swarm's global best (optimum best solution)
- Step 8: set the algorithm to a set of conditions so that, if  $t >$  maximum number of iterations, quite the program else do the following:
  - Adjust the parameter to adapt
  - Updates the swarm's original state and readjust the particles if the particles are out of search landscape
  - Calculate fitness values (optimized feature values) and update the information file
  - Check that the maximum capability of the file is not exceeded
  - Update the values of personal best pbest (of particle  $i$ ) and swarm's global best
  - For when  $t = t+1$ , (next iteration), then go back to check the loop count
  - The algorithm terminates the program when the maximum loop count is reached.

Based on the above-mentioned steps, the program was developed in MATLAB.

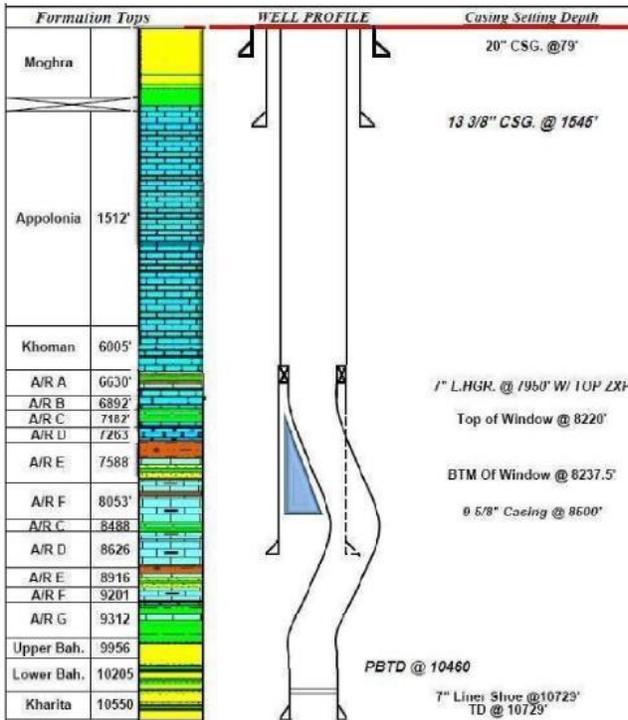


Figure 4. Well-3 final profile

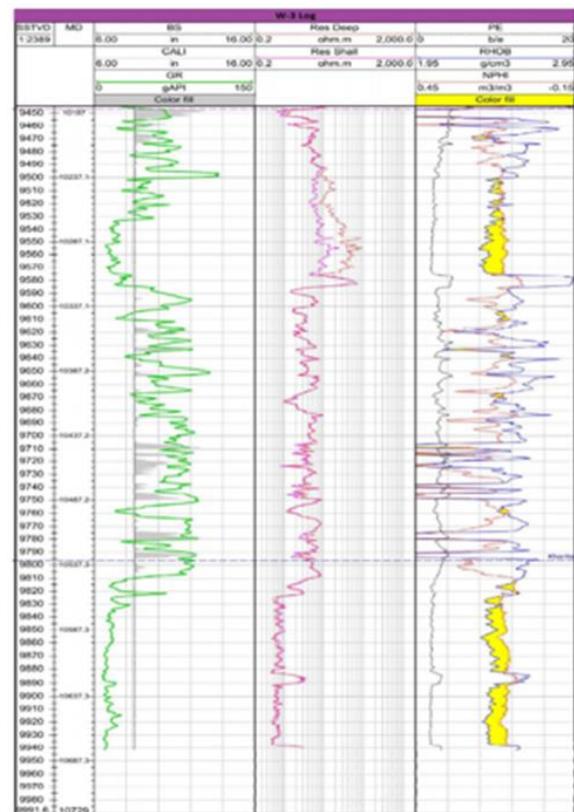


Figure 5. Well 3 Logging

### 7. Field data description

Well 3 is a side-tracking well of total depth 10729 ft. The well is drilled in order to reach 2 targets: Upper and Lower Bah. sandstone formations at 9956 ft., and 10205 ft. respectively. However, while drilling 12 1/4" hole and section from 8505 ft. to 8658 ft., and while reaming the hole section, the stand and the string got stuck at 8560 ft., 60 ft. in open hole in A/R-C shale formation. Worked on stuck with while applying 500-1500 psi, no progress was achieved. Apply maximum torque and slacked down all the string weight for 5 hrs. but still no progress was seen. Therefore, back off string and side-tracking the well was performed. Well 3 geological column, casing setting points, formation tops, and actual well profile are shown in Fig. (4). Well 3 logs are shown in Fig. (5).

### 8. Field data analysis, results and discussion

Data acquisition and statistical data analysis are implemented for well 3 for all drilling phases and operations in order to identify well problems, show operations time distribution, determine the NPT distribution, construct the well time–depth plot for suggested trajectories, and compare with the planned and actual well cost–depth plot (Figs 6&7). Fig.6 describes the NPT distribution and the total NPT percentage and the total operating time of well 3. The percentage of hole problems lost time is higher with a maximum of 55%. NPT is one of the major problems the drilling industry is facing. The greater the lost time the greater the cost of drilling. Understanding what causes NPT, is the key to reducing drilling cost and saving cash. The NPT is described as the period of stoppage of drilling or the rate of penetration is very small.

The time taken in fishing operation, stuck pipe, transportation of drilling tools, circulation lost and tripping in and out are all counted as NPT. Basically, it is difficult to drill a well without any difficulties or drilling problems. Controlling or handling the drilling risk, knowing when and where the problems may probably occur or are likely to occur minimizes the risk. In most



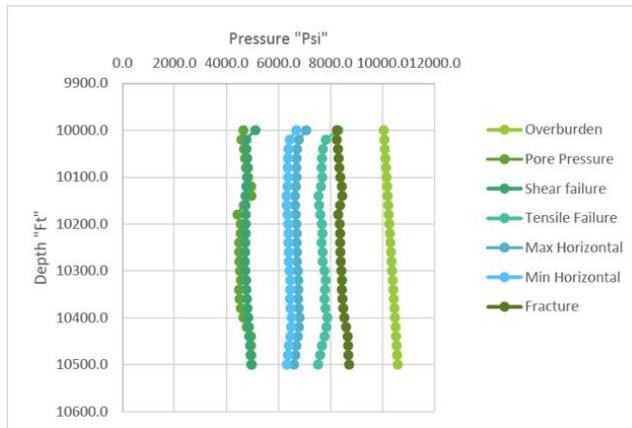


Figure 8. Wellbore in-situ stresses and pressure

During the actual trajectory drilling, while drilling the 12 1/4" hole, the driller got a high torque at 5149 ft. and pool out of casing to change the drilling bit. Also shoe plugged while displacing cement and the action was to washed down soft cement from 1500 ft. to 5270 ft. and drilled out soft to medium cement from 5550 ft to 8385 ft. Secondly, during drilling of the 8 1/2 ", the string was stuck at 8560 ft., the driller kept the string under compression & right hand torque for 30 min's then jarring down for 30 min's, but there is no progress. After that a 500-1500 psi is

applied and the results shows no progress, the driller apply maximum torque and slacked down all string weight for 5 hrs. and still no progress.

Due to poor well-bore stability identification, the string was back off and the well was side tracked. Drilling through the side-tracked hole also results in tight hole at 8560 ft. and 9700 ft. but the driller approaches the problem with 50 klbs MOP and 120 klbs slack off weight and got free.

However, using logging data, a new geometry and well-bore stability model was designed using the minimum curvature method to optimize and update the side-tracking profile to reach to the target without any other instability problems. After designing and updating the well trajectory, the target is reached at MD = 11052 ft, TVD = 10929 ft, and azimuth = 43.827 deg. The reason for well-3 side-tracking was due to poor well-bore stability determination, for this reason, LWD data and drilling data were used to build a new geo-mechanical model and well-bore stability model to be able to optimize and update mud profile so that instability zones can be drill easily. Firstly, the MEM is developed for instability zones of well 3 as illustrated in Figure 2. The MEM results for wellbore instability zone which include minimum horizontal stress ( $\sigma_h$ ), overburden stress ( $\sigma_v$ ), and pore pressure (Pp) are shown in Figs.(8&9). After the wellbore stability has been updated, the optimal wellbore trajectory can be drilled safely using the optimum mud weights determined in Table 3.

Based on geological data and target location and in order to prevent formation instability for well 3, two trajectory profiles are recommended and selected. The first is the build, hold and drop often called the "S-shape" and the second is the double build horizontal trajectory profile. For the S-curved type, explicit design and calculations have been carried out. For the double build horizontal trajectory, the model of optimum trajectory is programmed and run in MATLAB, the model was evaluated using different approaches.

**First scenario: S-Profile**

In order to prevent hard formation, the KOP was also selected based on the lithology of each formation. Also, the minimum curvature method was selected to build the optimum survey trajectory for the S-type profile. Considering the target location, the trajectory was designed to be drilled vertically from 0 ft. to 8300 ft. and the KOP was set at 8400 ft. reference to the target true vertical depth (TVD) and horizontal departure to the target. The results obtained from the surface location to the target location is presented in Table 4.

The well-bore trajectory initial and target coordinates are chosen to hit the target at Bahariya zones. The S-type profile is selected for this target location, the design output is described in Table 4. The design is to drill the conductor, surface and intermediate hole vertically to 8300 ft. depth. Then KOP at 8400 ft. depth at 0 inclination angle to 8500 ft. The build section begins at 8500 ft. with inclination angle of 2.86 deg. at 8499.96 ft. true vertical depth (TVD) and 2.495 ft. horizontal departure. The build section ends at 9190 ft. measured depth, with 22.604 deg. inclination, with 153.895 ft. departure at true vertical depth of 9170.02 ft. The

tangent (hold) section begins with constant inclination angle of 22.604 deg. at 9200 ft. measured depth, 9178.92 (ft) TVD and a departure of 157.6 up to 10100 ft. MD, TVD = 10102.1 ft., and HD = 541.965 ft. Drop section starts at MD = 10262 ft., TVD = 10158.98 ft and the target is hit at MD = 11052 ft, TVD = 10929 ft. respectively. Using the minimum curvature method (Table 5) to design the optimum survey trajectory for the S-type well, it resulted in MD = 11052 ft. and Azimuth = 43.827 deg. Design profile is shown in Figure 10. Based on the planned design output results, it indicated an optimum design because the target is hit successfully.

**Second scenario: Double build horizontal profile**

The double build horizontal trajectory design and optimization are programmed in MATLAB using the particle swarm optimization algorithm (PSO). Three algorithms were programmed using [50 & 200] particles in the search space at a cost of 1000 iterations. The first algorithm (PSO1) is programmed with [50] particles, inertia weight of [w = 0.721], velocity limitations and constraints handling approach using penalty method. The second algorithm (PSO2) is set with inertia weight of [w<sub>min</sub> = 0.2 and w<sub>max</sub> = 0.9], [50] candidate solutions in the search space and the third algorithm is programmed with [200] candidate solutions, and inertia weight of [w<sub>min</sub> = 0.2 and w<sub>max</sub> = 0.9]. All the 3 algorithms are set as constrained optimization algorithms. Each of the algorithms is run 5 times and each is programmed to display the average values at the end of runs. Based on the calculation algorithm, the values of the parameters to be optimized (depth of first and second kick of points, upper and lower build of rates, tangent angle) were obtained and the results are presented in Table 6.

The algorithms allow us to select a group of alternatives, each containing the highest optimal solution for one of the parameters. In order to evaluate the performance of each algorithm to know which algorithm performs significantly better than the other, a statistical test was applied using the WILCOXON RANKSUM test in MATLAB to compare the performance of the algorithms.

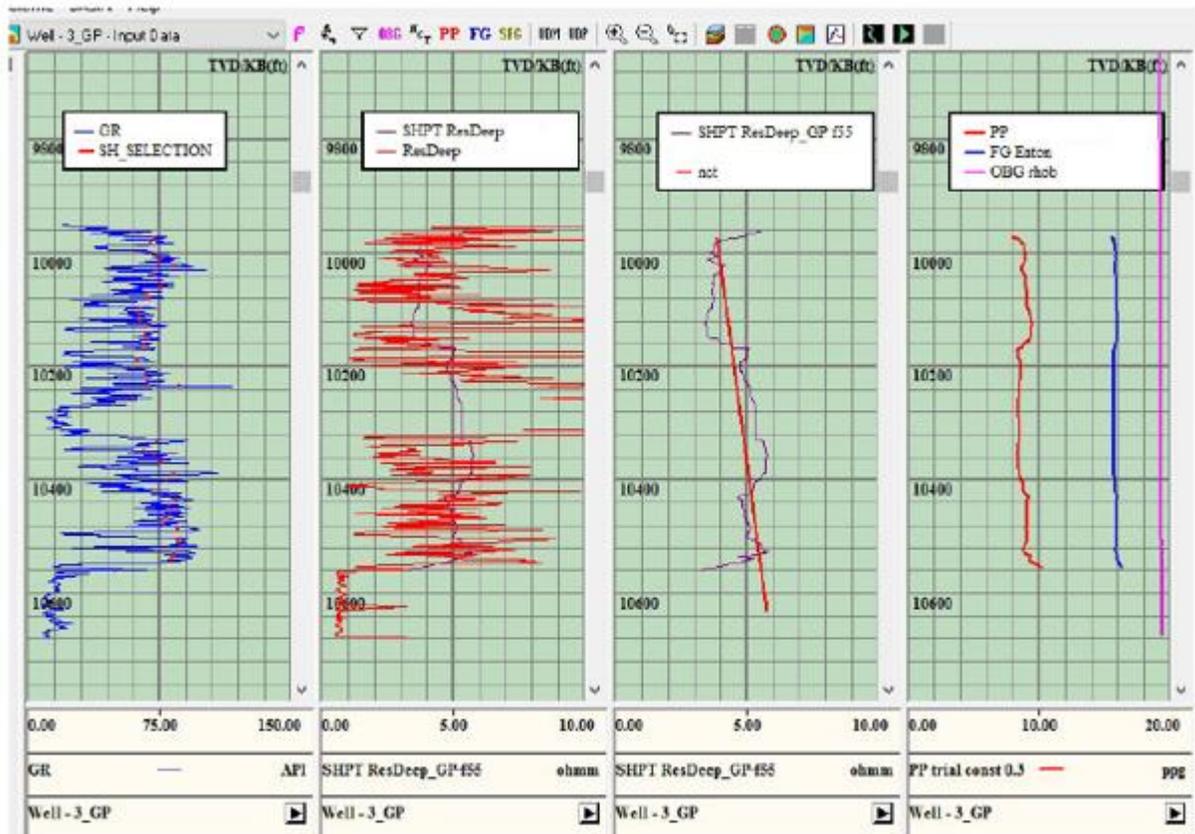


Figure 9. Pore pressure, overburden and fracture pressure of well-3

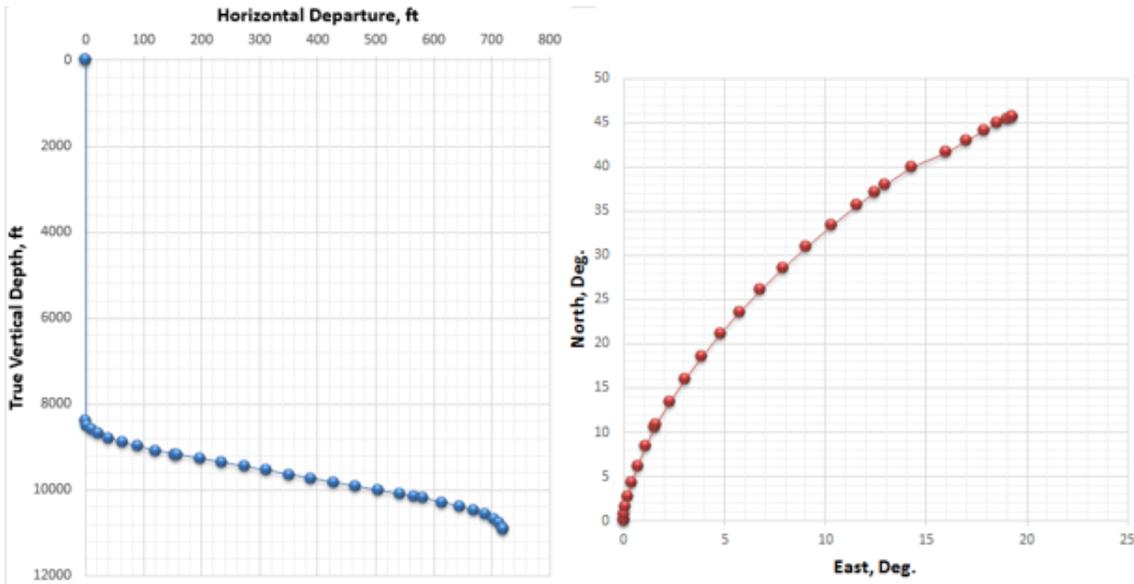


Figure 10. S-shape wellbore trajectory of well 3

To be able to perform this test, we specify two input usually vectors,  $P = \text{RANKSUM}(X, Y)$ . In order to call the input as a function, we set the programme to run two algorithms and put all the best solutions found in  $X$  at the end of runs and same approach for  $Y$ . That will give us a P-Value, and the P-value determines if one algorithm statistically performs better than the other and that depends on the significance level considered so that we can judge the outputs if one algorithm is significantly better than the other. This approach allows us to make a reasonable decision because one algorithm may have the shortest well trajectory length, but this results in high cost or low hitting accuracy. The results of the PSO algorithms PSO1, PSO2, and PSO3 presented in (Table 6) is a set of optimal alternatives for which any of the alternatives can be the optimal solution. The objective functions of the optimized parameters are presented in Table 7 and Figure 11 describes how the particles in the search space converge towards the global optimum solution as the iteration continues to find the optimum solution for the minimum trajectory length (MD) to hit the target successfully.

Table 4. Directional design trajectory for S-type well 3

Measured depth (ft)	Inclination (deg)	TVD (ft)	Horizontal Departure (ft)	Description
0	0	0	0	vertical drilling
100-8300	0	100-8300	0	vertical drilling
8400	0	8400	0	kick of point
8500	2.86	8499.96	2.495	Build
8600	5.72	8599.67	9.975	Build
8700	8.58	8698.88	22.42	Build
8800	11.44	8797.35	39.801	Build
8900	14.3	8894.83	62.072	Build
9000	17.16	8991.07	89.18	Build
9100	20.02	9085.84	121.056	Build
9190	22.604	9170.02	153.895	end of build
9200	22.604	9178.92	157.6	Hold
9300	22.604	9271.23	196.036	Hold
9400	22.604	9363.55	234.473	Hold
9500	22.604	9455.87	272.909	Hold
9600	22.604	9548.19	311.346	Hold
9700	22.604	9640.51	349.782	Hold
9800	22.604	9732.82	388.219	Hold
9900	22.604	9825.14	426.655	Hold
10000	22.604	9917.46	465.092	Hold
10100	22.604	10009.78	503.528	Hold
10200	22.604	10102.1	541.965	Hold
10262	22.604	10158.98	565.649	start to drop
10300	21.507	10194.55	580.061	Drop
10400	18.647	10288.47	614.385	Drop
10500	15.787	10383.98	643.981	Drop
10600	12.927	10480.84	668.774	Drop
10700	10.067	10578.83	688.703	Drop
10800	7.207	10677.68	703.718	Drop
10900	4.347	10777.17	713.782	Drop
11000	1.487	10877.03	718.87	Drop
11052	0	10929	719.544	Target

Table 5. Minimum curvature results for S-type profile

MD(ft)	$\theta$ (deg)	D1	D2	FC	$\Delta TVD(ft)$	$\Delta North(deg)$	$\Delta East(deg)$
8400	0	1	0	0	0	0	0
8500	1.565	0.998	0.049	0.069	6.976	0.174	0.004
8600	3.130	0.998	0.049	0.069	6.959	0.521	0.023
8700	4.695	0.998	0.050	0.069	6.924	0.866	0.061
8800	6.261	0.998	0.050	0.069	6.872	1.207	0.118
8900	7.826	0.998	0.050	0.069	6.803	1.542	0.192
9000	9.391	0.998	0.050	0.069	6.717	1.870	0.285
9100	10.956	0.998	0.050	0.069	6.615	2.189	0.395
9190	12.522	0.998	0.046	0.069	5.852	2.234	0.466
9200	14.087	0.999	0.010	0.069	6.644	0.261	0.061
9300	15.652	0.999	0.010	0.069	6.445	2.593	0.688
9400	17.217	0.999	0.010	0.069	6.445	2.573	0.759
9500	18.783	0.999	0.010	0.069	6.445	2.551	0.829
9600	20.348	0.999	0.010	0.069	6.445	2.528	0.898
9700	21.913	0.999	0.010	0.069	6.445	2.502	0.967
9800	23.479	0.999	0.010	0.069	6.445	2.475	1.035
9900	25.044	0.999	0.010	0.069	6.445	2.446	1.102
10000	26.609	0.999	0.010	0.069	6.445	2.415	1.168
10100	28.174	0.999	0.010	0.069	6.445	2.382	1.234
10200	29.740	0.999	0.010	0.069	6.445	2.347	1.299
10262	31.305	0.999	0.010	0.069	3.995	1.432	0.844
10300	32.870	0.999	0.021	0.069	2.458	0.844	0.528
10400	34.435	0.998	0.050	0.069	6.555	1.995	1.325
10500	36.001	0.998	0.050	0.069	6.666	1.688	1.688
10600	37.566	0.998	0.050	0.069	6.761	1.387	1.034
10700	39.131	0.998	0.050	0.069	6.839	1.092	0.861
10800	40.697	0.998	0.050	0.069	6.900	0.805	0.670
10900	42.262	0.998	0.049	0.069	6.943	0.527	0.463
11000	43.827	0.998	0.049	0.069	6.970	0.261	0.240
11052	43.827	0.999	0.025	0.069	3.629	0.033	0.032
				$\Sigma$	176.537	45.751	18.784

Table 6. PSO algorithm optimum solutions for the optimized parameters

Optimized parameters	PSO1	PSO2	PSO3
1 <sup>st</sup> KOP, ft.	6900	7182	7010
2 <sup>nd</sup> KOP, ft.	8115.4	8488	8219
Hold Angle, deg.	17.3	20.4	17.9
Upper BUR, deg./100ft.	1.5	2.9	2.13
Lower BUR, deg./100ft.	2.29	2.5	2.1

Table 7. Objective function of PSO optimized parameters

Optimized objective functions	PSO1	PSO2	PSO3
Target hitting accuracy, %	99.97	99.99	99.8
Horizontal trajectory length, ft	12030	12047	12404
Drilling cost, \$	88 8518.19	80 740	89 507.65

Table 8. P-value for WILCOXON RANKSUM test

Set	P-Value	Output
(PSO1, PSO2)	0.80727	Not Statistically Significant
(PSO2, PSO3)	0.0003182	Is Statistically Significant
(PSO1, PSO3)	0.00031821	Is Statistically Significant

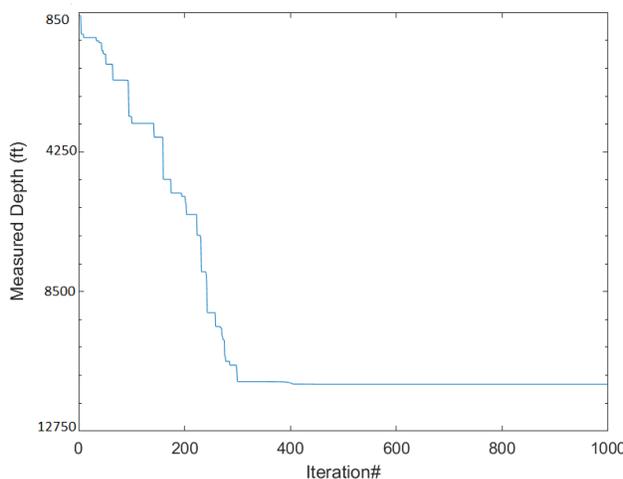


Figure 11. Measured depth (MD) at 1000 loop count (iterations)

The following are the input parameters used to compute the objective functions of Well 3: ROP for the vertical section = 25 ft/hr., rate of penetration (ROP) for the build sections = 15 ft/hr., cost of drilling the vertical section = \$100/hr. and for the build section = \$200/hr. and the offset of the target in vertical direction  $J = \pm 8.20$ ft. The previous parameters are selected based on the previous analysis done in previous section and cost of drilling section does not include casing, cementing, and logging operation cost. Theoretically, the best optimum solution can be any of the solutions (PSO1, PSO2, or PSO3).

Looking at the results from Table 7, the solution with the minimum trajectory length, however, it does not have the highest target hitting accuracy. The solution with the highest target hitting accuracy does not have the minimum trajectory length, however it is the solution with the minimum drilling cost. The solution with the maximum trajectory length is the one with the highest cost of drilling.

In order to select the best algorithm, we use the WILCOXON rank sum test technique. Let us build the technique and judge the significant performance of each algorithm. The equation for this technique is  $P = \text{RANKSUM}(X, Y)$ . It is a test for equal medians that performs a double-sided RANKSUM test of the hypothesis that two autonomous samples in vectors X and Y originate from distribution with the same median and return the p-value from the test. The probability of watching the specified outcome for the given results is P, if the null hypothesis (medians are equal) is valid ("true"). Small P values cast doubt on the null hypothesis validity. The two-information set are presumed to come from ongoing distributions that are identical but otherwise arbitrary except for a location change. The lengths of X and Y may vary. RANKSUM treats and removes -1s in X or Y as missing values. The two-sided p-value is calculated by doubling the most significant one-sided value.

Each of PSO1, PSO2 or PSO3 is ranked with one another to statistically test their performance. For the first vector X the algorithm will put all the best solutions found for a given algorithm and put all the best solutions found in X and best solutions of second algorithm in Y vector at a given number of runs and that will give us a P-value. The ranking procedure is as follows:

The program displays and average best solutions initially, so we need only to apply the following changes:

- `PSO1_Ave = mean (BestSolutions1);`
- `PSO2_Ave = mean (BestSolutions2);`
- `disp('wilcoxon ranksum test')`
- `P = ranksum(BestSolutions1, BestSolutions2)`

Bestsolution1, 2 are judgmental vectors were all best solutions of PSO1 and PSO2 will automatically run into and return the p-value. The significance level is  $0.05 = 5\%$  level. Therefore, and if statement should be written for the level of significance.

- `if P < 0.05`
- `disp(['p-value =',num2str(p)])`
- `disp('The results of best algorithm IS statistically significant')`
- `else`
- `disp(['p-value =',num2str(p)])`
- `disp('The results of best algorithm IS NOT statistically significant')`

Based on this approach, all the algorithms are compared, and the results are presented in Table 8.

The results show that PSO1 is not statistically better than PSO2, however PSO1 is statistically better than PSO3. Also, PSO2 is statistically better than PSO3. If PSO1 is not statistically better than PSO2 then the if condition is not satisfied and therefore PSO2 is better than PSO1. Therefore, PSO2 is significantly better than PSO1, and PSO3, and so PSO2 will be the optimum choice for the solution. Well-3 optimal solution is (PSO2) and the optimized parameters are as follows: Vertical depth of first KOP = 7182 ft.; vertical depth of the second KOP = 8488 ft., hold angle in degrees = 20.4; build-up rate of upper section = 2.9 deg. /100ft; and build-up rate for the lower section = 2.5 deg. /100ft.

## 9. Conclusions and suggestions

A wellbore trajectory profile was selected and optimized based on wellbore instability prediction and PSO algorithm in this article. The selected profiles and the recommended scenarios are compared with the actual profile provided from the company. Based on the results and analysis, the following conclusions and recommendations are extracted:

- Statistical data analysis of actual well data are relatively useful in order to review the history of the area, identify anticipated hole problems, to optimize drilling parameters, and to assist in future drilling activities planning.
- Building wellbore stability modelling is a key element to avoid major geological problems of well-3 and wells of the same field
- Using an adaptive value for the inertia weight in PSO algorithm is beneficial. Quantitatively, decreasing the inertia weight causes the particles to slowly converge towards a point and improved the accuracy of the solutions found in the exploration phase. Limiting the particle's velocity and using inertia weight from 0.2 to 0.9 provide optimum solution. Decreasing the number of candidates' solution to [50] improves the performance of the PSO algorithm.
- The method used in designing the well trajectory has been used in many trajectory designs and the results found were satisfactory, in those that did not operate efficiently, it could be a matter of changing and adjusting the values of the main PSO parameters.
- This study design and optimized well trajectories by studying a real field drilling data, however, it is essential to manually design and calculate the optimum trajectory parameters before the numerical simulation, this will let us judge the performance of the simulation. Furthermore, before approaching any drilling problems, it is essential to first determine the reasons behind the occurrence of the problem and then choose the best method to minimize, prevent or to mitigate the problems.

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