Petroleum and Coal

Article **Open Access**

An Analytical Model for Characterising Dynamic Filtration of Drilling Fluid in Vertical Well

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Received September 21, 2019; Accepted January 24, 2020

Abstract

Filtrate loss during drilling is one of the major challenges the oil and gas industry seeks to combat and control. Studies showed that dynamic filtration accounts for a greater proportion of these losses. It is, therefore, necessary to reduce the inevitable formation of damage caused by dynamic filtration.
In this study, an analytical model that characterizes the filtration through a compressible filter cake under dynamic conditions was developed. The rate at which mud cake builds up on the borehole wall, the distribution of solid pressure, the radius of invasion of the drilling fluid filtrate and the rate of filtrate flow through the formation and effect of mud shear stress on dynamic filtration rate were determined. Results showed that the cake thickness and cumulative filtrate volume vary with the square-root of time, and an increase in the non-linearity exponent from 0-2 gave a corresponding increase in the values obtained for the cake thickness and cumulative filtrate volume. The rate of filtration varies directly with the drilling fluid viscosity and the rate of drilling fluid circulation. Dynamic filtration equation should be treated as a non-linear equation and the rate of dynamic filtration can be reduced by reducing the drilling fluid viscosity and its rate of circulation.

Keywords: *Cake resistance; Dynamic filtration; Filtrate invasion; Mathematical model.*

1. Introduction

Formation damage is a generic term used to describe the impairment of permeability of petroleum bearing formations. It is majorly caused by fluids used during drilling, completion, and work over operations. This condition, as rightly put, is an expensive headache to the oil and gas industry, and as a result, a lot of effort has been dedicated to minimizing it. Most formation damages that occur during drilling have been attributed to the loss of fluids by the drilling mud and the subsequent deposition of solids on the wall of the borehole [1-2].

Liquid filtration is defined as a unit operation that is designed to separate suspended particles from a fluid media by passing the solution through a porous membrane or medium [3]. As the fluid or suspension is forced through the voids or pores on the filter medium, the solid particles are retained on the medium's surface or in some cases, on the wall of the pores, while the fluid, which is referred to as the filtrate passes through. Two classes of liquid filtration have been identified [4-5] as cake and deep bed filtrations. Cake filtration occurs when there is a sufficiently high concentration of solids in the liquid slurry. The solids bridged the filter medium pores and formed a cake. The minimum solid concentration required for cake filtration depends on the nature of the solids and the filter medium but is usually about 0.5% by volume. The filtrate has to overcome two resistances: cake resistance and filter medium resistance. Cake filtration is of primary importance to the petroleum engineer as it occurs during the drilling of wells.

In deep bed filtration, the solid particles become entrapped within the complex pore structure of the filter medium. The filtrate has to overcome only the resistance of the filter medium. There are two types of deep bed filtration: clarifying filtration and cross-flow filtration. In clarifying filtration, the slurry flow is perpendicular to the surface of the filter medium, and the solid particles are deposited within its pores. For the latter case, the suspension flows with a very high velocity across the filter medium, causing a thin layer of solids to form on the surface of the medium. But the high velocity liquid prevents the layer from building up as in cake filtration.

1.1. Filtration of drilling fluids in the borehole

In overbalanced drilling conditions, drilling mud pressure is maintained above the formation pressure to prevent the flow of formation fluids into the borehole. As the drilling bit penetrates the permeable formation, the liquid portion of the mud (filtrate) is forced into the formation as a result of this differential pressure and mud cake is deposited on the wall of the borehole. Three different classes of drilling fluids filtration can be distinguished from their occurrence in the borehole [6]: static filtration, dynamic filtration, and beneath bit filtration.

1.1.1. Static filtration

This occurs when the drilling mud is not being circulated, e.g. whenever drilling stops for pipe trips, BHA changes, etc. The thickness of the cake formed during this mode of filtration increases with time while the volume of filtrate decreases.

1.1.2. Dynamic filtration

This occurs when drilling mud is being circulated past the surface of the cake [7]. The forces governing the deposition and erosion of the cake continually increase with filtration time until they reach equilibrium, causing a constant cake thickness. The filter cake formed here is characterized by the presence of a low-shear strength transition zone. The continuous flow of mud past the filter cake erodes this transition zone and limits the filter cake thickness until equilibrium is reached i.e. the rates of deposition and erosion of the cake are equal.

1.1.3. Beneath the bit filtration

During rotary drilling of wells the formation beneath the bit is continually exposed as the bit grinds the rock beneath it. The cutting action of the bit and the flow of mud through its nozzles prevent the formation of the filter cake. Put in another way, the tendency of a cake to form is usually counter acted by the cutting action of the bit. As a result, this mechanism of filtration leads to a very high filtrate loss.

1.2. Dynamic filtration of drilling fluids

The first published study on the filtration of drilling muds under dynamic conditions was by [8]. They constructed the first laboratory tester designed to study the effects of temperature and pressure on the filter loss behaviour of clay-water drilling fluids. Their results showed that the filtration rate attained a substantially constant value at the end of about two hours. In their study, Ferguson and Klotz [9] plotted a curve of distance from filter surface versus solid content for compressible and incompressible cakes and reported that the solids content gradually increases from the slurry to the cake through a transition region which influences dynamic filtration and also accounts for the apparent equilibrium filter cake thickness observed in dynamic filtration. Their study established that dynamic filtration accounts for about 70% to 90% of the fluid loss during drilling. It is therefore very important to thoroughly understand the mechanisms governing the filtration process.

Krueger [10] investigated the relative effects on dynamic fluid loss rate as given drilling fluid is treated with increasing amounts of chemical additive to reduce the API filter loss. His results showed that the additives which produce low static fluid loss test values do not necessarily effect low dynamic fluid loss rates. Dynamic cake once formed is extremely difficult to erode and that dynamic filtration rate decreases when the drilling fluid becomes less viscous and if the rate of circulation is reduced. Peden *et al.* [6] conducted both static and dynamic filtration tests on two inhibited water based muds. The dynamic filtration experiment con-ducted showed that annular velocity (shear rate) has a pronounced effect on dynamic fluid loss and filtrate loss is highly dependent upon cake permeability.

The first analytical study on filtration was done by [11]. He proposed a theoretical-empirical non-linear diffusivity equation to model the mechanism of filtration through the compressible filter cakes formed in the borehole. From his developed model, he assumed that the observed non-linearity of the general non-linear diffusion equation was mild and could be neglected without a great loss of accuracy. This would lead to a linearized boundary value problem that can be solved for both static and dynamic conditions of flow. This linearized boundary value problem is analogous to a heat flow problem for which an explicit solution is known. Ershaghi and Azari [12] and Akinsete *et al.* [13] applied numerical simulation techniques to solve the dynamic filtration model proposed by Outmans. The former used a Crank-Nicholson approach while the latter used the Fully Implicit Finite Difference discretization scheme. They concluded that cake permeability and filtrate viscosity are two important factors controlling the filtration process. Isehunwa and Falade [14] applied an approximation technique in order to solve the general non-linear diffusivity equation that models filtration. Their study also shows that the assumption of a linearized filtration equation introduces errors into the cake thickness and filtrate volume computations. The correct estimation of drilling fluid filtration properties under dynamic conditions is very crucial as it helps the drilling engineers to prevent formation damage and other unwanted problems that may occur during drilling.

2. Methodology

In the development of this model that accurately describes the mechanism of filtration (under dynamic conditions) through a filter cake in a vertical well, the following assumptions were made:

- Compressible filter cake
- Isothermal conditions exist in the borehole.
- Laminar flow conditions are prevalent in the borehole
- The filtrate flows linearly through the filter cake

These assumptions were made so as to conveniently combine the laws (e.g., Law of conservation of mass and Darcy's law) and equations governing the fluid flow of porous media to obtain:

$$\frac{\partial}{\partial x}\left(-\frac{kA}{\mu}\frac{dP}{dx}\right)dx = -\frac{\partial}{\partial t}(dV) \qquad 1$$

Also, the theory of consolidation proposed by Terzaghi *et al.* [15] was employed in the derivation of the filtration equation. This theory was based on the following assumptions:

1. Fluid flow through a compressible porous medium is governed by Darcy's law
2. The total pressure on a surface normal to the line of flow is equal to the sum of the fluid pressure and the solid pressure at the surface. This total pressure is assumed to be constant.
3. The solid pressures are incompressible within the range of pressures considered.

Applying this theory to equation (1), gives:

$$\frac{1}{\mu}\frac{\partial}{\partial x}\left(k\frac{dP_s}{dx}\right) = \alpha\frac{\partial P_s}{\partial t} \qquad 2$$

Equation (2) models the consolidation of compressible soils. Expressing permeability and compressibility in terms of solid pressure using the expressions derived by Grace [4] gives in terms of stress function the general theoretical-empirical nonlinear diffusivity equation that models filtration across a compressible mud cake as:

$$\frac{\partial}{\partial x}\left((\vartheta)^{\infty}\frac{\partial\vartheta}{\partial x}\right) = \frac{\mu}{D}\frac{\partial\vartheta}{\partial t} \qquad 3$$

2.1. Mathematical development

2.1.1. Rate of mud cake buidup

The rate at which mud cake builds up in the borehole is proportional to the rate at which filtrate flows through it, expressed mathematically as:

$$\frac{dh}{dt} = bq \qquad 4a$$

In terms of stress function gives:

$$\frac{dh}{dt}=\frac{bD}{\mu}(\vartheta)^{\infty}\frac{\partial\vartheta}{\partial x} \quad 4b$$

2.1.2. The cumulative volume of filtrate during dynamic filtration

The cumulative filtrate volume at a given time with rate q at the filtrate through put as expressed by [2] is given as:

$$Q=\int_0^t q\,dt \quad 5a$$

Hence, the general equation in terms of stress function that describes the cumulative filtrate volume obtained during dynamic filtration of muds is given as:

$$Q=\frac{D}{\mu}\int_0^t\left(\vartheta^{\infty}\frac{\partial\vartheta}{\partial x}\right)dt \quad 5b$$

2.2. Solutions to developed mathematical models

Equation (3) is a non-linear second order partial differential equation. This equation is analogous to a non-linear heat equation, which was solved analytically by the method of separation of variables proposed by Parikh [16]. From inference made from the heat equation and from experimental observations, we assumed a solution of the form:

$$\vartheta(x,t)=\vartheta_s(x)+\vartheta_\tau(x,t) \quad 6$$

Where $\vartheta_s(x)$represents the steady state solution and $\vartheta(x,t)$represents the stress distribution after a long period of filtration time while $\vartheta_\tau(x,t)$ represents captures the transient (unsteady) effects that die down with time. Dynamic filtration occurs in three different stages as proposed by [11] Outmans (1963): In the first stage, cake deposition and erosion occur simultaneously. The transition zone is continuously removed by the hydrodynamic shear of the flowing mud, thereby limiting cake thickness and causing filtrate loss to be high. During the second stage, the filtration process reaches equilibrium as the rates of deposition and erosion of the filter cake become equal while the cake thickness becomes constant. In the third stage, filtration takes place at a steady state as the process would have gone on for a long time. The cake thickness and filtration rate become constant.

In order to define the stress distribution for the three stages, equation (3) was solved for the boundary and initial condition of the respective stages.

2.2.1. First stage of dynamic filtration

The boundary conditions are given as:

$$\vartheta(h,t)=0$$
$$\vartheta(0,t)=\vartheta_0 \quad 7$$

Steady state solution $\vartheta_s(x)$is given as:

$$\vartheta_s(x)=A+Bx \quad 8a$$

Applying the boundary conditions equation (7) to equation (8a) to eliminate canstants A and B, we have:

$$\vartheta_s(0)=A+B(0)=\vartheta_0 \Rightarrow \vartheta_s(0)=A=\vartheta_0 \quad 8b$$

Similarly,

$$\vartheta_s(h)=A+B(h)=0 \Rightarrow B=-\frac{\vartheta_0}{h} \quad 8c$$

Substituting equations (8b) and (8c) into (8a), gives:

$$\vartheta_s(x)=\vartheta_o\left(1-\frac{x}{h}\right) \quad 8d$$

Transient component $\vartheta_\tau(x,t)$ is given as:

$$\vartheta_\tau(x,t)=\left(C_1\cos\lambda(x-h)+C_2\sin\lambda(x-h)\right)e^{-(\lambda x+y)^2} \quad 9a$$

Substituting equations (8d) and (9a) into equation (6) and applying the boundary conditions to equation (9a) and eliminating constants C_1 and C_2 gives:

$$\vartheta(x,t)=\vartheta_o\left(1-\frac{x}{h}\right)+\sum_{n=1}^{\infty}C_n\sin\frac{n\pi}{h}(h-x)\,e^{-\left(\frac{n\pi x}{h}+y\right)^2} \quad 9b$$

where C_n is given by

$C_n = \frac{2}{\pi n^2}\vartheta_o e^{\left(\frac{2\pi nxy}{h}+y^2-\frac{2\pi x}{h}\right)}$ 9c

Substituting equation (9c) into (9b), gives:

$\vartheta(x,t) = \vartheta_o\left\{\left(1-\frac{x}{h}\right)+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin n\pi\frac{(h-x)}{h}e^{-\left(\frac{n^2\pi^2x^2}{h^2}+\frac{2\pi x}{h}\right)}\right\}$ 9d

2.2.2. Second stage of dynamic filtration

At this stage stress distribution $\vartheta(x,t')$ was determined, where,

$t' = t - T$ 10

T is the time deposition ends i.e. where the first stage ends, filtration takes place through a constant cake thickness *H* during this second stage.

At initial condition, $t' = 0$, hence $t = T$ i.e. $\vartheta(x,0) = \vartheta(x,T)$.

The boundary conditions are:

$\vartheta(0,t') = \vartheta_o and \vartheta(H,t') = 0$ 11

For steady state, equation (8d) applies, while the transient effect form is as follows:

$\vartheta_\tau(x,t') = \vartheta_o\left(1-\frac{x}{H}\right)+(C_1\cos\lambda x + C_2\sin\lambda x)e^{-\lambda^2 t'}$ 12

Applying boundary conditions equation (11) to equation (12), we have:

$\vartheta(x,t') = \vartheta_o\left(1-\frac{x}{H}\right)+\sum_{n=1}^{\infty}C_n\sin\frac{n\pi x}{H}e^{-\left(\frac{n\pi}{H}\right)^2 t'}$ 13

To evaluate C_n in the transient component of equation (13), which is a Fourier series, the initial boundary condition was applied, and the resulting expression was solved by Euler method to give:

$C_n = \frac{2}{H}\int_0^H \sin\frac{n\pi x}{H}\vartheta(x,T)dx$ 14

Substituting equation (14) into equation (13), gives:

$\vartheta(x,t') = \vartheta_o\left(1-\frac{x}{H}\right)+\sum_{n=1}^{\infty}\left(\frac{2}{H}\int_0^H \sin\frac{n\pi x}{H}\vartheta(x,T)dx\right)\sin\frac{n\pi x}{H}e^{-\left(\frac{n\pi}{H}\right)^2 t'}$ 15

For a longer time, eqaution (15) approaches the steady state:

$\vartheta(x,t') = \vartheta_o\left(1-\frac{x}{H}\right)$ 16

2.2.3. Third stage of dynamic filtration

The boundary conditions are: $\vartheta(0,t'') = \vartheta_o and \vartheta(H,t'') = 0$

Since filtration proceeds at steady state, then the stress distribution is given as:

$\vartheta(x,t'') = \vartheta_o\left(1-\frac{x}{H}\right)$ 17

2.3. Model application in determining relevant parameters

2.3.1. Rate of mud cake buildup

Integrating equation (3) gives

$\int_0^x \frac{\partial}{\partial x}\left(\vartheta(x,t)\right)^{\infty}\frac{\partial\vartheta(x,t)}{\partial x}dx = \int_0^x \frac{\mu}{D}\frac{\partial\vartheta(x,t)}{\partial t}dx$ 18a

$\left(\vartheta(x,t)\right)^{\infty}\frac{\partial\vartheta(x,t)}{\partial x} - \left(\vartheta(0,t)\right)^{\infty}\frac{\partial\vartheta(0,t)}{\partial x} = \frac{\mu}{D}\frac{\partial}{\partial t}\int_0^x \vartheta(x,t)dx$ 18b

Rearranging equation (4b) and substiuting into equation (18b) gives:

$\frac{\mu}{bD}\frac{dh}{dt} - \vartheta_o^{\infty}\frac{\partial\vartheta(0,t)}{\partial x} = \frac{\mu}{D}\frac{\partial}{\partial t}\int_0^x \vartheta(x,t)dx$ 18c

Rearranging equation (9d) and integrating using integration by parts and substituting into equation (18c) gives:

$$h^2 = \frac{4\pi^2 b\times 0.3863\vartheta_o^{\infty+1}D}{(\vartheta_o b(\pi^2+4)-2\pi^2)\mu}t$$

$h = \left(\frac{4\pi^2 b\times 0.3863\vartheta_o^{\infty+1}D}{(\vartheta_o b(\pi^2+4)-2\pi^2)\mu}\right)^{\frac{1}{2}}(t)^{\frac{1}{2}} \Rightarrow h = \sqrt{\varpi t}$ 19

Equation (19) defines the rate of mud cake buildup during the first stage of dynamic filtration.

During dynamic filtration, drilling mud, propelled by its shear strength, is continuously being circulated past the filter cake surface. According to Larsen [17], filtration is independent of the circulation past the filter cake surface as long as this circulation is unable to hydraulically erode the cake. It is generally assumed that the build-up of mud cake stops when the hydrodynamic shear stress exerted by the flowing mud is greater than or equal to the shear strength of the deposited cake. Ferguson and Klotz [9] explained that this condition occurs as a result of the presence of a low shear strength transition zone in the dynamic cake. As mud cake is being deposited, this transition layer is swept away continuously until eventually, the rates of deposition and erosion are equal and a constant cake thickness is obtained. Now, if we consider the filtration behavior at a distance below the cake surface-a transition layer with low strength, deposition stops at a time T given rise to a constant cake thickness H.

By analogy with (19) we can define this constant cake thickness as

$$H = \sqrt{\varpi T} \qquad 20$$

where *H* is the constant cake thickness, and *T* is the time deposition stoped. The cake thickness remains constant from the end of the first stage through the second and third stages.

2.3.2. Solid pressure distribution

For the First Stage of filtration, Terzaghi and Peck [18] in their study stated that cake compaction is s function of solid pressure, P_s while Akinsete *et al.* [13] gave an expression that relates solid pressure variation in mud cake to the stress function distribution. This expression was employed for use in this study and given as:

$$P_s = \left(\frac{\vartheta(x,t)\times B(\beta+\lambda)}{\lambda E}\right)^{-\frac{1}{\beta+\lambda}} \qquad 21$$

Substituting equation (9d) and applying initial boundary condition into equation (21) gives:

$$P_s = P_t\left(\left(1-\frac{x}{h}\right)+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin n\pi\,\frac{(h-x)}{h}\,e^{-\left(\frac{n^2\pi^2x^2}{h^2}+\frac{2\pi x}{h}\right)}\right)^{-\frac{1}{\beta+\lambda}} \qquad 22$$

The expression for the second and third stages of Filtration is given as:

$$P_s = \left(\frac{\vartheta(x,t')\times B(\beta+\lambda)}{\lambda E}\right)^{-\frac{1}{\beta+\lambda}} \qquad 23a$$

$$P_s = \left(\frac{\vartheta(x,t'')\times B(\beta+\lambda)}{\lambda E}\right)^{-\frac{1}{\beta+\lambda}} \qquad 23b$$

Substituting equation (16) into equations (23a) and (23b) and evaluating gives:

$$P_s = P_t\left(1-\frac{x}{H}\right)^{-\frac{1}{\beta+\lambda}} \qquad 24$$

Equation (24) defines the solid pressure distribution during the steady state filtration process.

2.3.3. Cumulative filtrate volume

Re-writing equation (5b) in the form:

$$Q = \frac{D}{\mu}\int_0^t\left(\vartheta_o^{\infty}\frac{\partial\vartheta(0,t)}{\partial x}\right)dt \qquad 25a$$

Substituting the evaluated form of equation (9d) into equation (25a) resulted in:

$$Q = \frac{D}{\mu}\int_0^t\left(\frac{0.3863\vartheta_o^{\infty+1}}{h}\right)dt \qquad 25b$$

Substituting equation (19) and integrating gives:

$$Q = -\frac{0.772\vartheta_o^{\infty+1}D}{\mu}\sqrt{\frac{t}{\varpi}} \qquad 25c$$

Equation (25c) defines the cumulative filtrate volume at the first stage of dynamic filtration.

Differentiating equation (17) to give:

$$\frac{\partial\vartheta(x,t''')}{\partial x} = -\frac{\vartheta_o}{H} \quad 26a$$

Substituting equation (26a) into equation (25a) and evaluating the integral gives cumulative filtrate volume at the end of the second stage of dynamic filtration as:

$$Q = -\frac{\vartheta_o^{\infty+1} D}{\mu\sqrt{\varpi T}} t \quad 26b$$

2.3.4. Extent of invasion

In order to estimate the extent (or depth) of invasion of the mud filtrate into the formation, we apply a simple material balance approach given by Akinsete *et al.* [13]. The following assumptions were made; the formation is homogeneous, isotropic, and highly permeable; the filtrate penetrates radially and uniformly into the formation and displaces the formation fluid ahead of it and that the saturation of the filtrate in the invaded zone is constant. Under these conditions, the material balance yields:

$$Q = \frac{1}{4}\pi\varepsilon h\eta\left(R_i^2 - R_w^2\right) \quad 27a$$

Hence, the radius of invasion is expressed as:

$$R_i^2 = \sqrt{R_w^2 + \frac{1.273Q}{\varepsilon h\eta}} \quad 27b$$

2.3.5. Mud shear stress on dynamic filtration rate

Considering a transition layer at a distance δbelow the surface of the filter cake, the solid pressure at this distance is given by equation (22), where $\delta$is the thickness of any particle deposited within this layer. After being deposited, a particle is able to withstand shearing stress given by $fP_s(h-\delta)$. Particles would continue to be deposited as long as the shearing stress within this transition layer is greater than the shear stress exerted by the flowing mud on the filter cake; deposition stops when these stresses are equal i.e. $fP_s(h-\delta)=\tau$.
Hence, equation (22) becomes:

$$\frac{\tau}{f} = P_t\left(\left(1-\frac{\delta}{H}\right) + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin n\pi\frac{(H-\delta)}{H}e^{-\left(\frac{n^2\pi^2\delta^2}{H^2}+\frac{2\pi\delta}{H}\right)}\right)^{-\frac{1}{\beta+\lambda}} \quad 28$$

For $\delta\rangle\rangle\rangle\rangle H \Rightarrow \frac{\tau}{f} = P_t$ 29

From equation (5a), q is given as:

$$q = -\frac{D}{\mu H}\left(\frac{\lambda E}{B(\beta+\lambda)}\left(\frac{\tau}{f}\right)^{-\beta-\lambda}\right)^{\infty+1} \quad 30$$

3. Results and discussion

The model developed was applied to a vertical Well X (Table 1) undergoing dynamic filtration.

Table 1. Data for case study vertical well X

Parameter	Value	Parameter	Value
Compressibility co-efficient, β	0.08	Specific cake resistance, α	1×10^6
Compressibility co-efficient, λ	-0.31	Specific cake volum,e_b	0.23
Initial cake porosity, ε_o	0.3	Total well depth, D	561ft
Mud solid density, ρ_s	12ppg	Viscosity of filtrate	0.8cP

The results obtained for the rate of mud cake build up on the borehole wall; cumulative filtrate volume and radius of invasion of the filtrate are discussed below.

Equations (19) models the rate of mud cake build-up during the first stage of filtration, while (20) models the rate of mud cake build up during the second stage and the third stage. The equilibrium cake thickness H is constant from the end of the first stage through put the second and third stages. This equilibrium cake thickness depends on time T at which the rates of deposition and erosion are equal. Figure (1) showed that cake thickness increases with increasing values of the non-linear exponent, and cake thickness increases with time initially then becomes constant.

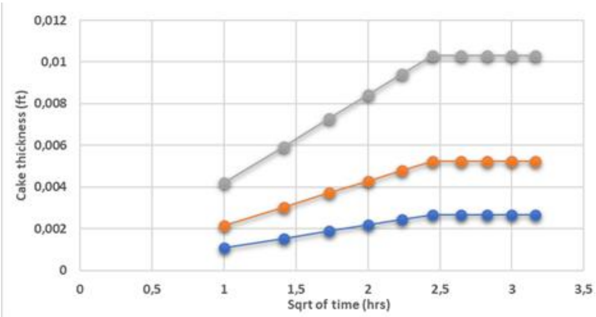

Figure 1. Cake thickness vs. square root of time

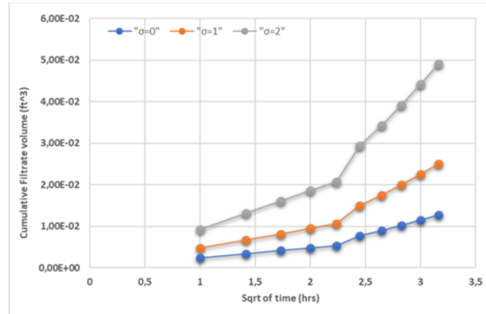

Figure 2. Cumulative filtrate volume vs. square root of time

Equations (25c) and (26b) model the cumulative filtrate volumes for the first stage and the end of the second stage (up to the third stage), respectively. The resulting plot in Figure (2) also showed that the cumulative filtrate volume is sensitive to the non-linear exponent; as we have increasing filtrate volumes for increasing values of ∞.

Equation (30) describes the relationship between the drilling fluid shear stress and the rate of filtration through a filter cake of thickness H. This colloborate Outmans [11] the drilling fluid shear stress is approximately proportional to its viscosity and the rate at which it is being circulated. Consequently, the effect of drilling fluid viscosity and the rate of drilling fluid circulation on the filtration rate can be evaluated from the relationship between drilling fluid shear stress and filtration rate. The plots in Figures (4) and (5) showed a proportional relationship between the rate of filtration on both the drilling fluid's viscosity and its rate of circulation.

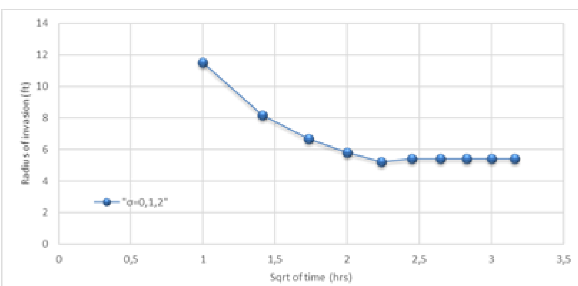

Figure 3. Radius of invasion vs. square root of time

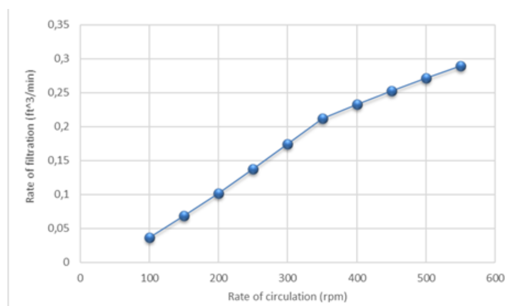

Figure 4. Rate of filtration vs. Rate of circulation

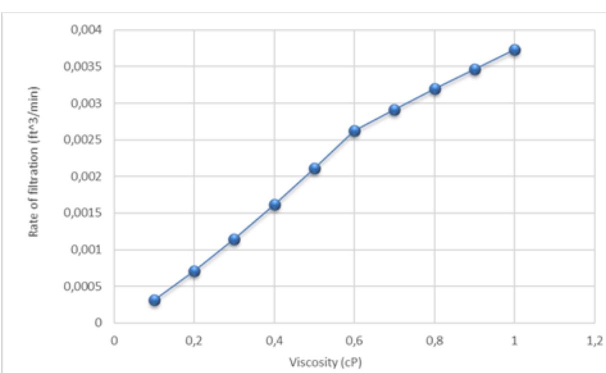

Figure 5. Rate of filtration vs. viscosity

4. Conclusion

A model that characterizes the filtration through a compressible filter cake under dynamic conditions of a vertical well was developed and the resulting non-linear second order partial differential equation was solved analytically by a modified method of separation of variables. The model showed that: cake thickness, filtration rate and cumulative filtrate volume are sensitive to changes in the non-linearity exponent. Consequently, the dynamic filtration equation must be treated as a non-linear equation. The rate of dynamic filtration is dependent on the viscosity of the drilling fluid. In order to achieve reduced rates of filtration, the mud viscosity should be reduced using viscosity retarders. Also, the rate of circulation of the mud should also be controlled to obtain reduced filtrate volumes.

References

[1] Rubel G. Filtrate in Permeable Reservoir. Trans. AIME, 1932; 98: 173-182.
[2] Akinsete OO, and Isehunwa SO. An Analysis of Formation Damage during the Drilling of Deviated Wells. Petroleum Science and Technology, 2013; 31(21): 2202-2210.
[3] Bezemer C, and Havenaar I. Filtration Behaviour of Circulating Drilling Fluids. Society of Petroleum Engineers Journal, 1966; 292-298.
[4] Grace HP. Resistance and Compressibility of Filter Cakes I and II. Chem. Eng. Prog., 1953;49: 303-318, 367-377.
[5] Richardson J, Harker J, and Backhurst J. Chemical Engineering: Particle Technology and Separation Processes. 2002; 5th ed. Massachusetts: Butterworth-Heinemann.
[6] Peden JM, Avalos MR, and Arthur KG. The Analysis of Dynamic Filtration and Permeability Impairment Characteristics of Inhibited Water Based Muds. Paper SPE 10655, presented at the SPE Formation Damage Control Symposium, held in Lafayette, Louisiana, USA. 24-25th March 1982.
[7] Peng SJ, and Peden J. Prediction of filtration under dynamic conditions. Paper SPE 23824 presented at the SPE Formation Damage Control Symposium, held in Lafayette, LA. 26-27th February 1992.
[8] Jones PH, and Babson E. Evaluation of rotary drilling muds. API Drilling and Production Practice, 1935; 22-23.
[9] Ferguson C, and Klotz J. Filtration from mud during drilling. Petroleum Transactions of AIME, 1952; 201: 30-31.
[10] Krueger RF. Evaluation of drilling fluid filter loss additives under dynamic conditions. Journal of Petroleum Technology, 1962; 90-98.
[11] Outmans HD. Mechanics of Static and Dynamic Filtration in the Borehole. Society of Petroleum Engineers Journal. 1963; 228: 236-244.
[12] Ershaghi I, and Azari M. Modelling of Filter Cake Build-Up under Dynamic-Static Conditions. Paper SPE 8902, 50th Annual California Regional Meeting of SPE held in Los Angeles, USA. 9-10th April 1980
[13] Akinsete OO, Raji AI, and Akpabio JU. Formation Damage Analysis due to Filtrate Invasion in Deviated Wells: A Numerical Approach. British Journal of Applied Science & Technology, 2016; 18(6): 1-10.
[14] Isehunwa SO, and Falade GK. An Approximate theory of static filtration of drilling muds in vertical wells. ARPN Journal of Engineering and Applied Sciences, 2012; 7(1): 26-31.
[15] Terzaghi K, Ralph PB, and Gholmreza M. Soil Mechanics in Engineering Practice. 1996; 3rd ed. New York: John Wiley and Sons.
[16] Parikh A. Generalized Separable Method to solve second order non-linear Partial Differential Equations. Indian Journal of Applied Research, 2015; 5:3.
[17] Larsen DH. Relationship between filtrate volume and filtrate time, Englewood, CO: American Institute of Mining and Metallurgical Engineers, 1938.
[18] Terzaghi K, and Peck RB. Soil mechanics in engineering practices. New York, NY: Wiley; 2010.

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